Broadband Butler Matrices with the Use of High-Pass LC Sections as Left-Handed Transmission Lines

Kamil STASZEK, Slawomir GRUSZCZYNSKI, Krzysztof WINCZA

AGH University of Science and Technology, Mickiewicza Str. 30, 30-059 Krakow, Poland

kstaszek@agh.edu.pl, slawomir.gruszczynski@agh.edu.pl, krzysztof.wincza@agh.edu.pl

Abstract. An application of left-handed transmission line sections in Butler matrices has been investigated. It has been shown, for the first time, that the utilization of both left-handed and right-handed transmission lines allows for broadband differential phase shifters’ realization, required in the Butler matrices. A complete theoretical analysis is given, for Butler matrices incorporating ideal transmission lines of both right- and left handed types and expressions for the achievable bandwidth and differential phase deviation are derived. The presented idea has been verified by the design of a 4 x 4 Butler matrix operating in a frequency range of 2.5 – 3.5 GHz. As an artificial left-handed transmission line, an equivalent high-pass LC circuit realized in a quasi-lumped element technique has been considered, and the resulting phase shift of such a circuit is given analytically. The obtained measurement results fully confirm the validity of the proposed idea of broadband Butler matrices’ realization.

Keywords
Butler matrices, coupled-line directional couplers, differential phase shifters, left-handed transmission lines.

1. Introduction
Butler matrices (BMs) are commonly known microwave networks widely described in literature [1] – [7]. They feature unique properties which make them suitable for wide range of applications in contemporary electronics, e.g. as beamforming networks of multibeam antennas [3] or in direction finding systems [4]. Butler matrices can be realized in different techniques, the choice of which affects the resulting bandwidth. The exemplary realizations can be found in [5], [6], in which branch-line couplers are used and, therefore, narrow operational bandwidth is obtained. The bandwidth of BMs may be significantly enlarged with the use of coupled-line directional couplers [7], [8]. Apart from bandwidth enhancement the utilization of coupled-line couplers allows for significant size reduction of BMs. Due to their relatively low complexity, 4 x 4 Butler matrices are most broadly utilized and are composed of four 3dB/90° directional couplers in conjunction with two 45° phase shifters. Such circuits provide ±45° and ±135° differential phases of the signals measured between adjacent output ports [2]. Higher order Butler matrices are rarely presented due to their complexity. An exemplary 8 x 8 BM realized in LTCC technology may be found in [9]. In the presented design 40 layers appropriately stacked and folded have been used. The realization of broadband Butler matrices requires not only application of broadband 3dB/90° couplers but also broadband constant-value phase shifters. Several methods for realization of phase shifters have been reported. The commonly used technique is the application of a transmission line section having appropriate electrical length (45° - in case of 4 x 4 BMs) [10]. However, such a technique results in the limited bandwidth of resulting BMs. Much wider bandwidth in terms of phase response can be obtained by applying a tandem connection of two 3dB/90° directional couplers and transmission-line sections in reference channels [11]. This solution ensures the required 45° phase shift and additionally serves as a transmission line crossover. On the other hand the well-known Schiffman network offers a broadband differential phase shift [12], [13] and at the same time compact size. The utilization of Schiffman ‘C’ sections in broadband BMs consisting of multisecti on directional couplers has been described in [7], [14], [15]. In [16] a BM has been shown in which 45° phase shifters are designed comprising half-wavelength open stubs to improve the differential phase characteristics between the output ports. Recently, left-handed transmission lines have been introduced, allowing for the design of miniaturized networks, dual-band networks and also differential phase shifters [17]. In [18] and [19] the composite right/left-handed transmission lines have been utilized for a significant size reduction of a branch-line coupler. Their application in a 4 x 4 Butler matrix has reduced the occupied area to 21% of a conventional design.

In this paper, a novel application of left-handed transmission lines in Butler matrices is presented. It has been shown, for the first time, that by application of both left-handed LH and right-handed RH transmission lines, the broadband phase shifters required in such networks may be realized. A theoretical analysis, of BMs incorporating ideal transmission lines of both types, is given. Moreover, design expressions for achievable bandwidth
are derived, once differential phase deviation has been chosen. The proposed application of left-handed transmission lines has been verified by the design of a 4 x 4 Butler matrix operating in frequency range 2.5–3.5 GHz. The ideal left-handed transmission line has been substituted with an equivalent circuit realized with the use of quasi-lumped high-pass LC elements. The analytical formulas are given allowing for the synthesis of an artificial LH section having the desired electrical length and ideal impedance match at a specified frequency. The designed 4 x 4 Butler matrix has been manufactured and measured. The measurement results fully confirm the usefulness of the proposed phase shifters realization with the use of left-handed transmission lines in application to broadband Butler matrices’ realization.

