Utilization of OIM for Measurement Selection in Multistatic Target Tracking

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Abstract. *The sensor management problem can be expressed as obtaining the state estimation with desired accuracy by utilizing the resources effectively. In the literature, there are two principal approaches to this problem, namely task-driven and information driven sensor management. Performance metrics for both task-driven and information driven sensor management frameworks suffer from the heavy computational burden due to the evaluation of expectations or are available only in simulation. In this paper, the Observed Information Matrix (OIM), which is widely used in statistical practice as a surrogate for the Fisher Information Matrix (FIM) in difficult problems, has been proposed as a metric that can be used in sensor management. Recursive computation of OIM has been derived for the cases with linear and nonlinear system dynamics corrupted with additive Gaussian noise. The usefulness of OIM in sensor selection in multistatic target tracking has been demonstrated via simulations.*

Keywords

Sensor management, observed information matrix, multistatic sensor networks, particle filter.

1. Introduction

Target tracking in multi-sensor environment comprises the problems of measurement to track association, state estimation through filtering and sensor management. Sensor management problem can be expressed as obtaining the state estimation with desired accuracy by utilizing the resources effectively. In the literature, there are two principle approaches to the sensor management problem. In the first approach, which is known as task-driven sensor management, the problem is formulated in terms of minimization of a risk function related to the true state and the estimated state. The second approach is called information-driven sensor management and the sensor management policy is constructed on improving information content of posterior distribution in some sense. Information-driven sensor management approach is based on determining control decisions that maximize some notion of information gain or minimize some notion of uncertainty [1].

Most of the studies on information-driven sensor management have focused on information gain between prior and posterior distribution. The information gain between two distributions has been expressed in terms of Kullback-Liebler (KL) divergence, which is a measure of similarity of two distributions, in [2]. In [3], more general Rényi divergence (α –divergence) was introduced instead of using KL divergence. α –divergence provides extra freedom by choosing the parameter α and this can lead to an opportunity to emphasize certain part of the distribution. Boers et al. have proposed to look at overall uncertainty, i.e., entropy instead of information gain and they showed a clever way to calculate entropy by using particle filter in [4]. Simulation based performance analysis of aforementioned information-driven sensor management measures have been analyzed in a multistatic sensor environment in [5]. Theoretical comparison of these measures was given in [1 and references therein].

In the task-driven sensor management, utilization of the resources is carried out by controlling a function related to the accuracy of the target state estimation. A study in which the function related to accuracy of the estimation has been chosen as Posterior Cramer-Rao Lower Bound was presented in [6]. PCRLB is defined to be the inverse of the Fisher Information Matrix (FIM) and provides a lower bound in terms of second order error on the performance of an unbiased estimator. In [6], the target tracking accuracy was quantified in terms of PCRLB and subsequently controlled. Sensor management framework presented in [6] was then extended to the case where sensor locations are not known precisely and sensors have uncertain movement in [7]. Another PCRLB based sensor management study was presented in [8] where the authors proposed a method for calculating multi target PCRLB and exploited PCRLB as a measure of estimation accuracy for selecting a subset of the available sensors in two scenarios where the total number of targets in surveillance region is known and fixed, and the number of targets is unknown and time varying. A study on determining a subset of available sensors where the number of sensors is large was presented in [9]. The study presented in [9] differs from the aforementioned papers by the challenge of optimally determining a subset of sensors from the large number of sensors on the basis that the selected set will provide reasonable PCRLB.

The authors proposed a convex optimization followed by a greedy local search as a near-optimal solution to solve the sensor selection problem, and achieved desired estimation accuracy measured in terms of PCRLB. This study was extended to the condition of limited communication bandwidth and average transmitting power in [10], and again estimation accuracy was controlled by quantifying PCRLB. An important study was presented in [11] where multistatic sensor placement problem has been surveyed. FIM has been exploited as the quantification of information gathered from the multistatic sensor network and global optimization based sensor placement strategy was introduced that maximizes the information provided to the tracker. PCRLB has also been exploited in distributed sensor networks for the aim of decentralized sensor selection. In [12], a recursive procedure to compute the fused FIM by using FIMs obtained from the distributed estimators was proposed. Decentralized sensor selection strategy based on distributed PCRLB has been presented in [13].

The above mentioned measures exploited for both information-driven and task-driven sensor management suffer from the necessity of taking statistical expectation to calculate them. Especially in the nonlinear state estimation problems, there is no closed form solution to these expectations and approximate solutions can only be given. Furthermore, PCRLB must be calculated around the true target state and which is only available in simulations. However, the papers that proposed PCRLB as a metric also proposed that PCRLB could be approximated via Monte Carlo integration. Consequently, exploiting these measures lead to heavy computational burden and approximate solutions.

In this paper, we propose to employ a statistical metric called Observed Information Matrix (OIM) as a sensor management tool. OIM is well known in the statistics literature and widely used in statistical practice as a surrogate for the FIM in difficult problems for which the FIM is not available [14, see references therein]. It is defined as the negative of the second derivative of the logarithm of the likelihood function evaluated at the Maximum A Posteriori (MAP) estimation [15]. The inverse of OIM is closely related to the PCRLB. However, since calculation of OIM involves no expectation, and it is evaluated at the MAP estimation, the OIM differs from the FIM. These two definitions of information coincide in a particular case where the estimation problem is modeled under linear Gaussian assumptions of the Kalman filter. It is shown in this paper that OIM and FIM are being exactly same and equal to inverse of the Kalman filter covariance under linear Gaussian assumptions. We have shown that the OIM can be exploited as the measure of information in multistatic target tracking problems in which range and range-rate measurements are available and state estimation is carried out by using central data fusion architecture. Since the state estimation using multistatic sensor measurements range and range-rate is a nonlinear problem, we have exploited SIR particle filter [16] as state estimator and shown that more accurate state estimation can be achieved by selecting sensor subset which provides more information in terms of OIM.

