Robust and Efficient Uncertainty Quantification and Validation of RFIC Isolation

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Abstract. *Modern communication and identification products impose demanding constraints on reliability of components. Due to this, statistical constraints more and more enter optimization formulations of electronic products. Yield constraints often require efficient sampling techniques to obtain uncertainty quantification also at the tails of the distributions. These sampling techniques should outperform standard Monte Carlo techniques, since these latter ones are normally not efficient enough to deal with tail probabilities. One such a technique, Importance Sampling, has successfully been applied to optimize Static Random Access Memories (SRAMs) while guaranteeing very small failure probabilities, even going beyond 6-sigma variations of parameters involved. Apart from this, emerging uncertainty quantifications techniques offer expansions of the solution that serve as a response surface facility when doing statistics and optimization. To efficiently derive the coefficients in the expansions one either has to solve a large number of problems or a huge combined problem. Here parameterized Model Order Reduction (MOR) techniques can be used to reduce the work load. To also reduce the amount of parameters we identify those that only affect the variance in a minor way. These parameters can simply be set to a fixed value. The remaining parameters can be viewed as dominant. Preservation of the variation also allows to make statements about the approximation accuracy obtained by the parameter-reduced problem. This is illustrated on an RLC circuit. Additionally, the MOR technique used should not affect the variance significantly. Finally we consider a methodology for reliable RFIC isolation using floor-plan modeling and isolation grounding. Simulations show good agreement with measurements.*

Keywords

Monte Carlo, importance sampling, tail probabilities, failure, yield estimation, uncertainty quantification, stochastic collocation, stochastic galerkin, sensitivity, variation aware, parameterized model order reduction, reliability, RFIC isolation, floor-plan modeling, isolation grounding.

1. Introduction

As transistor dimensions become smaller with each new technology generation, they become increasingly susceptible to statistical variations in their parameters. These statistical variations can result in failing memory. Additionally, unintended RF couplings can occur, which also downgrades the quality of the product and thus performance of end products or even safety of environment or of the end-user. Failures directly affect yield of the producing company and its fame for reliable products. Hence there is a general focus on reliability in IC design. In the Artemos project[1], NXP Semiconductors and Eindhoven University of Technology joined effort to tackle this topic. Mathematics was needed to accurately estimate low tail probabilities. Several novel methods were needed to do the simulations in an efficient way. Enhanced floor planning of a design was set up. In all the various tasks one was interested to determine the dominant parameters and the dominant sources that caused a certain effect.

[1]http://www.artemos.eu/

The paper is organized as follows. Section 2 overviews Monte Carlo simulation and comes down to estimation of tail probabilities, based on results from Large Deviation Theory. Section 3 considers the benefits of Importance Sampling. Section 4 gives some achievements obtained during the project. Section 5 describes statistics based on uncertainty quantification, where we combine the techniques with model order reduction and sensitivity analysis. Section 6 deals with reliable RFIC isolation.

2. Monte Carlo Simulations

We start this section with some general, well-known, results from statistics [2, 15]. We assume that N independent random observations Y_i ($i = 1, \ldots, N$) of Y are taken, each with mean μ and variance σ^2, based on a probability density function f. One can estimate the mean μ by the sample mean $\widehat{\mu}_N = \frac{1}{N}\sum_{i=1}^{N} Y_i$ and the variance σ^2 by the sample variance $\widehat{\sigma^2_N} = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \widehat{\mu}_N)^2$. Both, $\widehat{\mu}_N$ and $\widehat{\sigma^2_N}$, can be updated on-the-fly, using recursion. The Central Limit Theorem says that $\widehat{\mu}_N$ converges in distribution to a standard normal distribution, *i.e.*,

$$\lim_{N\to\infty} P\left(\frac{\widehat{\mu}_N - \mu}{\sigma/\sqrt{N}} \le x\right) = \Phi(x) \qquad (1)$$

where $\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy$ is the cumulative distribution function of the standard normal distribution, i.e., the normal distribution with mean 0 and variance 1. In fact, this theorem holds under much weaker conditions, but this is usually not important when performing simulations. Note that Φ is monotonically increasing and that, because of the symmetry of $\Phi(x)$ around 0, we have $\Phi(-x) = 1 - \Phi(x)$.

The Central Limit Theorem yields that we may use the following approximative confidence interval for μ. Let Z be a standard normal variable. In the sequel we will assume that $\alpha < 1/2$. We define z_α to be the unique number such that $P(Z > z_\alpha) = 1 - \Phi(z_\alpha) = \alpha$. Note that $z_\alpha > 0$ and $P(|Z| > z_\alpha) = 2\Phi(z_\alpha) = 2\alpha$. Combining this notation with (1), we obtain

$$\lim_{N\to\infty} P\left(-z_{\alpha/2} < \frac{\widehat{\mu}_N - \mu}{\sigma/\sqrt{N}} < z_{\alpha/2}\right) = \lim_{N\to\infty} P\left(-z_{\alpha/2} < Z < z_{\alpha/2}\right) = 1 - \alpha.$$

If we wish to estimate μ within absolute accuracy ε with $100(1-\alpha)\%$ confidence, then $N \ge z^2_{\alpha/2}\sigma^2/\varepsilon^2$. This result is not useful in practice, since we usually do not know σ. Although (1) also holds with σ replaced by $\widehat{\sigma}_N$ (this is not trivial, it requires Slutsky's Lemma [2, Section 7.7]), this only helps a posteriori unless we have some prior information, like lower and upper bounds.

Fig. 1 shows the powers of the tail accuracy, $\log_{10}(\alpha)$, versus the quantiles z_α of the normal distribution along a σ-scale. Clearly, the z_α vary moderately for $-12 \le \log_{10}(\alpha) \le -1$. For default statistics around 2σ, we have $z_\alpha = 2$. In the following, our interest will concern variations up to 6σ.