2. Theoretical Investigation

A conventional 4 x 4 Butler matrix requires the application of two 45° phase shifters as it is presented in Fig. 1. Therefore, the phase difference between appropriate signal paths is equal to -45°. Assuming an ideal 4 x 4 Butler matrix (Fig. 1), the exemplary phase relations may be expressed as follows:

\[ \arg(s_{s1}) - \arg(s_{s1}) = -90° - (-45°) = -45°, \]  

\[ \arg(s_{s1}) - \arg(s_{s2}) = -45° - 90° = -135°, \]  

\[ \arg(s_{s2}) - \arg(s_{s3}) = 90° - (-45°) - (-90°) = 135°, \]  

\[ \arg(s_{s3}) - \arg(s_{s4}) = -90° - (-90°) = 0°, \]  

\[ \arg(s_{s4}) - \arg(s_{s5}) = -90° - 45° - 90° = -225° \approx 135°, \]  

\[ \arg(s_{s5}) - \arg(s_{s6}) = -90° - (-90°) = 0°, \]  

\[ \arg(s_{s6}) - \arg(s_{s7}) = -90° - 45° - 90° = 135°. \]  

The same effect may be obtained with the use of phase shifters realized utilizing left-handed and right-handed transmission line sections. Since the sign of a phase shift in case of a LH transmission line is opposite to the RH transmission line, the LH sections constitute the inner connections of the BM, while RH section are placed in the outer channels, as it is shown in Fig. 2. The expressions for differential phases in case of the proposed Butler matrix take the following form:

\[ \arg(s_{s1}) - \arg(s_{s2}) = -90° - \Theta_{LH} - (-\Theta_{RH}), \]  

\[ \arg(s_{s2}) - \arg(s_{s3}) = -\Theta_{LH} - (-90° - \Theta_{RH}), \]  

\[ \arg(s_{s3}) - \arg(s_{s4}) = 90° - \Theta_{RH} - (-90° - \Theta_{LH}) = 135°, \]  

\[ \arg(s_{s4}) - \arg(s_{s5}) = -90° - (-90°) = 0°, \]  

\[ \arg(s_{s5}) - \arg(s_{s6}) = -90° - 45° - 90° = 135°. \]  

In order to obtain the phase distribution of an ideal Butler matrix the differential phases described by (7)-(12) have to be equal to the corresponding phase relations expressed by (1)-(6). The comparison of the two sets of equations leads to the following condition:

\[ \Theta_{LH} - \Theta_{RH} = -45°. \]  

Fig. 1. Schematic diagram of an ideal 4 x 4 Butler matrix consisting of four 3dB/90° directional couplers and two 45° phase shifters.

The ideal left-handed transmission line is described by two parameters, i.e. characteristic impedance \( Z_{0LH} \) and propagation constant \( \beta_{LH} \):

\[ Z_{0LH} = \sqrt{\frac{C}{L}}, \]  

\[ \beta_{LH} = -\frac{1}{\omega \sqrt{LC}}. \]  

As it is seen from (14) the characteristic impedance of a LH transmission line is expressed by the same formula as the characteristic impedance of ideal RH transmission lines. However the propagation constant of a left-handed transmission line is a hyperbolic function of frequency and has opposite sign in comparison to the right-handed transmission line. The differential phase \( \Delta \varphi \) between LH and RH transmission lines may be derived as follows:

\[ \Delta \varphi = \varphi_{LH} - \varphi_{RH} = \frac{l_{RH} - l_{LH}}{\omega \sqrt{L_{LH} C_{LH}}} - \frac{l_{RH}}{\omega \sqrt{L_{RH} C_{RH}}}. \]  

where \( l_{LH} \) and \( l_{RH} \) indicate the physical length of LH and RH transmission lines, respectively. Fig. 3 presents the phase difference between ideal left-handed and right-handed transmission lines with their electrical lengths satisfying (13). As it is seen, the equal electrical lengths of both transmission lines ensure the required -45° phase shift only at the center frequency. Further investigation reveals that by appropriate modification of electrical lengths of both RH and LH sections the optimum phase response in terms of a phase deviation \( \pm \delta \) within the specified frequency range can be obtained. Moreover, it has been found that ensuring the desired phase deviation in the specified frequency range requires unequal lengths of LH and RH sec-
tions. This fact is caused by the asymmetrical character of differential phase characteristics with respect to the center frequency, as it is presented in Fig. 3. The electrical lengths of left-handed and right-handed transmission lines may be expressed as follows:

