Emitter Location Finding using Particle Swarm Optimization

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Abstract. *Using several spatially separated receivers, nowadays positioning techniques, which are implemented to determine the location of the transmitter, are often required for several important disciplines such as military, security, medical, and commercial applications. In this study, localization is carried out by particle swarm optimization using time difference of arrival. In order to increase the positioning accuracy, time difference of arrival averaging based two new methods are proposed. Results are compared with classical algorithms and Cramer-Rao lower bound which is the theoretical limit of the estimation error.*

Keywords

Emitter location finding, time difference of arrival, time difference of arrival averaging, particle swarm optimization, Cramer-Rao lower bound.

1. Introduction

Many changes and new requirements have been made in people's lives to keep up with the technological improvements. One of the important requirements we encounter mostly is a necessity for localization like vehicle navigation systems or location based services. Source position can be determined by using characteristic properties of the transmitter signal. Currently there are many localization techniques whose accuracy, complexity, and hardware requirements are quite diverse. Received signal strength (RSS) method which requires the signal strength and attenuation of the medium is one of them [1]–[4]. Another technique, which uses the direction of arrival, has been published in several studies [5]–[7], where receivers need to be equipped with suitable antenna arrays. The time of arrival that utilizes the incident time of the received signal to localize the target is the most accurate and reliable method [8]–[10]. Using time differences at the receiver side an effective technique time difference of arrival (TDOA) is proposed [11]–[14]. This method is preferable for passive radar systems due to its nature which does not require any prior knowledge of the emitter signal and high localization accuracy. If any of either transmitter or receiver is mobile, in this case, the position of the source can be determined in frequency domain by using the frequency differences at the receiver side [15]–[17]. In order to increase the accuracy, different techniques can be combined and named as hybrid methods which have already taken their place in literature [18]–[20].

Using TDOA based algorithms, to localize the transmitter, time differences need to be obtained. Several techniques were proposed in literature for different cases, in particular, when the incident signals are emerged through line of sight, non-line of sight, and multipath medium [21]–[23]. After the time difference estimation processes, the target can be positioned using various algorithms such as nonlinear least square (NLS), maximum likelihood (ML), linear least square (LLS), or weighted linear least square (WLLS) [24]–[26]. Because of its invaluable properties such as simplicity, convergence speed, accuracy, and less immunity to local minima; particle swarm optimization (PSO), is an appropriate solution for different optimization problems [27]–[29]. In this manner, PSO is selected as an efficient estimator for source localization [30]–[33].

Time difference of arrival averaging (TDOAA) was presented by Ralph O. Schmidt for three receivers in 1972 and generalized in 1996 [34], [35]. The method claims that the sum of the range differences in a closed loop must be zero in the absence of measurement noise. Applying the TDOAA technique on the estimated time differences, measurement noise decreases, therefore the positioning accuracy increases. TDOAA can also be used with different algorithms to reduce the estimation error. In this concept, RSS and TDOAA are combined and this combination offers an error correction method [36], [37]. In the case of known noise covariance, considering the receivers position, the system performance could be improved [38]. In our study, PSO was combined with TDOAA and a significant decrement in positioning error was obtained. Consequently, results are compared to the Cramer-Rao lower bound (CRLB) which provides the theoretical limit for the estimation error.

The paper is structured as follows. In Section 2, how to localize the emitter using time differences and implementation of the TDOAA are explained. In Section 3, the theoretical error limit of any unbiased estimators is analyzed mathematically. TDOA based algorithms are presented in Section 4. In Section 5, the proposed methods are introduced. Finally in Section 6, we give the results and show the performance improvements of TDOAA based techniques. The conclusions are summarized in Section 7.

2. Source Localization using TDOA

Considering a single transmitter based positioning system, we need to discuss some details with respect to the number of receivers. It is well known that at least two synchronized receivers must be used in two dimensional positioning systems to obtain one TDOA and one hyperbolic line of position (LOP). For this scenario the emitter can be at any point on the defined LOP. In the case of three receives, the transmitter is generally located at a unique point obtaining three time differences and LOPs. However, to get rid of the possibility of two target points for one source, the number of receivers should be increased.