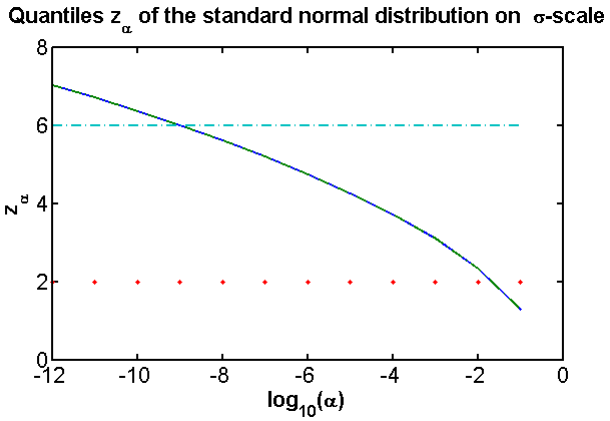

Fig. 1. Powers of tail accuracy, $\log_{10}(\alpha)$,versus quantiles z_α of the normal distribution along a σ-scale. Our interest goes to variations up to 6σ.

We now show how to estimate tail probabilities, rather than the mean, since this is relevant for determining reliability of electronic components. For a given set $A = (-\infty, x)$, we define the event indicator $X_i = I_A(Y_i)$ where $I_A(Y_i) = 1$ if $Y_i \in A$ and 0 otherwise. Then $p_f^{\mathrm{MC}} = \frac{1}{N}\sum_{i=1}^{N} X_i$ estimates $p = \int_{-\infty}^{x} f(z)dz = P(Y \in A)$. The X_i are Bernoulli distributed, hence $Np_f^{\mathrm{MC}} \sim \mathrm{Bin}(N, p)$ is binomially distributed, and thus for the expectation one has $\mathrm{E}(p_f^{\mathrm{MC}}) = \frac{1}{N}Np = p$ and for the variance $\sigma^2(p_f^{\mathrm{MC}}) = \frac{1}{N^2}Np(1-p) = \frac{1}{N}p(1-p)$. Note that, here, we can not directly approximate $\sigma^2(p_f^{\mathrm{MC}})$, like in the general Monte Carlo case. However, similarly to the general case we may replace p, in the expression for $\sigma^2(p_f^{\mathrm{MC}})$, by $\widehat{p} = p_f^{\mathrm{MC}}$, using Slutsky's Lemma [2, Section 7.7]. If we know p then we can estimate the number of Monte Carlo samples we have to take. Using (1), we derive

$$P(|p_f^{\mathrm{MC}} - p| > \varepsilon) = P\left(\frac{|p_f^{\mathrm{MC}} - p|}{\sigma(p_f^{\mathrm{MC}})} > z\right) \xrightarrow{N_{\mathrm{MC}}\to\infty} 2\Phi(-z) \le 2\Phi(-z_{\alpha/2}) \qquad (2)$$
$$= \alpha \qquad (3)$$

where $z = \varepsilon/\sqrt{p(1-p)/N_{\mathrm{MC}}}$ and $N = N_{\mathrm{MC}}$. The convergence holds for all points z in (2) for which the distribution is continuous. In our case it allows to derive an error estimate for a particular value of z, which leads to estimate the number of samples we have to take. Hence, if $z \ge z_{\alpha/2}$, we deduce

$$N_{\mathrm{MC}} \ge p(1-p)\left(\frac{z_{\alpha/2}}{\varepsilon}\right)^2 = \frac{1-p}{p}\left(\frac{z_{\alpha/2}}{\nu}\right)^2, \qquad (4)$$

for $\varepsilon = \nu p$. Here we assume $\nu = 0.1$ and $p = 10^{-10}$. Now let $\alpha = 0.02$. Then $z_{\alpha/2} \approx 2$ and (4) implies $N_{\mathrm{MC}} \ge 4 \cdot 10^{12}$. This is large, but it looks acceptable if we compare it to the small value of p. We see that N_{MC} may grow with $1/p$ and not necessarily with $1/p^2$. A problem arises if we do not

know p. Then more general estimates show up, in which indeed $N_{\mathrm{MC}} = O(1/p^2)$ [8, 16, 17]. To cover this gap in reasonable upper bounds for N_{MC}, results from Large Deviation Theory (LDT) can be used [4, 9, 10]. We assume the following lemma.

Lemma (LDT) The sequence of the Monte Carlo results $P_N(A) := p_f^{\mathrm{MC}}$ satisfies a "Large-Deviation Principle" [4, 9, 10], meaning that there is some "rate function" $I : \mathbb{R} \to \mathbb{R} \cup \{-\infty, +\infty\}$ such that

- (i) $\limsup_{N\to\infty} \frac{1}{N} \ln P_N(C) \leq -\inf_{x\in C} I(x)$ for all closed subsets $C \subset \mathbb{R}$,
- (ii) $\liminf_{N\to\infty} \frac{1}{N} \ln P_N(G) \geq -\inf_{x\in G} I(x)$ for all open subsets $G \subset \mathbb{R}$.

Note the first statement implies an upper bound for $\frac{1}{N} \ln P_N(C)$, for N large enough. From this, we can prove the following theorem [8, 16, 17].

Theorem (MC for tail probabilities) For $N = N_{\mathrm{MC}}$ large enough, the Monte Carlo results $P_N := p_f^{\mathrm{MC}}$ approximate p with a relative precision ν such that

$$P(|p_f^{\mathrm{MC}} - p| > \nu p) \leq \exp\left(-\frac{N_{\mathrm{MC}}}{2}\frac{p}{1-p}\nu^2\right). \tag{5}$$

The exponential type of bound in (5) is also valid from below and thus is sharp. For $\nu = 0.1$, $p = 10^{-10}$ and $\alpha = 0.02$, as above, we find: $N_{\mathrm{MC}} \geq 10^{12}$. Note that an extra k-th decimal in ν increases N_{MC} with a factor k^2. Indeed, this lower bound for N_{MC} is close to the one found with (4). Consequently, the result (5) is sharp. It means that in general one really needs $O(10^{12})$ samples, and, in general, $N_{\mathrm{MC}} \approx 1/p$.

