

# Diversity-Based Geometry Optimization in MIMO Passive Coherent Location

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**Abstract.** Applying the recently emerged technique, MIMO (Multiple Input Multiple Output) to PCL (Passive Coherent Location) is expected to improve performance of localization schemes. In this paper, we explore the application of MIMO technology to PCL schemes and see how it improves the spatial diversity of such systems. Specifically, we use the DVB-T stations as the illuminators of opportunity in the simulations, mainly because of their unique features which make them quite suitable for both MIMO and PCL application as will be demonstrated in this paper. In addition, we address the key problem of finding optimum locations for placement of receive antennas.

## Keywords

MIMO, passive coherent location, spatial diversity, illuminator of opportunity, DVB-T.

## 1. Introduction

### 1.1 Passive Coherent Location

Passive coherent location, using the existing commercial signals (e.g., FM broadcast and TV signals) has many advantages over conventional active radars. The energy emitted by the active systems can be used by the target under track to detect the radar's transmitter location. However, there is no such risk in a PCL system, since the transmitters are already in the environment for their intended purposes. This resilience to electronic countermeasures has attracted much attention as a good solution for localization at a lower cost and improved security [1].

The feasibility of different signals for PCL has been investigated earlier, for example, in the case of FM [2–4], Wireless LAN Transmissions [5], analog TV [6, 7], Digital TV (DTV) [8–11], satellite [12], and GSM [13–15] systems. New digital signals, such as Digital Audio/Video Broadcast (DAB/DVB), are excellent candidates for such purpose [16–18], as they are widely available.

In active systems, the target's range is defined by comparing the timing of transmit and receive pulses. However,

this is not available in the case of PCL. Instead, two sets of antennas are used, one for receiving the signal directly from its main source (reference antenna) and another one for collecting reflections arriving from the objects that are to be detected (reflection antenna). Figure 1 depicts the overall structure of such passive scheme.

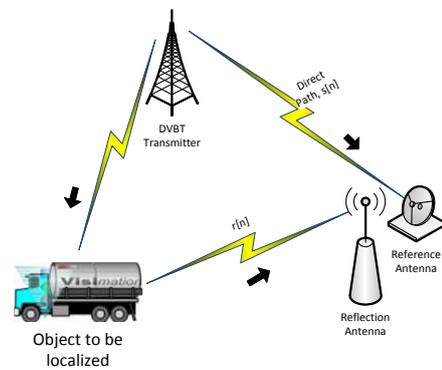


Fig. 1. Structure of a passive radar.

In such scenarios, detection is done through computation of CAF (Cross Ambiguity Function), as given in (1), which is a criterion of how much correlation exists between reference and reflected signals. A given CAF's peak in a range-Doppler cell is a representative of a potential object in that range and doppler frequency:

$$|\chi(\tau, \nu)|^2 = \left| \int_T r(t) s^*(t - \tau) e^{-j2\pi\nu t} dt \right|^2 \quad (1)$$

where  $r(t)$  is the received signal,  $s(t)$  is the reference signal,  $\nu$  is the Doppler shift,  $\tau$  is the delay shift, and  $T$  is the integration interval.

### 1.2 MIMO Technology

There has been significant interest recently towards use of MIMO in radar literature. Generally, MIMO radars can be divided into two main categories: systems based on use of widely separated antennas [19] and systems that use collocated antennas [20]. In the former case, multiple transmitters and receivers that are widely separated are used. The main point is that, by looking at an object from different angles, the probability of missed detection decreases, a concept known as spatial diversity in MIMO communication. In [21, 22], it is shown that such diversity gain is available

not only in the signal processing part but also in the data processing part. On the other hand, in the case of colocated antennas, transmitters and receivers are located at nearby positions. Such configuration is similar to phased array systems, with the difference that signals emitted by each antenna can be totally uncorrelated with the other antennas. Recent studies have shown that the widely separated antenna configuration leads to enhanced detection performance (Diversity Gain) [23–25], better tracking [26], and higher resolution (Spatial multiplexing Gain) [27]. On the other hand, improved parameter identifiability [28], better target identification and classification [29], direct applicability of adaptive arrays for detection and parameter estimation [30], and enhanced flexibility for transmit beam-pattern design [31, 32] are achieved by the colocated antennas configuration.

By exploiting simultaneously the two ideas of passive and MIMO localization, one can achieve the benefits of both schemes by using multiple receivers to detect objects illuminated by multiple noncooperative transmitters. One case of high interest is DVB-T SFN (Single Frequency Network), in which all TV transmitters are broadcasting the same data at the same frequency band. In the DVB-T-based passive scheme, the reflection antenna at the receiver side collects the echoes of the DVB-T signal in the environment. In general, a number of signal processing techniques have been developed at the receiver side in order to detect CW (Continuous Wave), OFDM (Orthogonal Frequency Division Multiplexing), and DVB-T echoes each object produces [9, 10]. In all such approaches, the correlator creates an output with a peak in the range-Doppler domain for each object to be localized.