Assuming four receivers positioning systems, as seen in Fig. 1, the number of time differences is calculated using (1), where N and M are the number of receivers and time differences respectively. Here, six time differences occur as Δ_{12}, Δ_{13}, Δ_{14}, Δ_{23}, Δ_{24}, and Δ_{34}. If the propagation speed is known, these time differences can be calculated using (2) and (3).

$$M = \frac{N(N-1)}{2}, \tag{1}$$

$$l_i = \sqrt{(x_i - x)^2 + (y_i - y)^2}, \tag{2}$$

$$\Delta_{ij} = \frac{l_i - l_j}{c}, \quad 1 \leq i < j \leq N. \tag{3}$$

Here, Δ is the exact time difference of arrival, c is the propagation speed, l_i is the distance between ith receiver and transmitter, (x, y) are the emitter coordinates and (x_i, y_i) are the coordinates of the ith receiver. As a result, the target can be positioned using N - 1 (independent/spherical set) or M (full set) time differences. In this study, we used the full set to increase the localization accuracy.

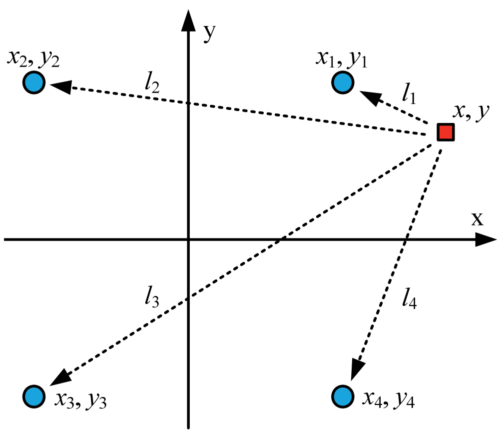

Fig. 1. Four receivers positioning system.

Applying the averaging method to the estimated time differences, the measurement error decreases and then the positioning accuracy increases. In this algorithm, time differences are assumed vectors as shown in Fig. 2.

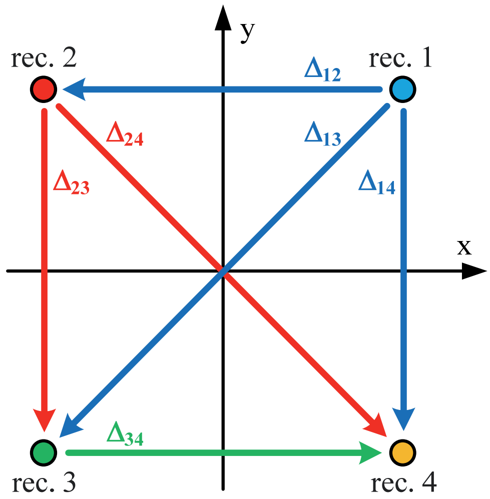

Fig. 2. TDOA vectors for four receivers positioning system

There are some simple rules for the implementation of the technique which are given in (4)–(6).

$$\Delta_{ii} = 0, \tag{4}$$

$$\Delta_{ij} = -\Delta_{ji}, \tag{5}$$

$$\Delta_{ij} + \Delta_{jk} = \Delta_{ik}. \tag{6}$$

Here i, j, and, k are receiver indexes. The mathematical expression of the method can be summarized as given below.

$$\tilde{\Delta}_{ij} = \hat{\Delta}_{ij} - \frac{1}{N}\sum_{k \neq i,j}\left(\hat{\Delta}_{ij} + \hat{\Delta}_{jk} + \hat{\Delta}_{ki}\right), 1 \leq i < j \leq N, \tag{7}$$

$$\hat{\Delta} = \Delta + n. \tag{8}$$

Here, $\hat{\Delta}$ shows the estimated time difference, $\tilde{\Delta}$ denotes the averaged TDOA, and n is the estimation error. Assuming four receivers, (7) and (8) can be turned into matrix form to obtain the averaged time differences as given in (9) and (10).