The rest of the paper is organized as follows: In Section 2, recursive computation of OIM is given and closed form expression for the posterior OIM in the presence of linear and nonlinear system dynamics corrupted with additive white Gaussian noise is derived. Theoretical background of particle filtering and SIR particle filter in particular is presented in Section 3. Multistatic radar network and simulation environment along with the obtained simulation results are presented in Section 4. Finally some concluding remarks are given in Section 5.

2. Recursive Computation of OIM

The OIM is calculated by using joint distribution of the state where state and measurement dynamics evolve with respect to linear/nonlinear stochastic processes. Assume that a target which moves according to discrete time state space model given in (1)

$$\begin{aligned} x_k &= f_{k-1}\left(x_{k-1}\right)+v_{k-1} \\ z_k &= h_k\left(x_k\right)+w_k \qquad\qquad k=1,2,... \\ x_0 &\sim p\left(x_0\right) \end{aligned} \tag{1}$$

where $x_k, z_k, x_0, f_{k-1}, h_k, v_{k-1}$ and w_k are $n_x \times 1$ state vector, $n_x \times 1$ measurement vector, initial state vector, linear/nonlinear state transition function, and linear/nonlinear measurement function, process and measurement noise processes, respectively. Then the OIM can be calculated by using the definition given in (2).

$$I_k = -\nabla_{X_k}\left[\nabla_{X_k} \log\left(p\left(X_k, Z_k\right)\right)\right]^T\Big|_{X_k=\hat{X}_k^{MAP}} \tag{2}$$

- $X_k = \{x_0, x_1, \ldots, x_k\}$, all the state vectors up to and including time *k,*
- $Z_k = \{z_1, z_2, \ldots, z_k\}$, all the measurements up to and including time *k,*
- $\hat{X}_k^{MAP}$, MAP estimations of the state vectors X_k

In (2), $p\left(X_k, Z_k\right)$ is the joint probability density function of the state and the measurements, and superscript T denotes matrix transpose. The operator ∇_{X_k} is the gradient with respect to X_k. The joint probability density function can be written in the form given in (3) by making some algebraic manipulations.

$$p\left(X_k, Z_k\right) = \left\{ \prod_{k=1}^{K} \begin{matrix} \dfrac{1}{C_k} p\left(z_k \mid x_k, X_{k-1}, Z_{k-1}\right) \\ \times p\left(x_k \mid X_{k-1}, Z_{k-1}\right) \end{matrix} \right\} p\left(x_0\right). \tag{3}$$

Logarithm of (3) gives the function where the gradient will be computed. The recursive formula for calculating OIM can be derived parallel to the method given in [14], [17] where recursively computed OIM is called as posterior OIM. One can obtain this recursive formula by taking the gradient of (4) with respect to X_k and making some algebraic manipulations. As it is seen in (2), the gradient must be taken two consecutive times. Hence, $K \times K$ gradient must be calculated. The posterior OIM at time k then can be computed recursively by using the formula given in (5)

$$\begin{aligned} &\log\left\{p\left(X_K, Z_K\right)\right\} = \\ &\log\left\{\left[\prod_{k=1}^{K} \frac{\frac{1}{C_k} p\left(z_k \mid x_k, X_{k-1}, Z_{k-1}\right) \times}{p\left(x_k \mid X_{k-1}, Z_{k-1}\right)}\right] p\left(x_0\right)\right\} \\ &= \sum_{k=1}^{K} \log\left\{p\left(z_k \mid x_k, X_{k-1}, Z_{k-1}\right)\right\} + \\ &\sum_{k=1}^{K} \log\left\{p\left(x_k \mid X_{k-1}, Z_{k-1}\right)\right\} + \\ &\log\left\{p\left(x_0\right)\right\} + \\ &\sum_{k=1}^{K} \log \frac{1}{C_k} \end{aligned} \tag{4}$$

$$\begin{aligned} J_k &= \beta_k - \Upsilon_{k-1,k}^T \left(\alpha_{k-1,k-1} + J_{k-1}\right)^{-1} \Upsilon_{k-1,k} \\ J_0 &= \eta \end{aligned} \tag{5}$$

where η represents the initial information which is inverse of the covariance of the prior distribution and β_k, $\alpha_{k-1,k-1}$, $\Upsilon_{k-1,k}$ are defined as follows:

$$\begin{aligned} &\beta_k = -\nabla_{x_k}\left[\nabla_{x_k} \log\left\{p\left(z_k \mid x_k, X_{k-1}, Z_{k-1}\right)\right\}\right]^T - \\ &\nabla_{x_k}\left[\nabla_{x_k} \log\left\{p\left(x_k \mid X_{k-1}, Z_{k-1}\right)\right\}\right]^T \\ &\Upsilon_{k-1,k} = \\ &-\nabla_{x_k}\left[\nabla_{x_{k-1}} \log\left\{p\left(z_k \mid x_k, X_{k-1}, Z_{k-1}\right)\right\}\right]^T \\ &-\nabla_{x_k}\left[\nabla_{x_{k-1}} \log\left\{p\left(x_k \mid X_{k-1}, Z_{k-1}\right)\right\}\right]^T \\ &\alpha_{k-1,k-1} = \\ &-\nabla_{x_{k-1}}\left[\nabla_{x_{k-1}} \log\left\{p\left(z_k \mid x_k, X_{k-1}, Z_{k-1}\right)\right\}\right]^T \\ &-\nabla_{x_k}\left[\nabla_{x_k} \log\left\{p\left(x_k \mid X_{k-1}, Z_{k-1}\right)\right\}\right]^T \end{aligned} \tag{6}$$

In the subsequent sections, posterior OIM is derived for two special cases, namely, *i)* System state-space model is linear and noise sequences are additive white Gaussian processes. *ii)* System state-space model is nonlinear and noises are additive white Gaussian processes.