### 1.3 Diversity Gain

Multipath fading is one of the most fundamental features of wireless channels. Because multiple received replicas of the transmitted signal sometimes combine destructively, there is a significant probability of severe fading. Without proper means to mitigate such fading scenarios, ensuring reasonable reliability requires large power margins [33]. One of the most powerful techniques to mitigate the effects of fading is to use diversity combining of independently fading signal paths. Diversity-combining uses the fact that independent signal paths have a low probability of experiencing deep fades simultaneously. Thus, the idea behind diversity is to send the same data over independent fading paths. These independent paths are combined in such way that the fading effect on the resulting signal is reduced. There are many ways of achieving independent fading paths in a wireless system. One method is to use multiple transmit or receive antennas, in an antenna array configuration, where array elements are separated enough in space [34]. Therefore, rather than making the success of a transmission entirely dependent on a single fading realization, the probability of failure is reduced by exploiting multiple such realizations [33], leading to spatial diversity.

Mathematically, in MIMO communication, diversity gain is defined at high SNR values as [35]:

$$\lim_{\text{SNR} \rightarrow \infty} \frac{\log P_e}{\log \text{SNR}} = -d \quad (2)$$

where  $P_e$  is the error probability and  $d$  denotes diversity gain. The same diversity gain can be obtained in radar by using multiple antennas. However, this time, this gain helps decrease the probability of missed detection ( $P_M$ ) instead of  $P_e$ . In other words, the dual diversity gain of MIMO communication's  $P_e$ , appears in MIMO radar's  $P_M$  [24].

It is well known that if a target is much greater than the wavelength of the illuminating signal, the received signal will be random and fluctuating in time. Signal fluctuations deteriorate detection performance [36], as object's cross section parallels the role of random wireless channel. The reason is that if an object's size is much greater than the wavelength of an illuminating signal, the difference in distances from scatterers to receiver antennas significantly exceeds the wavelength. Consequently, the phase of signals arriving from different scatterers may fluctuate significantly. Even small random rotations of a real object about its center of mass lead to significant changes in distance, and hence, sharp phase variations of signals received from different scatterers [36]. Correspondingly, parallel to MIMO communication, it is possible to obtain diversity by looking at an object from different angles.

### 1.4 Power Gain

In addition to diversity, receive antennas can also provide power gains. Opportunistic communication techniques primarily provide a power gain that can be quite significant at low SNR levels. In general, MIMO techniques can provide both power and degree-of-freedom gains, turning them into a primary tool to increase capacity of a wireless channel at high SNR levels [35].

As will be shown in the following, by using MIMO PCL, specifically a DVB-T based system, it is possible to achieve power gain at no extra cost.

### 1.5 DVB-T Based MIMO Passive Coherent Location

In this paper, we first explore the MIMO technology for the application of PCL, mainly based on DVB-T signals. Importantly, we will show that by adding the number of antennas, we can obtain diversity gain and reduce miss probability. Another gain that can be specifically obtained in MIMO localization by illuminators of opportunity, which is not present in conventional MIMO systems, is the power gain provided by using more transmitters. Although the results will be similar for other transmission schemes used in PCL, we mainly focus on the DVB-T stations and exploit unique properties of such signal for MIMO application.

Although MIMO localization is a well-known technique (e.g. [19, 24, 28, 30–32, 37]), the problem of positioning transmit and receive antennas has not been properly addressed in the literature [38, 39]. In [38], finding an appropriate position for a single receiver is addressed. There, the target can move just on a confined trajectory and the criterion for placing the receiver is improving the target positioning accuracy. Furthermore, the problem of positioning the antennas is considered in [39] assuming that the target has a known position in the region. Besides, their case is an active MIMO radar system. Also, in [40], probability of missed detection is chosen as the criterion for placing the receive antennas. Finally, in [41], it is shown that Cramer-Rao Bounds for target position and velocity estimations depend on the MIMO system geometry (antennas locations). There, it is suggested to use this CRB, in order to find optimal positioning of transmitters and receivers.

In this paper, we introduce a criterion and a new technique for proper placement of receive antennas. It should be noted that in comparison to receive antennas, positioning of transmit antennas is generally not as critical since in the case of PCL, the illuminators of opportunity are already installed in the environment and there is not much control over their locations. In the following, we simulate the case of a  $2 \times 2$  DVB-T based MIMO PCL and introduce a technique for finding the positions of the receive antennas.