$$\begin{bmatrix} \tilde{\Delta}_{12} \\ \tilde{\Delta}_{13} \\ \tilde{\Delta}_{14} \\ \tilde{\Delta}_{23} \\ \tilde{\Delta}_{24} \\ \tilde{\Delta}_{34} \end{bmatrix} = \begin{bmatrix} \hat{\Delta}_{12} \\ \hat{\Delta}_{13} \\ \hat{\Delta}_{14} \\ \hat{\Delta}_{23} \\ \hat{\Delta}_{24} \\ \hat{\Delta}_{34} \end{bmatrix} - \frac{1}{4} \begin{bmatrix} 2 & -1 & -1 & 1 & 1 & 0 \\ -1 & 2 & -1 & -1 & 0 & 1 \\ -1 & -1 & 2 & 0 & -1 & -1 \\ 1 & -1 & 0 & 2 & -1 & 1 \\ 1 & 0 & -1 & -1 & 2 & -1 \\ 0 & 1 & -1 & 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} \hat{\Delta}_{12} \\ \hat{\Delta}_{13} \\ \hat{\Delta}_{14} \\ \hat{\Delta}_{23} \\ \hat{\Delta}_{24} \\ \hat{\Delta}_{34} \end{bmatrix} \tag{9}$$

$$\tilde{\Delta} = \hat{\Delta} - \frac{1}{N}\boldsymbol{C}\hat{\Delta} \tag{10}$$

where $\boldsymbol{C}$ is $M \times M$ coefficient matrix which is obtained in [39] using a simple graph traversal algorithm.

3. Cramer-Rao Lower Bound

The CRLB is a lower bound on the variance of any unbiased estimator. For the time differences based localization techniques, CRLB is defined in [40] as given in (11).

$$\boldsymbol{\Phi} = c^2\left(\boldsymbol{G}_t^T \boldsymbol{Q}^{-1} \boldsymbol{G}_t\right)^{-1} \tag{11}$$

where $\boldsymbol{\Phi}$ is the inverse of the Fisher information matrix, $\boldsymbol{G}_t$

is the Taylor coefficient matrix, $\boldsymbol{Q}$ is the TDOA covariance matrix, $(.)^T$ and $(.)^{-1}$ indicates the matrix transpose and inverse respectively. Considering two dimensional plane and N receivers, $\boldsymbol{G}_t$ can be calculated as given in (12) and (13).

$$\boldsymbol{G}_t = \begin{bmatrix} \frac{x_1-x}{l_1} - \frac{x_2-x}{l_2} & \frac{y_1-y}{l_1} - \frac{y_2-y}{l_2} \\ \frac{x_1-x}{l_1} - \frac{x_3-x}{l_3} & \frac{y_1-y}{l_1} - \frac{y_3-y}{l_3} \\ \vdots & \vdots \\ \frac{x_1-x}{l_1} - \frac{x_N-x}{l_N} & \frac{y_1-y}{l_1} - \frac{y_N-y}{l_N} \end{bmatrix}_{N-1\times 2} \tag{12}$$

If the noise power spectral densities are similar at receivers, the coefficients of the TDOA covariance matrix may be assigned to 1.0 and 0.5 for diagonal terms and others respectively [40].

$$\boldsymbol{Q} = \sigma_d^2 \begin{bmatrix} 1.0 & \dots & 0.5 \\ \vdots & \ddots & \vdots \\ 0.5 & \dots & 1.0 \end{bmatrix}_{N-1\times N-1} \tag{13}$$

In (13) σ_d^2 is the variance of the TDOA noise. $\boldsymbol{\Phi}$ is calculated using equations (11)–(13) and then the sum of the diagonal terms gives the CRLB.

4. TDOA Based Source Localization Techniques

TDOA based positioning algorithms can be divided into two groups, as nonlinear and linear. Nonlinear methods, such as NLS and ML localize the emitter directly by using the equations between the target and the receivers. Although the accuracy of these techniques are quite high, convergence to the global minimum cannot be guaranteed since the optimization cost functions are multimodal. On the other hand, considering the linear algorithms such as LLS and WLLS, the transmitter position is determined by linearizing the nonlinear equation. Therefore, their optimization cost functions are unimodal and obtaining the global solution is always guaranteed [41]. However, their accuracy is lower than the nonlinear methods due to the linearization effect. In this section the TDOA based positioning techniques are going to be summarized.