2.1 Posterior OIM in Linear Gaussian Case

Linear Gaussian case is defined by stating some assumptions:

- State is a first order Markov process and evolves in time via stochastic linear equation given by (7).

$$x_k = F_{k-1} x_{k-1} + v_{k-1}, \qquad k = 1, \ldots \tag{7}$$

where v_k is zero mean white Gaussian noise process with known covariance

$$E\left[v_k v_j^T\right] = Q_k \delta_{k-j}, \qquad \delta_{k-j} = \begin{cases} 1, & k = j \\ 0, & k \neq j \end{cases}$$

- Measurements are a linear function of the state and they have been corrupted by additive white Gaussian noise.

$$z_k = H_k x_k + w_k, \quad k = 1, \ldots \tag{8}$$

where w_k is zero mean white Gaussian measurement noise process with known covariance

- The initial state x_0 is a random variable which is Gaussian distributed with known mean and covariance
- The noise sequences v_k, w_k and the initial state x_0 are independent

Under these assumptions, logarithm of the joint distribution can be written as follows:

$$\begin{aligned} &\log\left\{p\left(X_K, Z_K\right)\right\} = \sum_{k=1}^{K} \log\left\{p\left(z_k \mid x_k\right)\right\} + \\ &\sum_{k=1}^{K} \log\left\{p\left(x_k \mid X_{k-1}\right)\right\} + \log\left\{p\left(x_0\right)\right\} \end{aligned} \tag{9}$$

The probability density functions constituting (9) are all Gaussian.

$$\begin{aligned} &p\left(z_k \mid x_k\right) \sim N\left(z_k; H_k x_k, R_k\right) \\ &p\left(x_k \mid x_{k-1}\right) \sim N\left(x_k; F_{k-1} x_{k-1}, Q_{k-1}\right) \\ &p\left(x_0\right) \sim N\left(x_0; \mu_0, P_0\right) \end{aligned} \tag{10}$$

If we substitute these density functions into (6) and take the required derivatives, then we can write the posterior OIM by utilizing (5)

$$\begin{aligned} J_k &= \beta_k - \Upsilon_{k-1,k}^T \left(\alpha_{k-1,k-1} + J_{k-1}\right)^{-1} \Upsilon_{k-1,k} \\ &= H_k^T R_k^{-1} H_k + Q_{k-1}^{-1} - \\ &Q_{k-1}^{-1} F_{k-1} \left(F_{k-1}^T Q_{k-1}^{-1} F_{k-1} + J_{k-1}\right)^{-1} F_{k-1}^T Q_{k-1}^{-1} \end{aligned} \tag{11}$$

As it is seen in (11), posterior OIM at time k is independent of state vector as well as the measurements. It is also equal to the Fisher information Matrix calculated for the linear Gaussian case. If we replace J_{k-1} with the $P_{k-1|k-1}^{-1}$ (the estimation covariance at time $k-1$) and rearrange the equation by using matrix inversion lemma, then we will obtain the Kalman Filter covariance update equation in information filter form.

$$P_{k|k}^{-1} = \left(Q_{k-1} + F_{k-1} P_{k-1|k-1}^{-1} F_{k-1}^{T}\right)^{-1} + H_k^T R_k^{-1} H_k$$
$$P_{k|k}^{-1} = P_{k|k-1}^{-1} + H_k^T R_k^{-1} H_k \tag{12}$$

Consequently, since the posterior OIM is independent of both state and measurements, there is no more need to calculate the MAP estimation of the state. It is sufficient to know the probability densities given in (10) for calculating the posterior OIM.

2.2 Nonlinear State-Space Model with Additive Gaussian Noise Case

Suppose that the state transition function $f_k(.)$ and the measurement function $h_k(.)$ given in (1) are nonlinear and the noise sequences abide by the same set of assumptions defined in section 2.1. Then, one can calculate the posterior OIM by taking the required derivatives of the Gaussian densities and obtain the closed form recursive expression in the following way:

$$\begin{aligned} J_k &= \beta_k - \Upsilon_{k-1,k}^T \left(\alpha_{k-1,k-1} + J_{k-1}\right) \Upsilon_{k-1,k} \\ &= \left[\nabla_{x_k} h_k(x_k)\right]^T R_k^{-1} \nabla_{x_k} h_k(x_k) + Q_{k-1}^{-1} \\ &- \nabla_{x_k}\left[\nabla_{x_k} h_k(x_k)\right]^T R_k^{-1}\left[z_k - h_k(x_k)\right] - \\ & Q_{k-1}^{-1} \nabla_{x_{k-1}} f_{k-1}(x_{k-1}) \times \\ & \begin{Bmatrix} -\nabla_{x_{k-1}}\left[\nabla_{x_{k-1}} f_{k-1}(x_{k-1})\right]^T \times \\ Q_{k-1}^{-1}\left[x_k - f_{k-1}(x_{k-1})\right] + \\ \left[\nabla_{x_{k-1}} f_{k-1}(x_{k-1})\right]^T \times \\ Q_{k-1}^{-1}\left[\nabla_{x_{k-1}} f_{k-1}(x_{k-1}) + J_{k-1}\right] \end{Bmatrix}^{-1} \times \\ & \left[\nabla_{x_{k-1}} f_{k-1}(x_{k-1})\right]^T Q_{k-1}^{-1} \end{aligned} \tag{13}$$