The rest of the paper is organized as follows. Section 2 develops the structure detector for the DVB-T based MIMO PCL. In Section 3, we analyze the diversity and power gain achieved in MIMO PCL. In Section 4, we use diversity gain as a criterion for placing the receive antennas. The effect of increasing the number of receive antennas on system performance is analyzed in Section 5. Joint performance of receivers is studied in Section 6, and finally, Section 7 concludes the paper.

## 2. Detection in the DVB-T Based MIMO PCL

It is shown in [24] that in the case of MIMO localization, we can achieve a diversity gain similar to that in MIMO communication. However, in the approach in [24], transmitters send orthogonal waveforms that are easily separated at the receiver side. Naturally, in the case of non-cooperative transmitters, such orthogonality condition is not valid. Primarily, our goal is to investigate whether it is possible to achieve such diversity gain in the case of noncooperative opportunistic illuminators.

Assume that there are  $M$  illuminators of opportunity (e.g. broadcasting DVB-T signals in a Single Frequency Network), a single receiver (including a reference and a reflection antenna) and, an object to be localized. For simplification, we have assumed that the object to be localized has

no Doppler, although such assumption is not critical in our derivations. The reflection antenna is assumed to be omnidirectional, collecting signals arriving from all directions. At the receiver side, after DPI<sup>1</sup> cancellation, the signal is passed through a CAF processor to obtain the delays and Doppler frequencies of different echoes collected from the objects to be localized. The threshold at the output of CAF processor for declaring that an object is detected is determined by the desired false alarm rate ( $P_{fa}$ ). In the case of MIMO PCL, the signal received at a receive antenna is presented by [40]:

$$r(t) = \sqrt{\frac{E}{L}} \sum_{i=1}^M \frac{\alpha_i}{r_{T_i} r_R} s(t - \tau_i) + n(t) \quad (3)$$

where  $s(t)$  is the transmitted signal,  $r_{T_i}$  and  $r_R$  are the distance from the  $i$ 'th transmitter to the target and the distance from the receiver to the target respectively,  $M$  is the number of illuminators,  $\alpha_i$  is the cross-section gain of the object illuminated by the signal transmitted from the  $i$ th transmitter,  $E$  is the energy of the transmitted signal,  $L$  is the channel loss and  $\tau_i$  denotes its delay.

The output of the CAF processor (according to the (1)) is:

$$\chi(T_0) = \chi(T_0, 0) = \sqrt{\frac{E}{L}} \sum_{i=1}^M \left[ \int_T \frac{\alpha_i}{r_{T_i} r_R} s(t - \tau_i) s(t - T_0) dt \right] + \int_T n(t) s(t - T_0) dt. \quad (4)$$

In this case, our goal is to find the proper threshold level,  $\eta_0$ , that achieves the desired  $P_{fa}$ . Suppose we want to obtain the probability of false detection of an object at the delay  $T_0$ . Then, the term  $\sum_{i=1}^M \left( \int_T \frac{\alpha_i}{r_{T_i} r_R} s(t - \tau_i) s(t - T_0) dt \right)$  in (4) will be zero as there is no echo at delay  $T_0$ . As shown in [9], due to highly randomized nature of DVB-T symbols, the autocorrelation function of DVB-T only has one main peak at zero delay and the correlation of two DVB-T signals with different delays will be approximately zero. It should be noted that the DVB-T's CAF also has some ambiguities, the most important one of which is due to the cyclic prefix added in the beginning of each OFDM symbol to counter the multipath effect. In this paper, we use a scheme based on the one proposed in [9], which successfully removes such ambiguities.

The false alarm rate is also computed as:

$$P_{fa} = \Pr(\chi(T_0) > \eta_0) \simeq \Pr\left(\int_T n(t) s(t - T_0) dt > \eta_0\right). \quad (5)$$

With the assumption of  $n(t)$  being AWGN and normalizing  $s(t)$ , which is the input of the correlator obtained by decoding the reference signal, we have

$$n(t) \sim N(0, \sigma_n^2) \Rightarrow \int_T n(t) s(t - T_0) dt \sim N(0, E_s \sigma_n^2) \quad (6)$$

where  $E_s$  is the energy of the signal  $s(t)$ . Therefore,

$$P_{fa} = Q(\eta_0 / \sigma_n \sqrt{E_s}). \quad (7)$$

<sup>1</sup>Direct Path Interference

### 3. Diversity and Power Gain in MIMO PCL

In the previous works on MIMO radar, in order to get the full MIMO diversity gain, it is assumed to have active transmit antennas and the transmitted waveforms are orthogonal [24, 42]. In this paper we study the application of the MIMO technology to a passive radar system, especially multiple DVB-T transmitters which emit the same signal in a single frequency network (SFN).