3. Importance Sampling

There are several methods to speed up Monte Carlo sampling. In [5] antithetic variables, control variates, matching moment technique, and stratification are exploited. Here we will describe Importance Sampling. It fits the estimation of tail probabilities very well and is based on the observation that $p_f(A) = \int_{-\infty}^{x} f(z)dz = \int_{-\infty}^{x} \frac{f(z)}{g(z)} g(z)dz$ for any distribution function g (called design distribution) that satisfies $g(z) \neq 0$ on A. Hence, we sample the Y_i according to a different distribution function g rather than according to f and define a weighted success indicator $V = V(A) = I_A(Y)f(Y)/g(Y)$.

Then with the g-distribution we have for the expectation $\mathrm{E}_g(V) = \int I_A(y)\frac{f(y)}{g(y)} g(y)\,dy = \int_{-\infty}^{x} f(z)dz = p_f(A)$. Hence if we determine $V_i = I_A(Y_i)f(Y_i)/g(Y_i)$ from the g-distributed Y_i we can define $p_g^{\mathrm{IS}} = p_g^{\mathrm{IS}}(A) = \frac{1}{N}\sum_{i=1}^{N} V_i$. Its expectation becomes $\mathrm{E}_g\left(p_g^{\mathrm{IS}}\right) = \frac{1}{N}\sum_{i=1}^{N} \mathrm{E}_g\left(V_i\right) = p_f(A)$, which is unbiased. Note that this re-sampling may already be a benefit: sampling according to a known and simple g may be more efficient than sampling according to a density f that involves more calculations.
We can easily prove [16, 17]

$$\frac{f(z)}{g(z)} \leq 1 \text{ on } A \implies \mathrm{Var}_g\left(p_g^{\mathrm{IS}}\right) \leq \mathrm{Var}_f\left(p_f^{\mathrm{MC}}\right). \tag{6}$$

Thus we obtain variance reduction using the same number of samples as used for Monte Carlo sampling. We will sharpen the amount of reduction shortly, in (11). The variance reduction does not yet imply more efficiency. However, similarly to (5), we derive (7), in the next theorem (in which $N_{\mathrm{IS}} = N$), for N_{IS} large enough [16, 17].

Theorem (IS for tail probabilities) For $N = N_{\mathrm{IS}}$ large enough, the Importance Sampling results $P_N := p_g^{\mathrm{IS}}$ approximate p with a relative precision ν such that

$$P\left(\left|p_g^{\mathrm{IS}} - p\right| > \nu p\right) \leq \exp\left(-\frac{N_{\mathrm{IS}}\, p^2}{2\mathrm{Var}_g(V)}\nu^2\right). \tag{7}$$

Also this result is sharp as it was for (5). Comparing (5) and (7), we see the same type of exponential decay as a function of N. So an improvement for Importance Sampling should come from a proper choice of the distribution function g. Assuming the same upper bounds values in (5) and (7), comparing them we obtain

$$\frac{N_{IS}}{N_{MC}} = \frac{\mathrm{Var_g}(V)}{p(1-p)} = \frac{\mathrm{E}_g(V^2) - p^2}{p(1-p)}. \tag{8}$$

This expression can also be obtained by equating the normalized standard deviations $\sigma(p_f^{\mathrm{MC}})/\mathrm{E}(p_f^{\mathrm{MC}})$ and $\sigma_g(p_g^{\mathrm{IS}})/\mathrm{E}_g(p_g^{\mathrm{IS}})$. However, the way via (5) and (7) indicates the sharpness. Next, we consider the variance reduction (6) more closely. For this, suppose

$$\frac{f(z)}{g(z)} \leq \kappa < 1, \text{ on } A. \tag{9}$$

Then $p = \int_{-\infty}^{x} f(z)dz \leq \kappa \int_{-\infty}^{x} g(z)dz \leq \kappa$. With $q = 1 - p$, we obtain [16, 17]

$$\frac{N_{IS}}{N_{MC}} = \frac{\mathrm{E}_g(V^2)}{pq} - \frac{p}{q} \leq \frac{\kappa}{q} - \frac{p}{q} \leq \kappa(1+\zeta), \tag{10}$$

when $|(1-\frac{1}{\kappa})p + O(p^2)| \leq \zeta$. For $\kappa = 0.1$ and $p = 10^{-10}$ this means that $\zeta \leq 10^{-9}$. Hence, for $\kappa = 0.1$, we can take an order less samples with Importance Sampling to get the same accuracy as with regular Monte Carlo. This even becomes better with smaller κ. By Importance Sampling we gain efficiency; this is the main message. Also the asymptotic accuracy improves when compared to regular Monte Carlo, but the improvement is less impressive than for the efficiency. We can derive an enhanced variance reduction [16, 17]

$$\mathrm{Var}_g\left(p_g^{\mathrm{IS}}\right) \leq \kappa \mathrm{Var}_f\left(p_f^{\mathrm{MC}}\right) - \frac{1-\kappa}{N}p^2 \tag{11}$$

and thus $\sigma_g\left(p_g^{\mathrm{IS}}\right) \leq \sqrt{\kappa}\sigma_f\left(p_f^{\mathrm{MC}}\right)$, which for $\kappa = 0.1$ means that here not an order is gained, but a factor $\sqrt{\kappa} \approx 0.316$.