where J_k is calculated around the MAP estimates of the x_{k-1} and x_k. If one replaced the gradient and hessian terms given in (13) with the equivalences defined in (14) and applied the matrix inversion lemma, a more familiar form would be achieved.

$$\begin{aligned} \nabla_{x_k} h_k(x_k)\Big|_{x_k=\hat{x}_k^{MAP}} &= \tilde{H}_k, \\ \nabla_{x_k}\left[\nabla_{x_k} h_k(x_k)\right]^T_{x_k=\hat{x}_k^{MAP}} &= \bar{H}_k \\ \nabla_{x_k} f_k(x_k)\Big|_{x_k=\hat{x}_k^{MAP}} &= \tilde{F}_k, \\ \nabla_{x_k}\left[\nabla_{x_k} f_k(x_k)\right]^T_{x_k=\hat{x}_k^{MAP}} &= \bar{F}_k \end{aligned} \tag{14}$$

$$\begin{aligned} J_k &= \tilde{H}_k^T R_k^{-1} \tilde{H}_k + \\ & \left[Q_{k-1}^{-1} + \tilde{F}_{k-1}^T \begin{pmatrix} J_{k-1}^{-1} - \bar{F}_{k-1} Q_{k-1}^{-1} \times \\ \left[x_k - f_{k-1}(x_{k-1})\right] \end{pmatrix}^{-1} \tilde{F}_{k-1} \right]^{-1} - \\ & \bar{H}_k^T R_k^{-1}\left[z_k - h_k(x_k)\right] \end{aligned} \tag{15}$$

2.3 Posterior OIM in Multiple Sensor Case

As shown in preceding subsections, the posterior OIM is the negative of the hessian of the log-likelihood function evaluated at the MAP estimate of target state. One can easily extend the posterior OIM recursion to multiple sensors case under the assumption that each sensor measurements are mutually independent and MAP estimation of the target state is obtained via central data fusion. Assume that there are M sensors and all sensors generate target related measurement at each sampling time with probability of detection $P_D = 1$ and there is no clutter. The joint distribution required to derive the posterior OIM for multiple measurements is defined as follow

$$p(X_K, Z_K) = p\left(X_K, \left[Z_K^1, \ldots, Z_K^M\right]\right) \tag{16}$$

where Z_K^i represents all measurements belonging to ith sensor. The likelihood function given in (16) has more complex structure in comparison to one analyzed at the beginning of this section. However, it is reasonable to assume that the measurements of sensors are independent and (16) reduces to multiplication of likelihood function of each sensor. Additionally, log-likelihood function under this assumption becomes simply sum of the log-likelihood function of each sensor.

$$p(X_K, Z_K) = \prod_{i=1}^{M} p\left(X_K, Z_K^i\right), \tag{17}$$

$$\log p(X_K, Z_K) = \sum_{i=1}^{M} \log p\left(X_K, Z_K^i\right). \tag{18}$$

As we know from (2), the OIM is the negative of the hessian of the log-likelihood function evaluated at the MAP estimation. Multiple sensor OIM can be defined as the negative of the hessian of (18) as follows.

$$I_k = -\nabla_{X_k}\left[\nabla_{X_k} \log\left(\sum_{i=1}^{M} \log p\left(X_K, Z_K^i\right)\right)\right]^T\Bigg|_{X_k=\hat{X}_k^{MAP}} \tag{19}$$

By using linearity of the hessian, equation (19) can be written as

$$\begin{aligned} I_k &= \sum_{i=1}^{M} -\nabla_{X_k}\left[\nabla_{X_k} \log\left(\log p\left(X_K, Z_K^i\right)\right)\right]^T\Bigg|_{X_k=\hat{X}_k^{MAP}} \\ I_k &= \sum_{i=1}^{M} I_k^i \end{aligned} \tag{20}$$

where I_k^i is the OIM of each sensor. The posterior OIM of each sensor can be recursively calculated by using the recursion given in the preceding section. In other words, the posterior OIM of all sensors can be obtained by combining (5) and (20). Let J_k and J_k^i be the posterior OIM of all sensors at time k and the posterior OIM calculated by using measurement of i^{th} sensor at time k respectively. The relation between J_k and J_k^i can be expressed by using (20)

$$J_k = \sum_{i=}^{M} J_k^i \tag{21}$$

where

$$J_k^i = \beta_k^i - \Upsilon_{k-1,k}^T \left(\alpha_{k-1,k-1} + J_{k-1}\right)^{-1} \Upsilon_{k-1,k} . \tag{22}$$

Consequently, calculating the OIM in the presence of multiple measurement sources problem turns into summation of the OIM calculated of each measurement source. However, one should be careful at this point; each OIM must be evaluated at the MAP estimate computed by exploiting all available measurements but not at the MAP estimate of each sensor separately.