NLS algorithm is a simple solution if the noise characteristic is unknown. In NLS method, the least square based cost function is minimized and the emitter position is obtained using (14)–(18) [41].

$$\hat{\boldsymbol{x}}^{b+1} = \hat{\boldsymbol{x}}^b - \mu \nabla\left(J_{NLS}(\hat{\boldsymbol{x}}^b)\right), \tag{14}$$

$$\nabla\left(J_{NLS}(\hat{\boldsymbol{x}})\right) = 2\begin{bmatrix} \sum_{i=2}^{N} \frac{(c\hat{\Delta}_{1i} - \hat{l}_1 + \hat{l}_i)(x_i - \hat{x})}{\hat{l}_i} \\ \sum_{i=2}^{N} \frac{(c\hat{\Delta}_{1i} - \hat{l}_1 + \hat{l}_i)(y_i - \hat{y})}{\hat{l}_i} \end{bmatrix}, \tag{15}$$

$$\hat{l}_1 = \sqrt{(x_1 - \hat{x})^2 + (y_1 - \hat{y})^2}, \tag{16}$$

$$\hat{l}_i = \sqrt{(x_i - \hat{x})^2 + (y_i - \hat{y})^2}, \tag{17}$$

$$\hat{\boldsymbol{x}} = [\hat{x} \quad \hat{y}]^T. \tag{18}$$

Here b indicates the iteration index, $\hat{\boldsymbol{x}}$ is the estimated coordinate vector of the target, μ is the step size, $J_{NLS}(\hat{\boldsymbol{x}})$ is the cost function, ∇ shows the gradient operator, $\hat{l}_1$ shows the distance between the estimated position of the source and the first receiver, and $\hat{l}_i$ shows the distance between estimated position and ith receiver. In (14) the iterative procedure of the steepest descent technique is given where $\hat{\boldsymbol{x}}^{b+1}$ indicates the estimated coordinates. In order to make the NLS method converged, the appropriate initialization values $(\hat{\boldsymbol{x}}^b, b = 0)$ are needed. These values can be obtained by the LLS algorithm.

Assuming the noise distribution is known, ML method finds the transmitter location by maximizing the probability density function of the TDOA measurements [41]. If the TDOA noise is the zero-mean Gaussian distributed, then the target location can be determined using (19)–(23).

$$\hat{\boldsymbol{x}}^{b+1} = \hat{\boldsymbol{x}}^b - \mu \nabla\left(J_{ML}(\hat{\boldsymbol{x}}^b)\right), \tag{19}$$

$$\nabla\left(J_{ML}(\hat{\boldsymbol{x}})\right) = \begin{bmatrix} \frac{\partial J_{ML}(\hat{\boldsymbol{x}})}{\partial \hat{x}} \\ \frac{\partial J_{ML}(\hat{\boldsymbol{x}})}{\partial \hat{y}} \end{bmatrix}, \tag{20}$$

$$J_{ML}(\hat{\boldsymbol{x}}) = \boldsymbol{e}^T \boldsymbol{Q}^{-1} \boldsymbol{e}, \tag{21}$$

$$\boldsymbol{e} = \begin{bmatrix} c\hat{\Delta}_{12} - \hat{l}_1 + \hat{l}_2 \\ \vdots \\ c\hat{\Delta}_{1N} - \hat{l}_1 + \hat{l}_N \end{bmatrix}_{N-1} \tag{22}$$

$$\boldsymbol{Q}^{-1} = \frac{1}{\sigma_d^2}\begin{bmatrix} 2 - \frac{2}{N} & \dots & -\frac{2}{N} \\ \vdots & \ddots & \vdots \\ -\frac{2}{N} & \dots & 2 - \frac{2}{N} \end{bmatrix}_{N-1\times N-1} \tag{23}$$

Here $J_{ML}(\hat{\boldsymbol{x}})$ is the cost function, $\boldsymbol{e}$ is the estimation error vector, and $\boldsymbol{Q}^{-1}$ is the inverse of the convergence matrix of the TDOA noise. When the stopping criterion is satisfied, the $\hat{\boldsymbol{x}}^{b+1}$ gives the source coordinates. Like the NLS technique, the ML algorithm also needs the appropriate initialization values that can be obtained by the LLS method.