We define the probability of missed detection ( $P_M$ ) as the probability that we miss all echoes of the desired object. It should be noted that in order to find the location of the object by one receiver, we should have at least three echoes from three transmitters in the 2D plane. However, the reason that we do not consider the case of detecting one or two echoes as missed detection is that we can design the detector such that after detecting one or two echoes, the threshold can be reduced adaptively in order to detect sufficient number of echoes (in this case three). Although by such approach  $P_{fa}$  would increase, the data association algorithm developed in [21, 22], used to associate these echoes to targets, will eliminate such false echoes. Another reason is that we can localize the object by other techniques such as Direction-of-Arrival (DOA) estimation after detecting it at an acceptable  $P_M$  level. More importantly, data fusion schemes can be adopted to localize objects by multiple receivers, in which case it is not required to have three echoes at each receiver.

#### 3.1 Problem Formulation and Solution

In MIMO PCL, SNR is defined as the ratio of the signal power at the reference transmit station to noise power at receiver side (The reason for such definition will be clarified in subsequent sections):

$$\text{SNR} = \frac{E}{\sigma_n^2}. \quad (8)$$

In order to explore the diversity gain of MIMO PCL we have

$$\begin{aligned} \chi(T_0) &= \int_T r(t)s(t-T_0) \\ &= \sqrt{\frac{E}{L}} \sum_{i=1}^M \left[ \int_T \frac{\alpha_i}{r_{T_i} r_R} s(t-\tau_i)s(t-T_0) dt \right] \\ &\quad + \int_T n(t)s(t-T_0) dt, \end{aligned}$$

$$n(t) \sim N(0, \sigma_n^2) \Rightarrow k(T_0) \sim N\left(\sqrt{\frac{E}{L}} \frac{\alpha_k}{r_{T_k} r_R} E_s, E_s \sigma_n^2\right)$$

where  $E_s$  is the signal power of  $s(t)$ ,

$$P_M = \prod_{k=1}^M P_{M_k}$$

where  $P_{M_k}$  is the miss probability of each illuminator,

$$P_{M_k} = E\{Q(Ax - \varphi)\}$$

where  $x$  is RCS of the target with Rayleigh distribution,

$$\begin{aligned} Ax - \varphi &= \frac{\sqrt{\frac{E}{L}} \frac{x}{r_{T_k} r_R} E_s - \eta_0}{\sqrt{E_s \sigma_n}} \\ &= \sqrt{\frac{E_s \text{SNR}}{L}} \frac{x}{r_{T_k} r_R} - \frac{\eta_0}{\sqrt{E_s \sigma_n}} \\ \Rightarrow \begin{cases} A &= \sqrt{\frac{E_s \text{SNR}}{L}} \frac{1}{r_{T_k} r_R} = \frac{B}{r_{T_k} r_R} \sqrt{\text{SNR}} \\ \varphi &= \frac{\eta_0}{\sqrt{E_s \sigma_n}} \end{cases} \end{aligned}$$

where  $\eta_0$  is the threshold evaluated from (7),

$$\begin{aligned} P_{M_k} &= \frac{1}{\sqrt{2\pi}} \int_{x=0}^{\infty} \int_{u=-Ax-\varphi}^{\infty} x e^{-\frac{u^2}{2}} e^{-\frac{x^2}{2}} dx du \\ &= \frac{1}{\sqrt{2\pi}} \int_{u=-\varphi}^{\infty} e^{-\frac{u^2}{2}} \int_{x=0}^{\frac{u+\varphi}{A}} x e^{-\frac{x^2}{2}} dx du \\ &= \frac{1}{\sqrt{2\pi}} \int_{u=-\varphi}^{\infty} e^{-\frac{u^2}{2}} \left(1 - e^{-\frac{[(u+\varphi)]^2}{2A^2}}\right) du \\ &= Q(-\varphi) - \int_{u=-\varphi}^{\infty} e^{-\frac{1}{2}\left[u^2 + \frac{u^2}{A^2} + \frac{\varphi^2}{A^2} + \frac{2u\varphi}{A}\right]} du \\ &= Q(-\varphi) - e^{-\frac{\varphi^2}{2A^2} + \frac{\varphi^2}{2(A+1)^2}} \int_{u=-\varphi}^{\infty} e^{-\frac{1}{2}\left[u\left(1+\frac{1}{A}\right) + \frac{\varphi}{A\left(1+\frac{1}{A}\right)}\right]^2} du \\ &= Q(-\varphi) - Q\left(-\varphi\left[1 + \frac{1}{A} + \frac{\varphi}{1+A}\right]\right) \frac{A}{1+A} e^{-\frac{\varphi^2}{2}\left(\frac{1}{A^2} - \frac{1}{(1+A)^2}\right)}, \end{aligned}$$

$$f_M(\log \text{SNR}) = -\log(P_M) = -\log\left(\prod_{k=1}^M P_{M_k}\right). \quad (9)$$

From (9) it can be inferred that

$$\text{SNR} \rightarrow \infty : A \rightarrow \infty$$

$$\begin{aligned} \Rightarrow P_{M_k} &\simeq Q(-\varphi)\left[1 - \frac{A}{A+1}\right] \simeq \frac{1}{A} Q(-\varphi) \\ \Rightarrow P_M &= \prod_{k=1}^M P_{M_k} \simeq \left(\frac{Q(-\varphi)}{B}\right)^M (\text{SNR})^{-M/2} \prod_{k=1}^M r_{T_k} r_R \\ \Rightarrow \log P_M &\propto -M \log \text{SNR}. \end{aligned} \quad (10)$$

The result of (10) shows that the diversity gain is directly relative to the number of transmit antennas in our MIMO PCL system, which is consistent with dual concepts in MIMO communications [35].