$$\Theta_{\text{LH}} = \frac{(\Delta \varphi + \delta)^2}{2(\Delta \varphi - \delta)}, \quad \text{(17)}$$

$$\Theta_{\text{RH}} = -\frac{\Delta \varphi - \delta}{2}. \quad \text{(18)}$$

Assuming differential phase $\Delta \varphi = -45^\circ$ and phase deviation $\delta = 1^\circ$ the values of the electrical lengths are equal $\Theta_{\text{RH}} = 23.0^\circ$ and $\Theta_{\text{LH}} = -21.05^\circ$. Such an unequal division of both electrical lengths decreases the frequency at which the maximum phase difference $\Delta \varphi$ occurs. This frequency $f_m$ can be expressed as a function of the center frequency $f_c$ as:

$$f_m = f_c \frac{\Delta \varphi + \delta}{\Delta \varphi - \delta}. \quad \text{(19)}$$

which, in the discussed case, is equal $f_m = 0.96f_c$. Fig. 4 presents the differential phase obtained for the electrical lengths determined with the use of (17) and (18) in comparison to the phase difference corresponding to equal electrical lengths of both transmission lines satisfying condition (13).

The operational bandwidth $BW$, defined as a ratio of upper and lower frequency, in which the assumed phase deviation is guaranteed, for the considered differential phase shifter can be expressed as follows:

$$BW = \frac{f_2}{f_1} = \frac{\left(\Delta \varphi - \delta\right) + 2\sqrt{\delta \Delta \varphi}}{\left(\Delta \varphi - \delta\right) - 2\sqrt{\delta \Delta \varphi}}. \quad \text{(20)}$$

Fig. 5 shows the calculated operational bandwidth vs. the differential phase deviation for four values of the required differential phase shift.

The operational bandwidth $BW$, defined as a ratio of upper and lower frequency, in which the assumed phase deviation is guaranteed, for the considered differential phase shifter can be expressed as follows:

$$BW = \frac{f_2}{f_1} = \frac{\left(\Delta \varphi - \delta\right) + 2\sqrt{\delta \Delta \varphi}}{\left(\Delta \varphi - \delta\right) - 2\sqrt{\delta \Delta \varphi}}. \quad \text{(20)}$$

Fig. 5 shows the calculated operational bandwidth vs. the differential phase deviation. Since only two identical phase shifters are needed in case of an ideal 4 x 4 Butler matrix, neither differential phase is calculated between signal paths containing more than one phase shifter. Therefore, all the differential phases of such a matrix have the same shape as presented in Fig. 4, while their mean values are $\pm45^\circ$ and $\pm135^\circ$, depending on the excitation port. For the purpose of an 8 x 8 Butler matrix realization, phase shifters with three values are needed, i.e. $22.5^\circ$, $45^\circ$ and $67.5^\circ$. Exemplary values of electrical lengths of transmission line sections allowing for the design of such phase shifters are presented in Tab. 1.
TL_{LRH} provide -22.5°, TL_{RH} and TL_{LH} provide -45° and TL_{LRH} and TL_{LH} provide -67.5°). The differential phase characteristics of the three phase shifters are presented in Fig. 7.

<table>
<thead>
<tr>
<th>Differential phase</th>
<th>Transmission line type</th>
<th>$\delta = 1^\circ$</th>
<th>$\delta = 2^\circ$</th>
<th>$\delta = 5^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RH</td>
<td>11.75°</td>
<td>12.50°</td>
<td>13.75°</td>
</tr>
<tr>
<td>-45.0°</td>
<td>LH</td>
<td>-21.05°</td>
<td>-19.67°</td>
<td>-16.00°</td>
</tr>
<tr>
<td></td>
<td>RH</td>
<td>23.00°</td>
<td>23.50°</td>
<td>25.00°</td>
</tr>
<tr>
<td>-67.5°</td>
<td>LH</td>
<td>-32.28°</td>
<td>-30.87°</td>
<td>-26.94°</td>
</tr>
<tr>
<td></td>
<td>RH</td>
<td>34.25°</td>
<td>34.75°</td>
<td>36.25°</td>
</tr>
</tbody>
</table>

Tab. 1. Electrical lengths of transmission line sections for different choice of required differential phase and assumed phase deviation $\delta$.