In the LLS algorithm, the nonlinear equations formed according to the transmitter-receiver geometries, are translated into the linear equations. Then the emitter position is executed using only one step using (24)–(29). [41]. It should be noted that the estimated target coordinates using LLS can be assigned as the appropriate initialization values for NLS $(\hat{\boldsymbol{x}}^b)$, ML $(\hat{\boldsymbol{x}}^b)$, and WLLS (R_1) techniques.

$$\boldsymbol{\theta}_{LLS} = (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{b}, \tag{24}$$

$$\boldsymbol{A} = 2\begin{bmatrix} x_1 - x_2 & y_1 - y_2 & c\Delta_{12} \\ x_1 - x_3 & y_1 - y_3 & c\Delta_{13} \\ \vdots & \vdots & \vdots \\ x_1 - x_N & y_1 - y_N & c\Delta_{1N} \end{bmatrix}_{N-1\times 3} \tag{25}$$

$$\boldsymbol{b} = \begin{bmatrix} (c\Delta_{12})^2 - (x_1 - x_2)^2 - (y_1 - y_2)^2 \\ (c\Delta_{13})^2 - (x_1 - x_3)^2 - (y_1 - y_3)^2 \\ \vdots \\ (c\Delta_{1N})^2 - (x_1 - x_N)^2 - (y_1 - y_N)^2 \end{bmatrix}_{N-1} \tag{26}$$

$$\boldsymbol{\theta}_{LLS} = [x - x_1 \quad y - y_1 \quad R_1]^T, \tag{27}$$

$$\hat{\boldsymbol{x}} = \left[[\hat{\boldsymbol{\theta}}_{LLS}]_1 + x_1 \quad [\hat{\boldsymbol{\theta}}_{LLS}]_2 + y_1\right]^T, \tag{28}$$

$$R_1 = \sqrt{(x - x_1)^2 + (y - y_1)^2}. \tag{29}$$

In these equations, $\boldsymbol{\theta}_{LLS}$ is the solution vector, $\boldsymbol{b}$ is the square error vector, and R_1 is the distance between the source and the first receiver.

The WLLS method is a weighted version of the LLS algorithm, and can reach more accurate position estimates than LLS. However, in order to obtain the weight computation, the knowledge of the mean and convergence values of the linear equations errors are needed. Again, in this technique, the target location is estimated in one step using (30)–(33) [41].

$$\boldsymbol{\theta}_{WLLS} = (\boldsymbol{A}^T\boldsymbol{W}\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{W}\boldsymbol{b}, \tag{30}$$

$$\boldsymbol{W} \approx \left[4\text{diag}\left([\hat{\boldsymbol{\theta}}_{LLS}]_3 - c\Delta_{12}, \cdots, [\hat{\boldsymbol{\theta}}_{LLS}]_3 - c\Delta_{1N}\right)\boldsymbol{Q}\ \text{diag}\left([\hat{\boldsymbol{\theta}}_{LLS}]_3 - c\Delta_{12}, \cdots, [\hat{\boldsymbol{\theta}}_{LLS}]_3 - c\Delta_{1N}\right)\right]^{-1} \tag{31}$$

$$\boldsymbol{\theta}_{WLLS} = [x - x_1 \quad y - y_1 \quad R_1]^T, \tag{32}$$

$$\hat{\boldsymbol{x}} = \left[[\hat{\boldsymbol{\theta}}_{WLLS}]_1 + x_1 \quad [\hat{\boldsymbol{\theta}}_{WLLS}]_2 + y_1\right]^T. \tag{33}$$

Here, $\boldsymbol{\theta}_{WLLS}$ gives the solution vector and $\boldsymbol{W}$ is the weighting matrix. It is important to know that, to calculate the $\boldsymbol{W}$ matrix, $[\hat{\boldsymbol{\theta}}_{LLS}]_3$ distance must be estimated using the LLS algorithm.