3. Particle Filtering

Particle filters are numerical methods that being used to approximate the posterior density function. This class of filters is suboptimal filters that perform sequential Monte Carlo (SMC) estimation based on point mass (or particle) representation of probability densities [16]. Since in most cases, the true posterior density is unknown and drawing particles from the true density is impossible, importance sampling method [18] that is based on drawing samples from the importance density which has to have same support set with the true density is used while implementing particle filters. Assume that X_k is all the target states up to time *k*, $q(X_k | Z_k)$ is the importance density function and the $\{X_k^i, w_k^i, i = 1,..., N\}$ is the sample points and their associated weights that characterize the joint posterior density function $p(X_k | Z_k)$. Then one can write the discrete approximation of the $p(X_k | Z_k)$ as follows,

$$p(X_k | Z_k) \approx \sum_{i=1}^{N} w_k^i \delta\left(X_k - X_k^i\right) \tag{23}$$

where the $\delta(\bullet)$ is delta Dirac function and w_k^i is normalized weights computed according to importance density function.

$$w_k^i \propto \frac{p\left(X_k^i | Z_k\right)}{q\left(X_k^i | Z_k\right)} . \tag{24}$$

Discrete approximation of the posterior density given in (23) is required batch computation of the density function. A method for recursive computation of the posterior distribution $p(x_k | Z_k)$ by using principle of importance sampling is given in [16] and it is called sequential importance sampling (SIS). By using SIS framework, discrete approximation of the marginal density function can be computed as follow,

$$p(x_k | Z_k) \approx \sum_{i=1}^{N} w_k^i \delta\left(x_k - x_k^i\right) , \tag{25}$$

$$w_k^i \propto w_{k-1}^i \frac{p\left(z_k | x_k^i\right) p\left(x_k^i | x_{k-1}^i\right)}{q\left(x_k^i | x_{k-1}^i, z_k\right)} \tag{26}$$

where the importance weights only depend on samples drawn at time *k* and measurement z_k. In this method, the weights computed by using relation (26) must be normalized such that $\sum_{i=1}^{N} w_k^i = 1$.

In this paper, Sampling Importance Resampling (SIR) particle filter has been used as state estimator. This algorithm can be easily obtained by modifying Sequential Importance Sampling (SIS) algorithm. The algorithm is derived from the SIS algorithm by choosing importance density to be the transitional density $p(x_k | x_{k-1})$ and performing the resampling step at every time update. In the SIR particle filter, there are two simple assumptions: *i)* State dynamics and measurements functions are known. *ii)* It is possible to sample realizations from the prior and transitional distributions. If these two assumptions are satisfied, SIR particle filter is initiated by drawing samples from the prior distribution and time updates of the filter is recursively implemented as follow:

- Draw samples at time *k* $x_k^i \sim p\left(x_k | x_{k-1}^i\right)$
- Calculate weights $w_k^i = p\left(z_k | x_k^i\right)$
- Normalize weights
$$\tilde{w}_k^i = \frac{w_k^i}{\sum_{i=1}^{N} w_k^i}$$
- Calculate state estimation and its covariance
$$\hat{x}_k = \sum_{i=1}^{N} \tilde{w}_k^i x_k^i, \qquad \hat{P}_k = \sum_{i=1}^{N} \tilde{w}_k^i \left(\hat{x}_k - x_k^i\right)\left(\hat{x}_k - x_k^i\right)^T$$
- Resample samples generated at time *k* with respect to the weights.

3.1 Modification of SIR Particle Filter for Multiple Sensor Case

As it can be seen from weight calculation step of the SIR particle filter (second step), measurements only have contribution on calculating the particle weights. Therefore, one can easily extend the SIR particle filter to multiple sensor case under the assumption that each sensor measurements are mutually independent. In this study, it is assumed that target of interest is observed by the multistatic sensor network consists of *M* sensors and measurements related to *M* targets can be acquired with probability of detection 1. It is also assumed that there is no clutter. Let $z_k^j, j = 1,..., M$ be the measurement sequence observed at time *k*. Then, the likelihood function of *M* measurements can be expressed under the independence assumption as follows:

$$p\left(z_k^1, z_k^2, \ldots, z_k^M \mid x_k^i\right) = p\left(z_k^1 \mid x_k^i\right) \times p\left(z_k^2 \mid x_k^i\right) \times \ldots \times p\left(z_k^M \mid x_k^i\right) = \prod_{j=1}^{M} p\left(z_k^j \mid x_k^i\right) \quad (27)$$

As it was mentioned in the preceding section, particle weights of the SIR particle filter are calculated directly proportional to the likelihood function. Therefore, particle weights would be easily evaluated by using independence of the measurements as given in (28).

$$w_k^i \propto \prod_{j=1}^{M} p\left(z_k^j \mid x_k^i\right). \quad (28)$$

4. Multistatic Radar Network and Simulation Results

In this study, results of information analysis in the passive radar network given in [19] have been used for modeling a multistatic radar network. A multistatic radar network consisting of two transmitters and 10 receivers has been modeled by preserving the assumption given in previous sections. The multistatic radar network is assumed to output bistatic range and range rate measurements that are generated by each transmitter-receiver pair and those measurements are corrupted by additive white Gaussian noise with known mean and variance.