In the following we assume that $A = (-\infty, x)$ with $x < 0$ and $|x|$ not too small. The condition (9) can usually easily be satisfied if f is a Gaussian probability density distribution and g has a broader or a shifted Gaussian distribution, with enough density on A. Let us consider $f(z) \sim N(0,1)$ and $g_\sigma(z) \sim N(0,\sigma^2)$, with $\sigma \geq 1$ (hence a broader distribution than f).

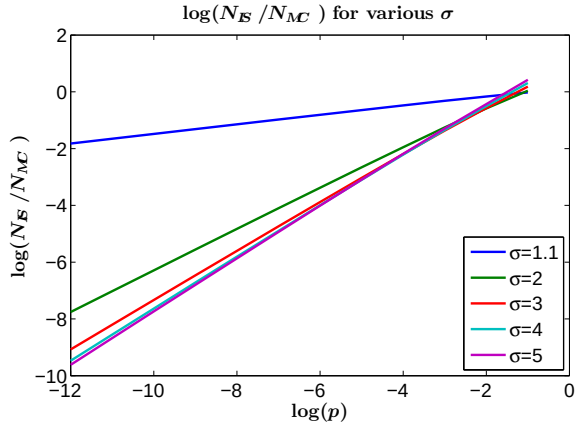

Fig. 2. $\log_{10}(N_{\rm IS}/N_{\rm MC})$ versus $\log_{10}(p)$ for $f(z) \sim N(0,1)$ and $g(z) \sim N(0,\sigma^2)$.

Fig. 2 shows the speed up that one can obtain. The figure also indicates convergence with respect to increasing σ. Indeed $\tilde{h}(\sigma,x) = \frac{f(x)}{g_\sigma(x)}$ has a minimum when $\sigma^2_{\rm opt} = x^2 = z^2_\alpha$. For $\alpha = p = 10^{-10}$, we find $z_a = 6.4$, giving $\log_{10}(N_{IS}/N_{MC}) = -8.5$ and thus an optimal speed up of $3.2 \cdot 10^8$. This is an example of parameterized Importance Sampling, in which an additional parameter (here σ) is used to optimize the outcome. Note that with $g_{\sigma_{\rm opt}}(x)$ still a significant fraction

$$\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma_{\rm opt}}\int_{-\sigma_{\rm opt}}^{\infty} e^{-\frac{x^2}{2\sigma^2_{\rm opt}}}\,dx = \frac{1}{\sqrt{2\pi}}\int_{-1}^{\infty} e^{-\frac{x^2}{2}}\,dx = 0.8413$$

is sampled outside $A = (-\infty, x)$; $g_{\sigma_{\rm opt}}(-z_\alpha) = \frac{1}{\sqrt{z_\alpha e}}$. Only 15 % falls within A. Surprisingly even this results in a much higher efficiency for Importance Sampling over ordinary Monte Carlo sampling. Note that in this example the simple choice $\sigma = 2$ already gives good results (thus we may use a sampling with a distribution that is double as wide as that of f).

Of course, all this reasoning assumes that each sample requires the same amount of cpu.

4. Parameterized Quantities

In several simulations the nonlinear output response $Y(\mathbf{p})$ depends on independent input parameters $\mathbf{p} = (p_1, \ldots, p_P)^T$ with known density distribution functions f_k for each p_k (in most cases a normal distribution). In this case the ratio "$f(\mathbf{p})/g(\mathbf{p})$" has to be considered in $\mathbf{p}$-space, where f is known and thus the ratio can easily be calculated. Of course, in a multi-dimensional parameter space the definition of $g(\mathbf{p})$ that should cover the area of parameters for the rare events of interest, requires more attention. With increasing dimension of the parameter space, importance sampling can have more impact. Assuming $\mathbf{p} = (p_1, p_2)^T$, a scalar function $Y(\mathbf{p})$ and samples $Y_i = y(\mathbf{p}^i) = y(p^i_1, p^i_2)$ in which the input parameters p^i_k are chosen according to density f_k. If the input parameters p_k are independent, we have $p_f(A) = \iint_A f_1(p_1) f_2(p_2) d\mathbf{p}$, in which A is identified with a 2-D $\mathbf{p}$-area such that $Y(\mathbf{p}) > Y_{\rm lim}$ (or $< Y_{\rm lim}$).

The indicator function is now defined by

$$I_A(y) = I_A(\mathbf{p}) = \begin{cases} 1 & \text{if} \quad \mathbf{p} \in A, \text{ i.e. if } Y(\mathbf{p}) > Y_{\rm lim} \\ 0 & \text{else} \end{cases} \tag{12}$$

and similar as before one can estimate $p_f(A)$ by

$$p_f^{\rm MC}(A) \approx \frac{1}{N}\sum_{i=1}^{N} I_A(Y_i), \tag{13}$$

$$p_g^{\rm IS}(A) \approx \frac{1}{N}\sum_{i=1}^{N} I_A(Y_i) * \prod_{k=1}^{P} \frac{f_k(p^i_k)}{g_k(p^i_k)}. \tag{14}$$

Note that, in practice, on A, not all factors f_k/g_k in (14) may be less than or equal to 1. In [6, 8, 17] SRAM (Static Random Access Memory) cells were considered. The threshold voltages V_t of the six transistors in an SRAM cell are the most important parameters causing variations of the characteristic quantities of an SRAM cell [6] like Static Noise Margin (SNM) and Read Current ($I_{\rm read}$). Hence $\text{SNM} = \text{SNM}(V_{t,1}, \ldots, V_{t,6})$ and $I_{\rm read} = I_{\rm read}(V_{t,1}, \ldots, V_{t,6})$. In [6, 16] Importance Sampling (IS), using Gaussian distributions with a $\sigma = 3\sigma_{V_t}$ for the V_t-s in each transistor in the SRAM cell, was used to accurately and efficiently estimate low failure probabilities for SNM and $I_{\rm read}$. $\text{SNM} = \min(\text{SNM}_h, \text{SNM}_l)$ is a measure for the read stability of the cell. SNM_h and SNM_l are identically Gaussian distributed. The min() function provides a non-linear operation after which the distribution of SNM is no longer Gaussian. However, in this particular case, one can argue that $\text{SNM} = 2\,\text{SNM}_h$ [6, 16]. For results, see [8].