Fig. 6. Schematic diagram of the proposed 8 x 8 Butler matrix with the use of left-handed transmission lines and 3dB/90° directional couplers.

Fig. 7. Differential phases of three phase shifters (-22.5°, -45° and -67.5°) realized with the use of left-handed and right-handed transmission lines calculated assuming phase deviation $\delta = 1^\circ$.

Fig. 8. Frequency response of an 8 x 8 Butler matrix shown in Fig. 6 calculated assuming phase deviation $\delta = 1^\circ$ for each phase shifter. Differential phase characteristics when port #1 is fed (a), port #3 is fed (b), port #5 is fed (c) and port #7 is fed (d).
The assumed phase deviation \( \delta = 1° \) allows one to achieve bandwidth \( \text{BW} = 1.61 \). It is worth mentioning, that in case of an 8 x 8 Butler matrix differential phases are calculated between signal paths containing different combinations of the phase shifters depending on the chosen excitation ports. Therefore, the final phase deviation of the entire network can be two times greater than the assumed phase deviation of a single phase shifter. The theoretically calculated differential phase characteristics of the 8 x 8 Butler matrix assuming phase deviation \( \delta = 1° \) for each phase shifter are presented in Fig. 8.

3. Butler Matrix Design

The proposed in Section 2 application of left-handed transmission lines for realization of broadband differential phase shifters required in Butler matrices has been experimentally verified by the design of a 4 x 4 Butler matrix. As it is seen in Fig. 1 and Fig. 2 a standard configuration of the matrix requires two transmission line crossovers. However, in applications in which the matrix is not integrated with linear antenna array, a relaxed configuration without crossovers, as shown in Fig. 9, can be used allowing for fully planar realization.

![Fig. 9. A fully planar configuration of the proposed 4 x 4 Butler matrix utilizing left-handed transmission line.](image)

For the purpose of practical realization ideal left-handed transmission lines may be substituted with high-pass LC networks. Two equivalent circuits presented in Fig. 10 have been considered.

![Fig. 10. Two investigated structures of the high-pass LC networks as artificial left-handed transmission lines: \( \Pi \)-shaped (a) and T-shaped (b).](image)

In order to provide ideal impedance match of these structures the following conditions must be fulfilled:

\[
C_{\Pi} = \frac{Z_0^2 + \omega_C^2 L_{\Pi}^2}{2Z_0^2 \omega_C^2 L_{\Pi}^2} \quad (21)
\]

in case of \( \Pi \)-structure and

\[
L_T = \frac{C_T}{2} \left( \frac{Z_0^2 + \frac{1}{\omega_C^2 C_T^2}}{Z_0^2 \omega_C^2 C_T^2 - 1} \right) \quad (22)
\]

T-structure. The calculated reflection coefficient of the circuit presented in Fig. 10b has been shown in Fig. 11. As it is seen an ideal impedance match is obtained at the specified angular frequency \( \omega_C \). Although the reflection coefficient of the discussed circuit has resonant character, it ensures sufficient impedance match in a wide frequency range. The argument of transmission coefficient \( \phi_T \) under the condition (22) is expressed by the following formula:

\[
\phi_T = \arctan \left( \frac{2Z_0 \omega_C C_T}{Z_0^2 \omega_C^2 C_T^2 - 1} \right) \quad (23)
\]

hence, the \( C_T \) value can be expressed as follows:

\[
C_T = \frac{1 + \cos \phi_T}{Z_0 \omega_C \sin \phi_T} \quad (24)
\]

![Fig. 11. Reflection coefficient of an artificial left-handed transmission line realized as T-shaped LC circuit presented in Fig. 10b.](image)

The proposed 4 x 4 Butler matrix has been designed in a homogenous dielectric structure (ARLON 25N laminate with \( \varepsilon_r = 3.38, \tan \delta = 0.0025 \)) shown in Fig. 12 consisting of two 1.52 mm thick laminates, between which two thinner laminates have been placed: 0.15 mm thick for the design of directional couplers and 0.025 mm allowing for easy realization of series capacitors of artificial left-handed transmission lines. The 3dB/90° directional couplers designed in a broadside coupled line technique, previously shown in [20], have been used. The couplers’ traces have been placed on metallization layers \( m_2 \) and \( m_3 \). The application of artificial left-handed transmission lines requires an additional layer of metallization \( m_1 \), to allow for series capacitors’ realization.

![Fig. 12. Cross-sectional view of the stripline layers used for the design of a broadband 4 x 4 Butler matrix.](image)
For practical realization the circuit presented in Fig. 10b has been chosen since the input and output of the LH section can be made on the same metallization layer (two series capacitors realized on metallization layers $m_1$ and $m_2$) unlike for the case of the circuit presented in Fig. 10a.