In (10), it is shown that higher diversity is obtained by adding more transmit antennas. However, the problem with the strict definition of diversity is that it only considers the case of SNR values tending to infinity, in other words as  $A \rightarrow \infty$ . However, such definition is not applicable in reality, where the bistatic range parameters  $r_{T_i} r_R$  can also

be very large. Consequently, assuming that  $A \rightarrow \infty$  is not valid even at high SNR regime. As a result, in the following we will derive a so-called “finite-high-SNR” diversity gain that addresses system behavior at high but finite values of SNR. Such definition depends on the configuration of transmitters, receivers and objects to be localized. Therefore, we can choose this new criterion to identify proper placement of receivers in the MIMO localization configuration.

For simplicity assume that in derivations of (10), we instead assume that SNR is large but finite, while  $r_{T_k} r_R \rightarrow \infty$ .

$$\left. \begin{array}{l} \text{SNR : finite} \\ r_{T_k} r_R \rightarrow \infty \end{array} \right\} \Rightarrow A \rightarrow 0^+$$

$$\Rightarrow \left\{ \begin{array}{l} Q(-\phi(1 + \frac{1}{A} + \frac{\phi}{1+A})) \rightarrow Q(\frac{-\phi}{A}) \rightarrow Q(-\infty) \rightarrow 0 \\ \frac{A}{A+1} \rightarrow 0 \\ e^{-\frac{\phi^2}{2}(\frac{1}{A^2} - \frac{1}{(1+A)^2})} \rightarrow e^{-\infty} \rightarrow 0 \end{array} \right.$$

$$\Rightarrow P_{M_k} \rightarrow Q(-\phi). \quad (11)$$

Thus, if  $r_{T_i} r_R \rightarrow \infty$  and SNR is assumed finite, no diversity gain will be achieved by increasing the number of transmitters. But in a real situation, at high but finite SNR regime,  $r_{T_i} r_R$  is not absolutely  $\infty$ . In other words, although it is expected that at high SNR regime diversity gain be directly proportional to the number of transmit antennas, as the number of illuminators in a MIMO detection system is increased, such argument is not true if the added transmitter leads to a large bistatic range ( $r_{T_k} r_R$ ) with respect to the location of the object to be localized and the receiver.

### 3.2 Simulations

In our simulations, DVB-T stations are assumed as noncooperative illuminators. The configuration of the scenario is shown in Fig. 2. In this scenario, the reference signal is obtained from the transmitter located at (-12.45, 6.15) km, and the Swirling I model [43], with  $\sigma_{av} = 2m^2$  is assumed for RCS.

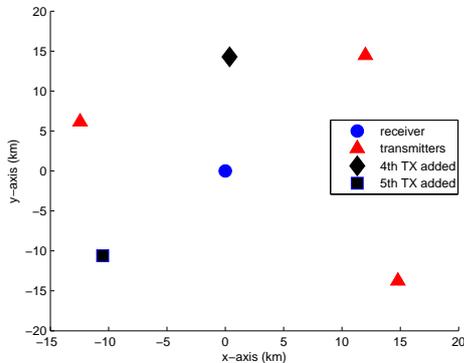


Fig. 2. Scenario configuration.

The parameters of the DVB-T stations and the DVB-T signal used in the correlator are shown in Tab. 1.

Parameter	Value
$P_T$	10 kW
$G_T = G_R$	0 dB
$\lambda_f$	0.6 m
$\sigma_{ave}$	$2 m^2$
$T_{int}$	20 ms

Tab. 1. Simulation Parameters.

Figure 3 shows  $P_M$  as a function of SNR for the case of 1, 3, 4, and 5 transmitters. In the case of one transmitter, only the noncooperative transmitter also used for the reference signal is used for detection. In this figure, the SNR value is defined as the ratio of signal to noise power at the reference antenna of the receiver, which, as expected, is much stronger than the signal received at the reflection antenna (due to being closer to a Line-of-sight signal). The diversity gain obtained by increasing the number of transmitters is clearly observed in this figure.