$$R = \sqrt{(x-x_r)^2 + (y-y_r)^2 + (z-z_r)^2} + \sqrt{(x-x_t)^2 + (y-y_t)^2 + (z-z_t)^2} + w_R \quad (29)$$

$$\dot{R} = \frac{(x-x_r)\dot{x} + (y-y_r)\dot{y} + (z-z_r)\dot{z}}{\sqrt{(x-x_r)^2 + (y-y_r)^2 + (z-z_r)^2}} + \frac{(x-x_t)\dot{x} + (y-y_t)\dot{y} + (z-z_t)\dot{z}}{\sqrt{(x-x_t)^2 + (y-y_t)^2 + (z-z_t)^2}} + w_{\dot{R}} \quad (30)$$

where R and $\dot{R}$ stand for bistatic range and range-rate measurements respectively. The arguments given in the measurement functions are defined as follow:

$$\begin{aligned} L_t &= \left[x_t \quad y_t \quad z_t\right]^T : \text{Location of the transmitter} \\ L_r &= \left[x_r \quad y_r \quad z_r\right]^T : \text{Location of the receiver} \\ \mathrm{x} &= \left[x \quad \dot{x} \quad y \quad \dot{y} \quad z \quad \dot{z}\right]^T : \text{State of the target} \end{aligned} \quad (31)$$

The additive noises w_R and $w_{\dot{R}}$ are assumed to be white Gaussian sequences with zero means and known variances. It is also assumed that these noise sequences are independent. Time and transmitter-receiver pair numbering has been omitted for convenience in (29) and (30). Locations of the transmitters and the receivers have been given in Tab. 1 and their distributions on the XY plane has been represented Fig. 1. As it is seen in Fig. 1, radar network consists of two hexagonal cells where cell structure comprises of one transmitter and six receivers. The hexagonal cells share an edge, thus the receivers labeled with R1 and R11 denote the same receiver, and R2 and R10 are the labels for the same receiver.

	R1	R2	R3	R4	R5	R6
X	30000m	15000m	-15000m	-30000m	-15000m	15000m
Y	0m	25981m	25981m	0m	-25981m	-25981m
Z	0m	0m	0m	0m	0m	0m
	R7	R8	R9	R12	T1	T2
X	75000m	60000m	30000m	60000m	0m	45000m
Y	25981m	51962m	51962m	0m	0m	25981m
Z	0m	0m	0m	0m	0m	0m

Tab. 1. Locations of transmitters and receivers.

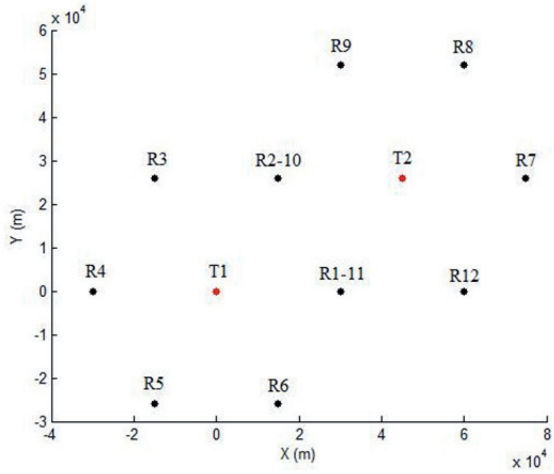

Fig. 1. Locations of the transmitters and receivers on XY plane.

It is assumed that all the receivers are capable of listening to signals coming from all available transmitters. It is also assumed that a target flies over the multistatic radar network with initial position (–40, 20, 10) km and velocity (150, 0, 0) m/s. Target moves according to the nearly constant velocity model [20] about 800 seconds. In the first half of its motion, target flies over the first cell of the network and it moves through the second cell in the last 400 seconds. Trajectory of the target is illustrated in Fig. 2.

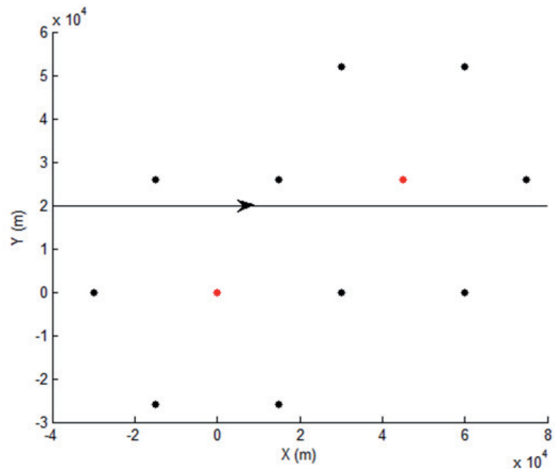

Fig. 2. Target trajectory.

4.1 Measurement Selection w.r.t. Posterior OIM

The aim of this study can be explained as determining how to make a decision on which measurement set should be used for target state update in multistatic radar network. If there are numerous measurement sources (transmitter-receiver pairs) that generate measurements related to the same target, one should choose a subset of these multiple measurements for updating the state instead of using all of those for track update. Utilizing a subset of the all available measurements would save the computational power. Furthermore, the sensors that provide consistently less information can be determined and put into stand-by state according to appropriate sensor management strategy. This would decrease the number of measurements acquired per scan and make measurement to track association problem easier, which is a hard problem to solve in real time depending on increasing number of measurements. Such a sensor management strategy would also reduce the communication cost and lead to save the power resources of sensors.