Assuming the determined in Section 2 electrical length of the LH transmission line $\Theta_{LH} = -21.05^\circ$ and the center frequency $f_c = 3$ GHz the calculated capacitances of high-pass LC network equal $C_T = 5.7$ pF. From (22) the calculated lumped inductance $L_T = 7.4$ nH. In our case the bandwidth of the applied coupled-line directional couplers, defined by the coupling/transmission imbalance equal $\delta_C = \pm 0.1$ dB, is narrower than the bandwidth of the differential phase shifter. Therefore, the electrical lengths of transmission lines determined using (17) and (18) are not critical. Hence, for a more convenient design of quasi-lumped LC elements, the derived capacitance and inductance values may be slightly changed, what leads to the shift of differential phase characteristic vs. frequency.

Fig. 13 presents the phase difference between right-handed and left-handed transmission lines and reflection coefficient of the LH section. Result of electromagnetic simulation.

Fig. 14. Layout of the proposed broadband 4 x 4 Butler matrix utilizing left-handed transmission lines.

4. Experimental Results

(a)

(b)

(c)
The designed BM has been manufactured and measured. For assembling Radiall R125.462.000 W SMA connectors have been used at which the measuring reference planes have been set. Measurement results in comparison with the calculated ones are presented in Fig. 15 and are in good agreement. The measured amplitude imbalance equals \( \pm 0.4 \) dB within the frequency range of 2.5 – 3.5 GHz and the maximum phase error is not greater than \( \pm 4^\circ \). The return losses are better than 22 dB and isolations are better than 24 dB within entire operational bandwidth.

5. Conclusions

In this paper a novel concept of broadband Butler matrices’ realization utilizing artificial left-handed transmission lines has been presented. In order to obtain broad operational frequency range in terms of amplitude characteristics coupled-line directional couplers have been utilized. The required constant phase shift within operational bandwidth is ensured by the application of artificial left-handed transmission lines designed with the use of quasi-lumped elements. The analytical expressions for the electrical lengths of LH and RH sections have been derived for arbitrary bandwidth and phase imbalance.

The proposed network has been experimentally verified in a planar configuration without crossovers. The obtained measurements results are in very good agreement with the theoretical ones in terms of amplitude and phase imbalance over frequency range 2.5 – 3.5 GHz. Additionally, good return losses and isolation of the 4 x 4 Butler matrix have been obtained.

Acknowledgements

This work was supported in part by the Polish National Research Centre under grant no. UMO-2011/01/D/ST7/00789 and in part the statutory research of the Faculty of Electronics AGH.

References


About Authors ...

Kamil STASZEK received his M.Sc. Tech. degree in Electronics Engineering from AGH University of Science and Technology, Cracow, Poland in 2011. He has coauthored 14 scientific papers. Currently he is working toward Ph. D. degree at the same university in the field of Microwave Engineering, focusing on microwave measurement techniques.

Sławomir GRUSZCZYNSKI received the M.Sc. degree and the Ph.D. degree in Electronics and Electrical Engineering from the Wrocław University of Technology, Poland, in 2001 and 2006, respectively. From 2001 to 2006 he was with the Telecommunications Research Institute, Wrocław Division. From 2005 to 2009, he worked at the Institute of Telecommunications, Teleinformatics and Acoustics, Wrocław University of Technology. In 2009, he joined the Faculty of Informatics, Electronics and Telecommunications at AGH University of Science and Technology where he became a Head of the Department of Electronics in 2012. He has coauthored 45 journal and 52 conference scientific papers. He is a member of the IEEE, and a member of the Young Scientists’ Academy and the Committee of Electronics and Telecommunications at the Polish Academy of Sciences (PAN).

Krzysztof WINCZA received the M.Sc. degree and the Ph.D. degree in Electronics and Electrical Engineering from the Wrocław University of Technology, Poland, in 2003 and 2007, respectively. In 2007, he joined the Institute of Telecommunications, Teleinformatics and Acoustics, Wrocław University of Technology. In 2009, he joined the Faculty of Electronics at AGH University of Science and Technology becoming an Assistant Professor. Dr. Winca was the recipient of The Youth Award presented at the 10th National Symposium of Radio Sciences (URSI) and the Young Scientist Grant awarded by the Foundation for Polish Science in 2001 and 2008, respectively. He has coauthored 43 journal and 50 conference scientific papers.