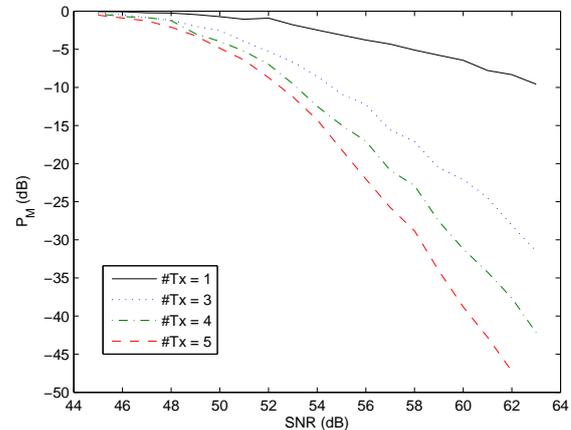


Fig. 3. Diversity of MIMO PCL can be seen in  $P_M$  plots.

Also, note that at low SNR values,  $P_M$  decreases as the number of antennas is increased, a characteristics referred to earlier as power gain. Generally, in a MIMO communication system, increasing number of antennas increases diversity gain, but, simultaneously it causes reduced performance at low SNR values [35]. Also in [24] a similar reasoning is given for MIMO detection where at low SNR values the disadvantage of the phased-array radar system turns into its advantage. In fact, in such scenarios, the instantaneous SNR becomes high compared with the average SNR level. Consequently, as the received SNR level can not deviate considerably from the average received SNR, the probability of detection of the MIMO system becomes lower than that of the phased-array system. In other words, in a MIMO system, we distribute power on different transmitters and view the object of interest from different angles at a fixed transmit power level. Then, by observing  $E\{\sigma_{rcs}\}$ , instead of instantaneous  $\sigma_{rcs}$ , at low SNR values, distributing the power on different transmitters with the goal of achieving higher diversity gains results in reduced performance. For a more detailed analysis of this effect refer to Fig. 4 in [24].

On the other hand, the interesting point is that such situation does not occur in the case of PCL. In PCL, as transmitters are noncooperative, we do not pay extra cost to add a transmitter. As mentioned earlier, the value of SNR at the reference antenna was used as a metric in Fig. 3. In fact, by adding more transmitters, the received SNR level will increase, while the reference power is fixed.

To summarize, at low SNR values  $P_{M_k} < 1$ , therefore

$$\begin{aligned} \prod_{k=1}^M P_{M_k}(\text{SNR}) &= \\ P_{miss}^M \text{TX}(\text{SNR}) &> P_{miss}^{M+1} \text{TX}(\text{SNR}) \\ &= \prod_{k=1}^{M+1} P_{M_k}(\text{SNR}), \end{aligned} \quad (12)$$

a result that is not generally true in a MIMO system, where by increasing the number of transmitters, in order to maintain the same power, the SNR value will change and the above inequality would not be valid.

In the next step, we validate our claim that the amount of the “finite-high-SNR” diversity gain depends on the configuration of the transmitters, receiver and the object to be localized, through simulations. We consider two different scenarios as shown in Fig. 4.

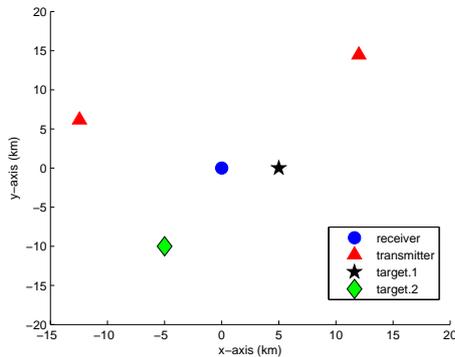


Fig. 4. Two different object positions for localization.

Figure 5 shows the resulting diversity gain as the position of the object illuminated by the DVB-T stations changes.

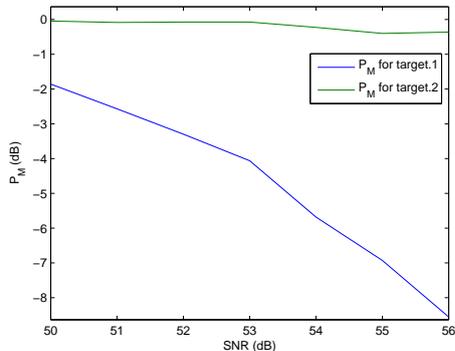


Fig. 5. Diversity gain of the two scenarios of Fig. 4 can be seen in their  $P_M$  plots.

Regarding the second object, the diversity gain obtained by adding the second transmitter will be less than the case for illuminating the first object. Mathematically, we showed that we may not get diversity gain due to large bistatic range. The same conjecture can also be justified intuitively as follows. Assume that the path loss in a wireless channel increases with the square of the distance [34]. In Fig. 4, considering the second object’s position, the second transmitter added to improve the detection, is far away from the object, in comparison with the first transmitter. Therefore, in this configuration we do not get a good diversity gain by adding the second transmitter. However, for the first object, the second illuminator plays a significant role in the detection process and results in a good diversity gain.