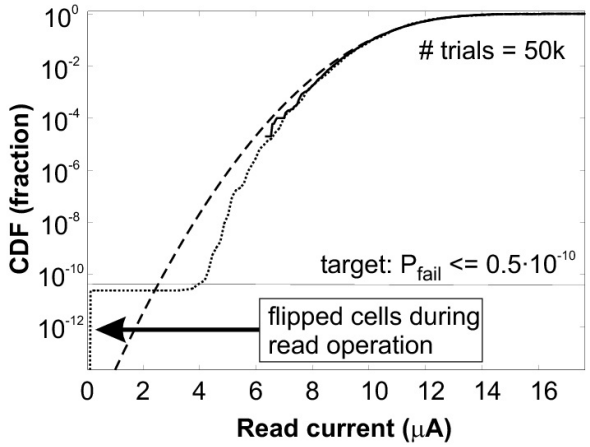

Fig. 3. Cumulative distribution function for the Read Current $I_{\rm read}$ based on extrapolated MC (dashed), regular MC (solid) and IS (dotted) [8]. Extrapolation assumes a normal distribution.

The Read Current I_{read} is a measure for the speed of the memory cell. This quantity has a non-Gaussian distribution and the cumulative distribution is shown in Fig. 3 [8]. Here IS is essentially needed for sampling I_{read} appropriately. Regular MC can only simulate down to a failure probability $P_{\text{fail}} = p_f(A) \leq 10^{-5}$.

In [7, 8, 17] the access time optimization for an SRAM active column in combination with a sense amplifier was considered. For robust design of SRAM memories, it is not sufficient to only guarantee good statistical margins on the individual SRAM cell parameters. The additional sense amplifier also needs sufficient input signal before it can reliably sense the data, while the SRAM cell requires sufficient time to develop that input signal. Here Importance Sampling was used for the sampling in generating the voltage differences ΔV_k at the output of the SRAM cells. As constraint one can guarantee a yield target set by the designer. Using this method, the access time of a 45 nm high performance SRAM could be improved 6 %, while simultaneously reducing the size of the sense amplifier.

5. Uncertainty Quantification

We consider Uncertainty Quantification (UQ) by expanding the solution in so-called generalized Polynomial Chaos expansions. In these expansions the solution is decomposed into a series with orthogonal polynomials in which the parameter dependency becomes an argument of the orthogonal polynomial basis functions. The time and space dependency remains in the coefficients. In UQ two main approaches are in use: Stochastic Collocation (SC) and Stochastic Galerkin (SG). In SC the coefficients in the expansion are approximated by quadrature and thus lead to a large series of deterministic simulations for several parameters. In SG one assumes a finite sum of the expansion as approximation to the solution and requires that the vector of residuals is orthogonal to all basis functions used in the finite expansion (using an inner product in parameter space), which leads to one big, but coupled, system. Also here quadrature can be applied but this does not automatically lead to decoupling as happens for SC.

We will denote parameters by $\mathbf{p} = (p_1, \ldots, p_P)^T$ again and assume a probability space $(\Omega, \mathcal{A}, \mathcal{P})$ given where $\mathcal{A}$ represents a σ-algebra, $\mathcal{P} : \mathcal{A} \to \mathbb{R}$ is a measure and $\mathbf{p} = \mathbf{p}(\omega) : \Omega \to B \subseteq \mathbb{R}^P$. Here we will assume that the p_i are independent.

For a function $f : B \to \mathbb{R}$, the mean or expected value is defined by

$$\mathbf{E}_p[f(\mathbf{p})] =< f >= \int_\Omega f(\mathbf{p}(\omega)) \mathrm{d}\mathcal{P}(\omega) = \int_B f(\mathbf{p})\, \rho(\mathbf{p}) \mathrm{d}\mathbf{p}. \tag{15}$$

The specific probability distribution density is defined by the function $\rho(\mathbf{p})$. A bilinear form $< f, g >$ (with associated norm L^2_ρ) is defined by

$$< f, g >= \int_B f(\mathbf{p})\, g(\mathbf{p})\, \rho(\mathbf{p}) \mathrm{d}\mathbf{p} =< f\, g > . \tag{16}$$

We assume a complete orthonormal basis of polynomials $(\phi_i)_{i\in\mathbb{N}}$, $\phi_i : B \to \mathbb{R}$, given with $< \phi_i, \phi_j >= \delta_{ij}$ $(i, j, \geq 0)$. When $P = 1$, ϕ_i has degree i. To treat a uniform distribution (i.e., for studying effects caused by robust variations) Legendre polynomials are optimal in some sense; for a Gaussian distribution one can use Hermite polynomials [12, 28]. A polynomial ϕ_i on $\mathbb{R}^P$ can be defined from one-dimensional polynomials: $\phi_i(\mathbf{p}) = \prod_{d=1}^P \phi_{i_d}(p_d)$. Actually i orders a vector $\mathbf{i} = (i_1, \ldots, i_P)^T$.