The idea of exploiting the only a subset of all available measurements stems from the fact that the increment rate of information provided by each additional measurement decreases with the increasing number of measurements in circular type of multistatic networks [19]. Therefore, using all available measurements would yield less improvement on state estimation than it is expected. This gives an opportunity to use available resources effectively by exploiting particular number of measurements. It was proposed in [19] that state estimation could be achieved with an acceptable accuracy level by exploiting six measurements. Thus, we have limited the number of measurements that will be used for target state update at each time with 6 in this study. At this point, the crucial question is "Which subset of all measurements should be exploited?". We have proposed that the posterior OIM can be utilized to determine the measurement subset containing the most information about target state. The proposed method is based on determining the predicted value of the posterior OIM at time k by using the calculated posterior OIM at time k-1, predicted state and available measurements at time k, and finding the measurement subset which maximizes the determinant[1] of the predicted posterior OIM. Let $\hat{x}^{MAP}_{k-1|k-1}$ be MAP estimation of the target state at time k-1 then MAP estimation of the predicted state at time k under linear dynamic state equation assumption would be

$$\hat{x}^{MAP}_{k|k-1} = F_{k-1}\hat{x}^{MAP}_{k-1|k-1} . \qquad (32)$$

Assume that n measurements acquired from all available sources are reported at time k and $m<n$ measurements can be used to update the target state. There would be $l = C(n,m)$ subsets where C stands for the combination and each m-combination of the measurements would constitute subsets $S_i, i = 1,\ldots,l$. Predicted value of the posterior OIM can be computed by using each measurement subset as follow:

$$J_{k|k-1,i} = \sum_{z_k \in S_i} \left\{ \begin{array}{l} -\nabla_{x_k}\left[\left(\nabla_{x_k} h_k(x_k)\right) R_k^{-1}\left(z_k - h_k(x_k)\right)\right]^T \Big|_{x_k=\hat{x}_{k|k-1}} + \\ \left[Q_{k-1}^{-1} + F_{k-1}^T \left(J_{k-1}\right)^{-1} F_{k-1}\right]^{-1} \end{array} \right\}, \quad i = 1,\ldots,l \qquad (33)$$

In (33), J_{k-1} and $J_{k|k-1,i}$ are posterior OIM at time k-1 and predicted value of posterior OIM based on the i^{th} measurement subset at time k respectively. The measurement subset that will be used to update the target state can now be determined by taking the determinant of (33) for $i = 1,\ldots,l$ and choosing the subset that maximizes the determinant.

4.2 Simulation Results

Performance of the posterior OIM based sensor management approach has been analyzed through simulation where the Root Mean Square (RMS) error has been used as the performance metric. SIR particle filter based MAP estimator has been used as the state estimator and the posterior OIM has been evaluated at the value that MAP estimation has pointed. Some assumptions[2] related to the SIR particle filter and the scenario are given below:

- There is only one target
- There is no clutter
- Target is detected at every sampling time with $P_D = 1$
- Target has been initialized and the initial state estimation and covariance are defined as follows:

$$x_0 \sim N\left(x_0^{true}, P_0\right)$$

$$P_B = \begin{bmatrix} 10^4 m^2 & 0 \\ 0 & 15^2 (m/sn)^2 \end{bmatrix} \text{ and } P_0 = \begin{bmatrix} P_B & 0 & 0 \\ 0 & P_B & 0 \\ 0 & 0 & P_B \end{bmatrix}$$

- Target dynamic model is linear and target states evolve according to

$$x_k = F_{k-1}x_{k-1} + v_{k-1}$$

where F_{k-1} is state transition matrix that is a function of sampling interval T and given below. v_{k-1} is the process noise sequence with zero mean and known covariance.

[1] One can also use trace instead of determinant. In physical meaning, determinant is related to volume of the hyper-ellipsoid described by a matrix and it contains statistical relations between axes. On the other hand, trace is related to diagonal elements of the matrix and the correlations between axes are not taken into account.

[2] Note that these assumptions are made to demonstrate the employability of the proposed method and can easily be lifted at the cost of increased computational load and complexity.

$$F_k = \begin{bmatrix} 1 & T & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & T & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- Process noise of the SIR particle filter is modeled as

$$Q = \begin{bmatrix} 5m^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1(m/sn)^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5m^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1(m/sn)^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5m^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1(m/sn)^2 \end{bmatrix}$$

- The number of particle utilized in particle filter is 12000.
- Measurement noise covariance is modeled as
$$\Sigma = \begin{bmatrix} 10000m^2 & 0 \\ 0 & 1(m/sn)^2 \end{bmatrix}$$
- 100 Monte Carlo simulations have been carried out.

Four cases have been investigated in the simulations with the described scenario, namely: *i)* Exploit the six measurements that produce maximum information in terms of predicted posterior OIM. *ii)* Exploit the six measurements that produce minimum information in terms of predicted posterior OIM. *iii)* Exploit the six measurements that produce maximum information in terms of predicted posterior FIM. *iv)* Exploit all measurements and do not use neither OIM nor FIM. The predicted posterior FIM is an information measure that has been used in the aforementioned studies for various sensor management problems. As mentioned before, the FIM must be calculated at the true state or must be approximated via Monte Carlo integration where the former is not possible in real life applications and the latter has significantly high computational burden.