5. Proposed Methods

PSO is a robust stochastic optimization algorithm that is inspired by the behavior of birds flocking or fish schooling [42]. Being fast, less parameter necessity, and low probability of local minimum convergence property are some of the advantages of the technique. PSO composed of particles which are individually the solution of the problem. Every particle has its own location and velocity vectors. The vector size depends on the number of the parameter of the problem. These vectors show the instantaneous position and velocity.

$$\boldsymbol{v}_a^{b+1} = \boldsymbol{v}_a^b + c_1 rand_1(\boldsymbol{pbest}_a^b - \boldsymbol{x}_a^b) + c_2 rand_2(\boldsymbol{gbest}^b - \boldsymbol{x}_a^b) \tag{34}$$

$$\boldsymbol{x}_a^{b+1} = \boldsymbol{x}_a^b + \boldsymbol{v}_a^{b+1}. \tag{35}$$

The particle velocity is calculated as given in (34) where $\boldsymbol{v}$ is the velocity vector, $\boldsymbol{x}$ is the position vector and a is the particle index. The number of the iteration can be either predefined or dynamically changed according to the defined fitness function's convergence. $rand_1$ and $rand_2$ are uniformly distributed random floating point numbers, which vary between zero to one and give the ability of moving to the particle around the problem space. While ***pbest*** is the best position vector for a particle that has been achieved so far, ***gbest*** is the best position vector for the whole swarm. c_1 and c_2 are learning factors selected between zero and four, generally two. If the c_1/c_2 rate increases, the particle movement is determined by its own experience rather than swarm's experience and vice versa. The next position is calculated as given in (35). When the stopping criterion is satisfied the ***gbest*** becomes the solution of the problem.

Due to its unique capabilities in optimization, PSO is found as an attractive algorithm for positioning problems [30]–[33]. While an emitter location is determined by PSO, firstly, the particles are distributed arbitrary or in a specific order to the search space. Then the cost function is defined in regard to the localization technique. Consequently, ***gbest*** restores the estimated source position at the end of the iterations. In our work the fitness function is given in (39), at a two-dimensional plane. The intervals between distributed particles are kept the same.

$$\boldsymbol{x}_a = [x_a \quad y_a]^T, \quad a = 1 \dots N_p, \tag{36}$$

$$l_{ai} = \sqrt{(x_a - x_i)^2 + (y_a - y_i)^2}, \quad a = 1 \dots N_p, \quad i = 1 \dots N, \tag{37}$$

$$\Delta_{aij} = \frac{l_{ai} - l_{aj}}{c}, \ a = 1 \dots N_p, 1 \le i < j \le N, \tag{38}$$

$$f(\boldsymbol{x}_a) = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left|\hat{\Delta}_{ij} - \Delta_{aij}\right|, \ a = 1 \dots N_p \ . \tag{39}$$

In (36)–(39), N_p is the number of the particle, x_a and y_a are the coordinate of the ath particle, l_{ai} is the distance between ath particle and ith receiver, Δ_{aij} is the time difference between ith and jth receivers depending on the position of the ath particle, $\hat{\Delta}_{ij}$ is the estimated time difference between ith and jth receivers, $f(\boldsymbol{x}_a)$ is the fitness function of the ath particle, and |.| indicates absolute value.