A solution $\mathbf{x}(t, \mathbf{p}) = (x_1(t, \mathbf{p}), \ldots, x_n(t, \mathbf{p}))^T$ of a dynamical system (which we do not further specify) becomes a random process. We assume that second moments are finite: $< x_j^2(t, \mathbf{p}) > < \infty$, for all $t \in [t_0, t_1]$ and $j = 1, \ldots, n$. We express $\mathbf{x}(t, \mathbf{p})$ in a Polynomial Chaos expansion

$$\mathbf{x}(t, \mathbf{p}) = \sum_{i=0}^{\infty} \mathbf{v}_i(t)\, \phi_i(\mathbf{p}) \tag{17}$$

where the coefficient functions $\mathbf{v}_i(t)$ are defined by

$$\mathbf{v}_i(t) =< \mathbf{x}(t, \mathbf{p}), \phi_i(\mathbf{p}) > . \tag{18}$$

Here the inner product is considered component wise. A finite approximation $\mathbf{x}^m(t, \mathbf{p})$ to $\mathbf{x}(t, \mathbf{p})$ is defined by

$$\mathbf{x}^m(t, \mathbf{p}) = \sum_{i=0}^{m} \mathbf{v}_i(t)\, \phi_i(\mathbf{p}). \tag{19}$$

When exploiting Stochastic Collocation (SC), the integrals (18) are computed by (quasi) Monte Carlo, or by multi-dimensional quadrature. We assume quadrature grid points $\mathbf{p}^k$ and quadrature weights w_k, $0 \leq k \leq K$, such that

$$\mathbf{v}_i(t) =< \mathbf{x}(t, \mathbf{p}), \phi_i(\mathbf{p}) > \approx \sum_{k=0}^{K} w_k\, \mathbf{x}(t, \mathbf{p}^k)\, \phi_i(\mathbf{p}^k). \tag{20}$$

Typically, for low numbers of random parameters, Gaussian quadrature is used with corresponding weights. We solve the dynamical system for $\mathbf{x}(t, \mathbf{p}^k)$, $k = 0, \ldots, K$ $(K + 1$ deterministic simulations). By post-processing we determine the $\mathbf{v}_i(t)$ in (20).

As alternative to SC, Stochastic Galerkin (SG) can be used. One puts the approximation (19) in the equations of the dynamical system and makes the residuals orthogonal to each basis function used. The result is a big system that involves all coefficients $\mathbf{v}_i(t)$, $0 \leq i \leq m$, as unknowns. For linear dynamical systems one can determine all integrals over B exactly, in advance. For nonlinear systems one may approximate these again by quadrature, similar as done for SC.

After determining the approximation (19) by SC or by SG, the expansion provides a response surface facility from which the solution can be determined for any values of t and $\mathbf{p}$. It also provides (fast) information about mean, variance and sensitivity. In [1, 18, 20] efficient methods are described to determine the coefficients by SC. In [18, 23, 25] also the combination with (parameterized) Model Order Reduction (MOR) was studied.

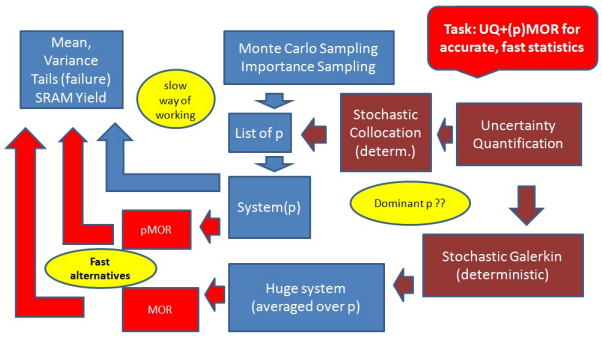

Fig. 4. The various ways to obtain statistic information.

In Fig. 4 at the top-left Monte Carlo Sampling and Importance Sampling generate a list of samples of $\mathbf{p}$ for which the dynamical system has to be solved, after which statistical analysis can be done. At the top-right the alternative path by UQ is indicated. Stochastic Collocation provides a list of deterministic values $\mathbf{p}$ for which the dynamical system has to be solved. In both cases parameterized MOR (pMOR) can be of help to faster provide approximations. The path via Stochastic Galerkin results in a huge system that involves all coefficients. The system is independent of $\mathbf{p}$, due to averaging. Here normal MOR can be of use - it may be even necessary to reduce the huge system to be able to obtain approximative solutions.

In [13, 18, 22] (response surface) approximations or the combination between them and more accurate solutions was studied for Failure Analysis. In [20] the method can shift the (probability density) weighting function in the inner product to the area of interest (shifted Hermite chaos). One can also use a windowed Hermite chaos. The shift is tuned by some optimization procedure. The windowed Hermite chaos looks to be the most accurate alternative. In [21] various multi-dimensional integration methods have been studied for the purpose of efficient reliability analysis.

Central in Fig. 4 is the question on dominant parameters. In [23, 24] the sensitivity coefficients of parameters to the variance of the solution have been studied via a Sobol decomposition [26] and using uniform distributions. Assuming a scalar solution x in (17), the variance of x (at time t) reads as

$$\mathrm{Var}_x(t) = \sum_{i=1}^{\infty} v_i^2(t).$$

The total normalized sensitivity of the j-th random parameter can be written as

$$S_j := \frac{V_j}{\mathrm{Var}_x}, \text{ with } V_j := \sum_{i\in I_j} v_i^2, \text{ for } j = 1,\ldots,P. \qquad (21)$$

Here $i \in I_j$ if and only if ϕ_i varies with respect to the random variable p_j , i.e., ϕ_i includes a non-constant univariate polynomial in p_j . Clearly the bounds $0 \le S_j \le 1$ apply for each j. One obtains approximations of these total normalized sensitivities by a truncated expansion

$$V_j^D := \sum_{i\in I_j^D} v_i^2, \text{ with } I_j^D := \{i \in I_j \,:\, \mathrm{degree}(\phi_i) \le D\}.$$

Although the bounds $1 \le S_1 + \cdots + S_P \le P$ hold, the sum of the total normalized sensitivities is often close to the lower bound. In view of this variability of the sum of sensitivities, we further normalize

$$S_j^* := S_j \left(\sum_{l=1}^{P} S_l\right)^{-1}, \quad j = 1,\ldots,P. \qquad (22)$$

We now have $S_1^* + \cdots + S_P^* = 1$. This facilitates to compare the S_j^*. After determining the dominant S_j^* [23] we only deal with the S_j. Note that Var_x and the S_j vary with t. If we assume a partitioning (possibly after re-ordering) $\mathbf{p} = (\mathbf{q},\mathbf{r})$, where $\mathbf{q}$ are P_{red} parameters that will be allowed to vary, while $\mathbf{r}$ are the parameters set to a fixed value $\mathbf{r}_0$, we obtain, for the error $\delta(t,\mathbf{r}_0)$ in doing this, the estimate in the following theorem [23, 24].