In another scenario, in order to show the dependence of the “finite-high-SNR” diversity gain on the target’s position and also obtain an insight on how such gain changes according to (9), we compute the gain for different target’s positions illuminated by two DVB-T transmitters at a receiver. The system’s parameters are the same as Tab. 1. In addition,  $P_{fa}$  of (7) is set to 0.001. Diversity gain obtained at high but finite SNRs (e.g. 60 dB) for different target’s position, according to (9), is shown in Fig. 6. The effect of the distance of the target from the transmitters and the receiver on the defined diversity gain is clear from the figure.

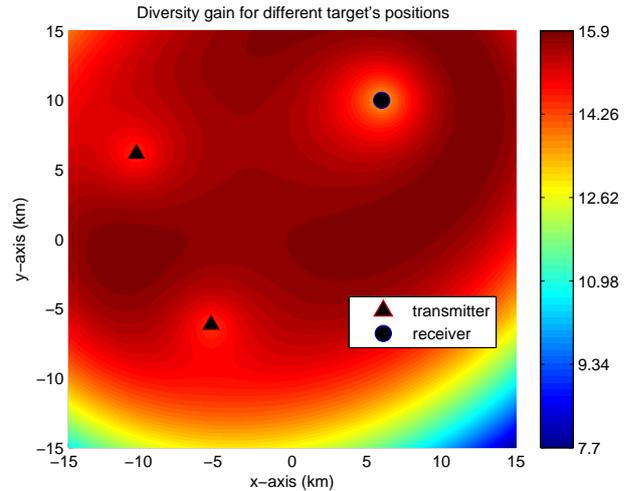


Fig. 6. “Finite-high-SNR” diversity gain for different target’s positions.

## 4. Diversity Gain as a Criterion for Receive Antenna Placement

Next, we use the aforementioned diversity gain as a criterion to find the best position for placing the receivers. We assume a square region with sides equal to 30 km. Earlier, we described how diversity gain changes when the object of interest’s position is changed. In fact, for each receiver placement, the diversity gain changes according to where the object of interest is located. We choose the mean value of the resulting gains (for each receiver’s position) as the crite-

tion. In other words, for a receiver’s position in the region of interest, we compute the average diversity gain of different target’s positions in this region. Then, we select the position with the highest average gain as the position for placing the first receiver. Figure 7 shows the resulting average gain by changing the receiver’s position in the whole region. Now, by choosing this gain as the criterion, it can be seen from the data of Fig. 7 that the best position among these candidates to place the receiver is (-8.4 km, -14.8 km), leading to better detection and subsequently, less missed detections in the region of interest, which is  $30 \times 30$  km.

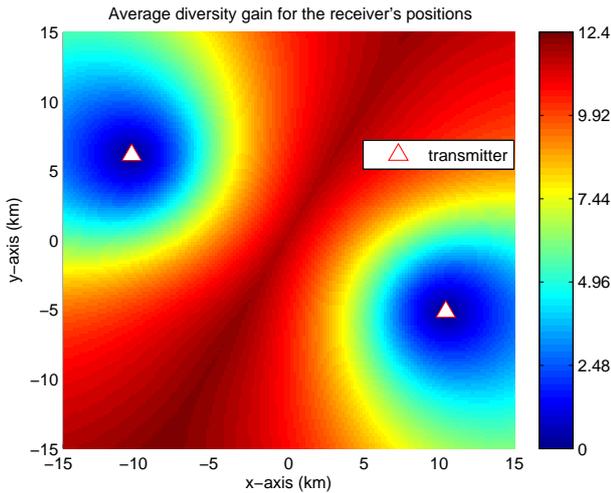


Fig. 7. Average diversity gain obtained by changing the receiver’s position.

Figure 8 shows the diversity gain obtained by placing the receiver at (-8.4 km, -14.8 km) for different target positions.

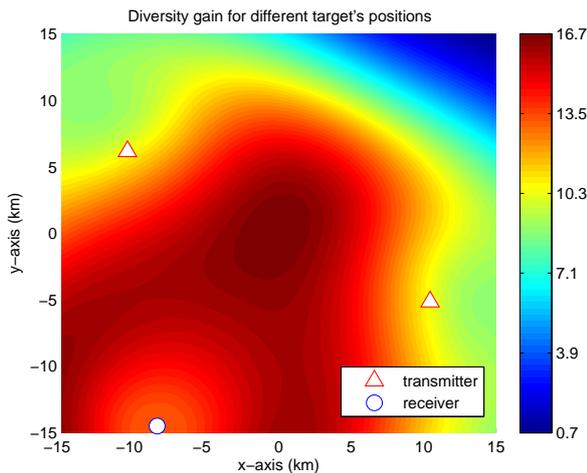


Fig. 8. Diversity gain obtained for different target positions after placing the first receiver.