In this study, the predicted posterior FIM has also been exploited as a sensor selection measure for the purpose of comparing its performance with the predicted posterior OIM. FIM based sensor selection has only been applied to the most informative measurement selection case where the simulation results of all four cases have been presented. Variation of the RMS error in the X axis is shown in Fig. 3. The simulation results have revealed that the tracking filter working with the measurement subset producing maximum information with respect to predicted posterior OIM has less RMS error in comparison to the minimum information filter and has comparable error level with the filter utilizing all measurements. It has been also revealed that similar RMS error level has been obtained by utilizing the predicted posterior FIM as a sensor selection measure in comparison to predicted posterior OIM. Especially, after the 100th sampling time, where target begins to move in the area bounded by the first hexagonal cell, RMS error decreases and remains around the 6.5*m* for both first and third cases. However, RMS errors of the tracking filter that exploits less informative measurement subset, fluctuates during the tracking process and it varies between 6.5*m* and 15*m* after the 100th sampling time. The two filters exploiting OIM have achieved similar RMS error performance, where filters share the same measurement subsets, for only a short period of the total tracking process. Similar results have been obtained for the position error in Y axis that is presented in Fig. 4. Utilizing the prediction of posterior OIM utilizing the most informative measurement subset has produced less RMS error in comparison to the second case. Similar error levels have been achieved for the first, third and fourth cases in Y axis. Furthermore, information based measurement selection has provided stable RMS error level both in X and Y positions which is a desirable property for a state estimator. If there are no other error sources (maneuver or disturbance of any kind) in the estimation process, it is desired to obtain a stationary estimation error. Advantage of using predicted posterior OIM in measurement selection has been clearly observed in simulation results obtained for Z axis and variation of RMS error in Z position has been given in Fig. 5.

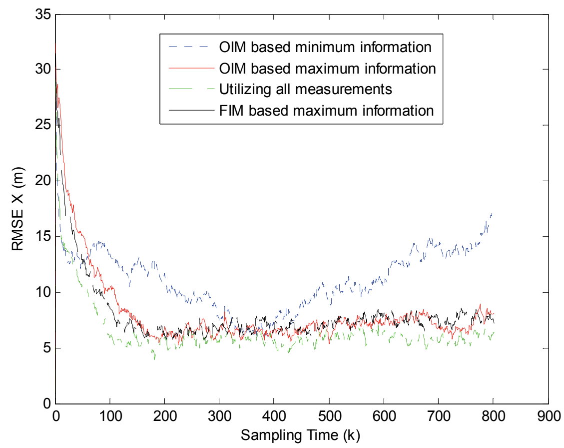

Fig. 3. Variation of RMS error in X.

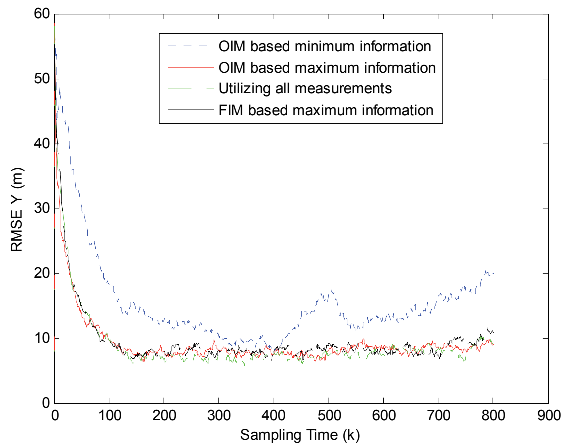

Fig. 4. Variation of RMS error in Y.

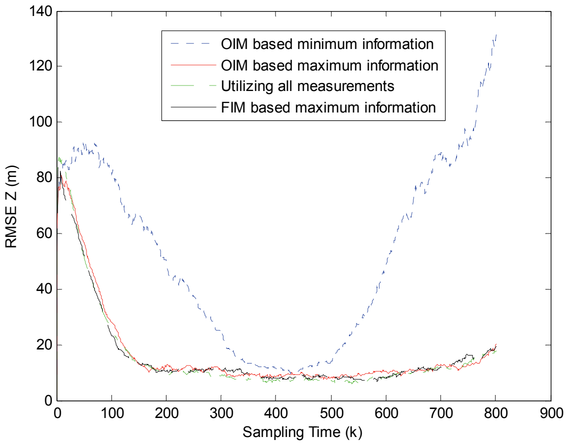

Fig. 5. Variation of RMS error in Z.

In the scenario, target does not change its position in Z and this leads to measurement process to produce less information about Z in comparison to X and Y. However, selecting most informative measurement subset for track update by using OIM based measure has provided substantial improvement on RMS error in comparison with the less information case and comparable results with the all measurements case. Stable RMS error level around 10*m* has been achieved for Z position as well. The FIM based sensor selection strategy has also led to lower RMS error level in comparison to less informative case and, it has similar RMS error level with the OIM based most information and the utilization of all measurements cases.

5. Conclusion

In this paper, we have introduced the well-known statistical metric Observed Information Matrix (OIM) as a sensor management criterion. The underlying reason to propose a metric different from the other metrics appeared in the open literature is that metrics exploited in either task-driven or information-driven sensor management frameworks have heavy computational burden due to the evaluation of the expectations or they can only be obtained via simulation. However, OIM can be calculated in all cases where MAP estimation of the state is available.

In this study, we have shown that the posterior OIM can be computed recursively where target and measurement dynamics are corrupted with additive white Gaussian noises. It has been shown that prediction of posterior OIM can be exploited in measurement selection process in the single target tracking in a multistatic sensor network problem. It has been shown that the measurement subset maximizing information in terms of OIM leads to less RMS position errors during the state estimation and quantifying predicted posterior OIM as a measure on information is reasonable.

Consequently, utilization of predicted posterior OIM as an information measure in sensor networks gives an opportunity to achieve lower estimation errors in all three axes and it is also possible to obtain comparable position errors with the case where well-known measure predicted posterior FIM is exploited.

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