Theorem (Scaled approx. error after fixing parameters)

$$\begin{aligned} \delta^2(t,\mathbf{r}_0) &= \frac{< \left(x(t,\mathbf{q},\mathbf{r}) - x(t,\mathbf{q},\mathbf{r}_0)\right)^2 >}{\mathrm{Var}_x(t)}, \\ &\le (1+\varepsilon^{-1}) \sum_{j=P_{\mathrm{red}}+1}^{P} S_j(t). \end{aligned} \qquad (23)$$

In (23), ε is a confidence parameter, and $\mathbf{q} \in \mathbb{R}^{P_{\mathrm{red}}}$ and $\mathbf{r}_0 \in B_\varepsilon$ with $\mathcal{P}(\mathbf{p}^{-1}(\mathbb{R}^{P_{\mathrm{red}}} \times B_\varepsilon)) \ge 1-\varepsilon$ (here $\mathbf{p}^{-1}$ is the inverse mapping in the sense of $\mathbf{p}^{-1}(C) := \{\omega \in \Omega : \mathbf{p}(\omega) \in C\}$). Note that $\delta^2(t,\mathbf{r}_0)$ is scaled by Var_x. It assumes that $\mathrm{Var}_x(t)$ is bounded away from 0. Clearly, if $\mathrm{Var}_x(t)$ is bounded, small S_j, corresponding to the parameters $\mathbf{r}$ set to $\mathbf{r}_0$, lead to an upper bound for the mean of the squared approximation error. In [23] the parameter reduction is considered for the transfer function $H(s,\mathbf{p})$ where $s \in i\mathbb{R}$ on the imaginary axis. Now, first an approach similar to (23) is applied to the transfer function $H(s,(\mathbf{q},\mathbf{r}_0))$ after splitting $\mathbf{p}$, resulting in an error estimate for $\delta_H^2(s,\mathbf{r}_0)$ and leading to a mean squared error $< |H(s,\mathbf{q},\mathbf{r}) - H(s,\mathbf{q},\mathbf{r}_0)|^2 >$. From this an upper bound for $\max_{t>0} < x(t,\mathbf{q},\mathbf{r}) - x(t,\mathbf{q},\mathbf{r}_0) >$, for the solution $x(t,\mathbf{p})$ in the time domain, can be derived.

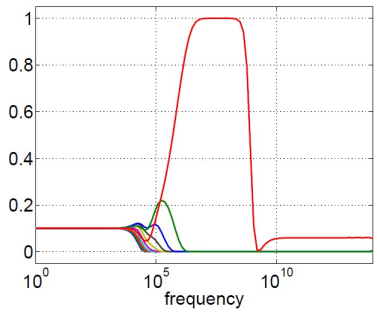

Fig. 5. Normalized variation sensitivities of $H(s,\mathbf{p})$ for conductances as random parameters [23].

For an RLC-circuit, Fig. 5 shows a typical outcome for the variation sensitivities of $H(s,\mathbf{p})$ of various conductances as random parameters. Similar results can be shown for capacitances and for inductances. By this we obtain error estimates for the coefficients in the generalized polynomial

chaos expansion by which we can provide error plots of the mean and of the variance (as functions of time).

Next, parameterized MOR on $H(s,(\mathbf{q},\mathbf{r}_0))$ can be applied, which leads to an additional error. In [23] a typical Krylov-subspace MOR-technique was used after first applying Stochastic Galerkin. As an alternative, Balanced Truncation MOR techniques could be applied, which provide error estimation. This type of MOR-technique leads to an L_2-norm estimate (in time). In [25] the authors did focus on Stochastic Galerkin and considered first reducing the original system by parameterized MOR, followed by SG, versus first applying SG on the original system, followed by a (global) MOR.

At Eindhoven University of Technology, several Ph.D. theses on Model Order Reduction have been done during the last years, with emphasis on sparsity and multi-terminal problems [11, 27], with application to coupled systems by exploiting low-rank approximation [14], with application to nonlinear problems [3], and use of MOR within optimizaton [19]. The sensitivity technique described in this section can lead to variation-aware MOR approaches. Clearly, MOR should not lead to reduced models that do not preserve the main statistical characteristics of the full model.

6. Reliable RFIC Isolation

In order to minimize interference issues and coupling effects in RF products, it is essential to apply proper floorplanning and grounding strategies. The interaction of the IC with its physical environment needs to be accounted for, so as to certify that the final packaged and mounted product meets the specifications.

The first focus was on the key requirements to address physical design issues in the early design phases of complex RF designs. Typical physical design issues encountered, such as on-chip coupling effects, chip-package interaction, substrate coupling and co-habitation, were investigated.

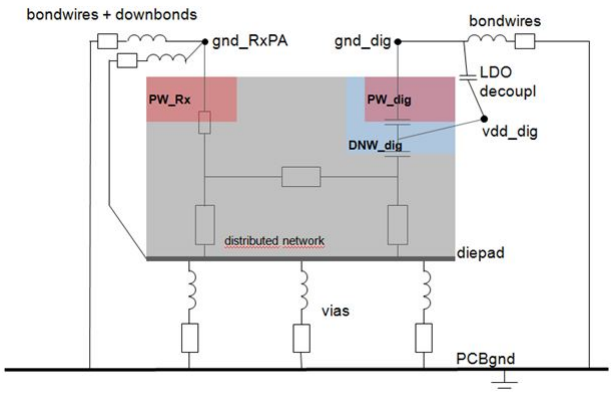

Fig. 6. Floorplan model for isolation and grounding strategies.