The effect of path loss is clearly seen in this figure, a factor that has not been considered in earlier works on diversity in MIMO localization. This factor gets more important as widely separated antennas are used for MIMO local-

ization and detection. From Fig. 8, it can be noted that as the path from transmitter to target to receiver becomes longer, SNR values decrease leading to larger values of  $P_M$ .

### 5. Improving Detection by Increasing Number of Receive Antennas

In the next step, we consider the effect of adding the second receive antenna in order to improve the detection performance and do the localization by jointly processing of the two receivers. Our goal is then to find the best position for this second receiver. Using an argument similar to the one given in the previous section, we use the diversity gain as the criterion. However, the procedure of finding a good position for the second receiver differs from the earlier case, as will be discussed in the following.

From Fig. 8, it can be seen that after placing the first receiver, high gains at some locations and poor gains at others will be observed. Our strategy in placing the second receiver is to complement the first receiver in terms of covering the desired area. Therefore, we choose the second receiver’s position such that it results in high gains at target positions where the first receiver provides a low gain. Consequently, we can measure the mean diversity gain at the whole region for placing the second receiver. However, the main difference is that we compute a weighted mean gain, weighted by the reciprocal of the gain obtained from the first receiver. In other words, before averaging over the diversity gains obtained by the second receiver, we weight each one with the reciprocal of the gain obtained from the first receiver. In this manner, we try to give more importance to points where the first receiver gives poor diversity. By computing such weighted mean for the region of interest, we get the gains shown in Fig. 9 for different receiver positions. It can be seen that the best position for the second receiver, by this criterion is (4.8 km, 9.4 km).

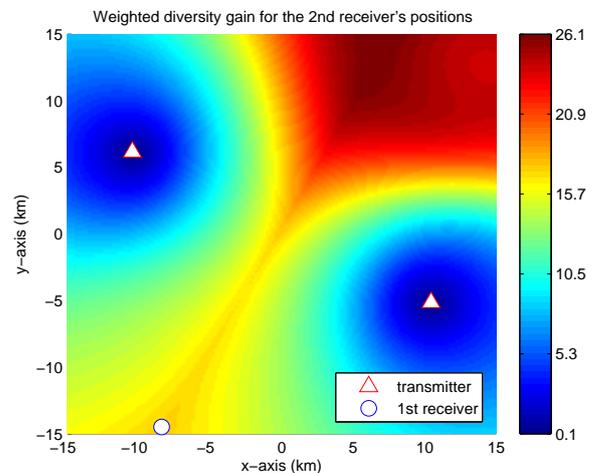


Fig. 9. Weighted diversity gain for second receiver’s position.

## 6. Joint Diversity Gain of Two Receive Antennas

Finally, we explore the joint diversity gain obtained by using both receivers, placed at the positions determined in the previous sections. Again, the probability of missed detection is obtained in a way similar to the earlier approach, for various target positions. The results are shown in Fig. 10. Comparing this figure with the plot of  $P_M$  in the case of only one illuminator (Fig. 8), one can easily observe the diversity gain improvement by adding the second receiver.

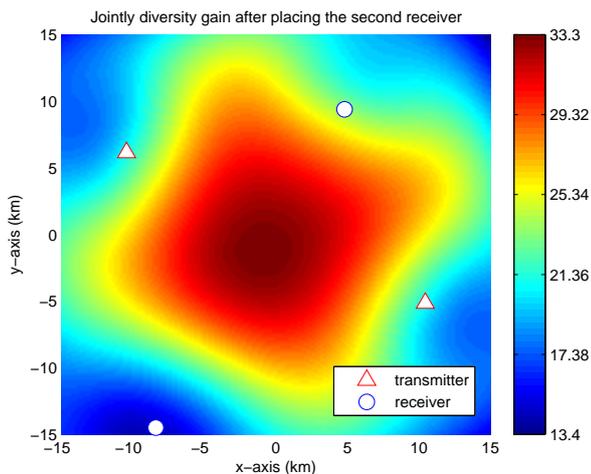


Fig. 10. Joint diversity gain of the two receivers.

## 7. Conclusion

We showed that in MIMO passive coherent location schemes, by using widely separated antennas, we get a diversity gain due to random nature of the target's cross section. In addition, for MIMO PCL, power gain was also obtained without an extra cost. However, the resulting "finite-high-SNR" diversity gain is highly dependent on the relative position of transmitters, receiver and the object to be localized. Since in the PCL case, the illuminators of opportunity are fixed, we introduced a new criterion for finding the best position for the receiver's antenna. The joint diversity gain of using both receive antennas in comparison with just using one receiver was also investigated.

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