In this study, PSO was combined with TDOAA and a significant reduction in positioning error is observed. As it is shown in Fig. 3, the averaging is applied on the estimated time differences ($\hat{\Delta}$) and therefore the averaged time differences ($\tilde{\Delta}$) are obtained. Then, the target is located by PSO using these time differences. Two new methods are

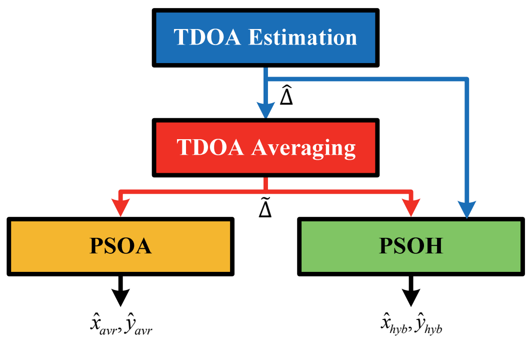

Fig. 3. Block diagram of the proposed algorithms.

proposed here: The first one uses only averaged time differences and is called as particle swarm optimization averaged (PSOA). Differently in the second algorithm both estimated and averaged time differences are used and the algorithm is called as particle swarm optimization hybrid

(PSOH). In Fig. 3, $(\hat{x}_{avr}, \hat{y}_{avr})$ indicate the target position which is localized by PSOA. Similarly $(\hat{x}_{hyb}, \hat{y}_{hyb})$ show the emitter coordinates determined by PSOH.

Assuming N receivers based system, there are M estimated and M averaged time differences. Therefore, from the perspective of computational complexity, PSOA is approximately the same as PSO. On the other hand, the PSOH technique covers both estimated and averaged time differences. For this reason, its computational complexity is higher than PSO and PSOA.

6. Simulations and Results

In this section, the proposed PSOA and PSOH methods are compared with PSO, classical techniques, and CRLB. Both arbitrary and circular distributed receivers have been taken into account. It is observed that the PSO based algorithms are exceeding the theoretical limit.

6.1 Localization with Arbitrary Array

In order to determine the source location, in this part, receivers are placed arbitrarily and the simulation parameters are kept same with [40] to make a reliable comparison. The emitter coordinates are (x = 8, y = 22) and sensor positions are (0, 0; -5, 8; 4, 6; -2, 4; 7, 3; -7, 5; 2, 5; -4, 2; 3, 3; 1, 8). The variance of the TDOA estimation error is set to $0.001/c^2$ and mean square error (MSE) is averaged over 100000 independent runs using (40). CRLB is obtained as given in (11)–(13).

$$MSE = E\{(\hat{x} - x)^2 + (\hat{y} - y)^2\}. \tag{40}$$

Here $E\{.\}$ indicates the expectation operator. In order to obtain the estimated time differences, correlated Gaussian noise with covariance matrix given by $\boldsymbol{Q}$ is added to the exact TDOAs [40]. The number of particles is selected as 36. The maximum iteration number is determined as 1000, learning factors c_1 and c_2 are set to two, and the search space is limited to $-150 \leq x \leq 150$, $-150 \leq y \leq 150$ for PSO. The step size (μ) is selected as 0.0001 and $0.0009\sigma_d^2$ for NLS and ML respectively. The maximum iteration number is 250 for both methods. While the estimated target position ($\hat{\boldsymbol{x}}$) obtained by LLS algorithm is used as the initializing value ($\hat{\boldsymbol{x}}^b$) for NLS and ML techniques, the (R_1) is assigned as the initial value of the distance for the first receiver of the WLLS algorithm. After all particles are placed in the same interval within the search space as given in (41), (42), PSO, PSOA, PSOH, and classical methods start running under the same TDOA measurements. At the end of the simulation, the obtained results are given in Tab. 1.

$$x_{\mathrm{a}} = 50(a\%6) - 125, \quad a = 0 \ldots 35, \tag{41}$$

$$y_{\mathrm{a}} = 50(a\ /\ 6) - 125, \quad a = 0 \ldots 35 \tag{42}$$

where (%) and (/) show the integer remainder and division respectively.

Considering Tab. 1, the MSE of the PSO based algorithms (PSO, PSOA, PSOH) is lower than the classical methods (NLS, ML, LLS, WLLS) [41] and CRLB [40]. The reason of this result is to utilize the independent set for CRLB and classical techniques, while PSO and variants use the full set. Because of the increased number of time differences, PSO's knowledge about the source also increases. From the simulation results, it is also clearly seen that the proposed PSOA and PSOH algorithms reach approximately 15% and 17% lower MSE respectively comparing to the PSO.

N	Mean Square Error							
	CRLB	PSO	PSOA	PSOH	NLS	ML	LLS	WLLS
4	0.688	0.705	0.657	0.610	1.574	1.639	1.566	1.566
5	0.145	0.113	0.103	0.099	0.161	0.169	0.160	0.159
6	0.133	0.115	0.085	0.092	0.152	0.154	0.152	0.137
7	0.114	0.071	0.049	0.050	0.125	0.124	0.126	0.114
8	0.105	0.057	0.047	0.045	0.122	0.121	0.122	0.109
9	0.103	0.050	0.046	0.044	0.119	0.118	0.120	0.106
10	0.094	0.056	0.052	0.051	0.119	0.107	0.121	0.096

Tab. 1. Comparison of MSE for the PSO based techniques, classical methods, and theoretical limit; arbitrary array.

6.2 Localization with Circular Array

In the second part of this section, receivers are located circularly to estimate the target position. Here, the transmitter location is (x = 61, y = -34) and the first receiver position is (x_1 = 10, y_1 = 0). Other receivers are located at equal intervals on a circle of radius 10. TDOA noise power is set to $0.0001/c^2$ and MSE is obtained from the average of 100000 independent trails. Using the same parameters with the previous simulation, new results are given in Tab. 2.

Comparing the previous results, we can easily realize the similarity. The performances of the PSO based techniques are still higher than the classical algorithms and CRLB. Moreover, PSOA and PSOH reach 23% and 22% lower MSE comparing to PSO method.

N	Mean Square Error							
	CRLB	PSO	PSOA	PSOH	NLS	ML	LLS	WLLS
4	1.670	1.044	0.697	0.705	1.776	1.772	1.776	1.776
5	0.971	0.398	0.287	0.289	0.993	0.990	0.994	0.978
6	0.666	0.342	0.255	0.265	0.719	0.715	0.719	0.671
7	0.544	0.315	0.241	0.251	0.609	0.603	0.610	0.541
8	0.472	0.259	0.215	0.216	0.544	0.537	0.545	0.470
9	0.419	0.230	0.192	0.191	0.496	0.488	0.497	0.419
10	0.377	0.208	0.176	0.174	0.459	0.449	0.460	0.376

Tab. 2. Comparison of MSE for the PSO based algorithms, classical techniques, and theoretical limit; circular array.

Finally, the number of time differences used by the PSO based algorithms, classical techniques, and the theoretical lower bound are given in Tab. 3. It is known that the more TDOAs are included in the method, the more computational complexity but better positioning accuracy can be obtained. It is clear that the accuracy of the proposed algorithms is better than the classical methods. However, since the proposed techniques are based on the PSO, their computational complexities are higher than the classical methods [43].

N	Mean Square Error							
	CRLB	PSO	PSOA	PSOH	NLS	ML	LLS	WLLS
4	3	6	6	12	3	3	3	3
5	4	10	10	20	4	4	4	4
6	5	15	15	30	5	5	5	5
7	6	21	21	42	6	6	6	6
8	7	28	28	56	7	7	7	7
9	8	36	36	72	8	8	8	8
10	9	45	45	90	9	9	9	9

Tab. 3. Comparison of number of TDOAs for the PSO based methods, classical algorithms, and theoretical limit.

7. Conclusions

In this study, a significant increase on the positioning accuracy is achieved by using PSO and TDOAA together. As an alternative to PSO on determining the emitter location, the techniques PSOA that uses only averaged time differences and PSOH that uses both estimated and averaged time differences are proposed. Their performances are compared with classical methods and the theoretical limit. Presented simulation results show that the positioning accuracy of PSO based algorithms is better than the classical estimators. MSE of the PSOA and PSOH algorithms are lower than 20% of the PSO. From the point of view of computational complexity, PSOA is approximately the same as PSO. Furthermore, the MSE of the proposed methods are lower than CRLB.

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