The main challenges are the first order prediction of cross-domain coupling. Therefore we apply a floorplan methodology to quantify the impact of floorplanning choices and isolation grounding strategies. This methodology is based on a very high level floorplan EM/circuit simulation model, including the most important interference contributors and including on-chip, package and PCB elements, to be applied in the very early design phases (initial floorplanning).

The overall model of a complete RF product contains the following parts (see Fig. 6):

- On-chip: domain-regions, padring, sealring, splittercells, substrate effects.
- Package: ground and power pins, bondwires/downbonds, exposed diepad.
- PCB: ground plane and exposed diepad connections.

The effect of the number of parameter variations on the impact of noise from digital parts on the isolation sensitive RF domains has been investigated, i.e., the number of downbonds, the number of ground pins, the domain spacing and shape, the application of deep-Nwell and exposed diepad, and the number of exposed diepad vias.

Key to the investigations is the cross-domain transfer function from the digital to RF domain. First a reference situation (I_{core} to Voltage at domain grounds) has been chosen, with which all other situations have been compared.

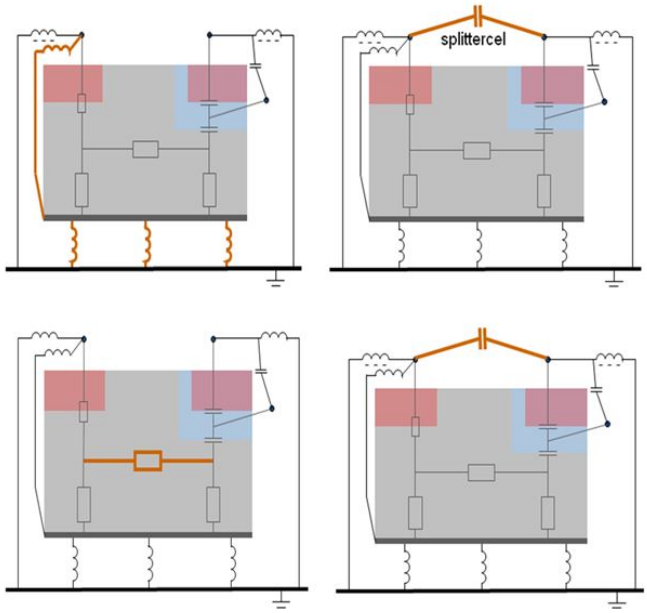

Fig. 7. Coupling paths: 1. (Top-Left) Exposed diepad & downbonds 2. (Top-Right) Splittercells 3. (Bottom-Left) Substrate 4. (Bottom-Right) Air.

The specific coupling paths that have been identified and investigated are (see Fig. 7, row-wise ordered)

1. Via the exposed diepad and downbonds.
2. Via the splitter cells.
3. Via the substrate.
4. Through the air.

We describe these couplings in more detail in the next subsections.

6.1 Exposed Diepad & Downbonds

From the investigations, the following conclusions can be drawn. Without downbonds the isolation is independent of the diepad impedance (up to 300 MHz). With downbonds we find that the isolation is determined by the diepad–PCB ground impedance. In this case as many PCB ground - exposed diepad vias as possible should be used. Furthermore we see that the frequency range of isolation increases.

6.2 Splittercells

The main conclusion from the investigation is that a splitter cell only impacts the domain to domain coupling (between neighbours). There is no impact on the coupling of the digital domain to RF (< 1 GHz).

6.3 Substrate

A number of investigations have been performed. First of all the substrate can be described by a resistive network. This is mainly fixed by the specific technology choice. Coupling can be minimized by maximizing the lateral resistance. Several options can be used to ensure this, such as minimizing the domain boundary length, usage of so-called "pwell-prot" at the boundary between domains to ensure high resistivity between domains, and increasing the domain spacing. However, the latter has very limited impact on domain-domain coupling and no impact on the coupling of digital to RF.

6.4 Air

The capacitive coupling via air (plastic mold, $\varepsilon \approx 4$) between domains is determined (from electromagnetic simulation) to be negligible compared to the other coupling elements in the network. Simulations show some impact when the coupling capacitances would be 100-1000 times higher.

Additional coupling paths and measures are via sealring downbond to diepad (see Fig. 8) and using domain buffers, but investigation shows negligible effect.

Overall conclusions of the coupling path investigation are:

- The digital - RF main coupling path is via the exposed diepad and downbonds.
- Downbonds are effective only in combination with a low-ohmic connection of exposed diepad to PCB ground. When this impedance is too high, downbonds have an adverse effect, serving as a coupling path to the RF; removing the downbonds then improves the isolation.
- Isolation of digital - RF is not impacted by inter-domain spacing, downbonding the sealring, splittercel capacitance (< 1 GHz) or domain buffers (< 1 GHz).

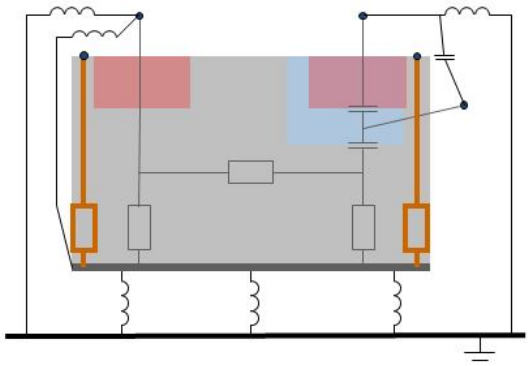

Fig. 8. Coupling paths: 5. Sealring downbond to diepad.

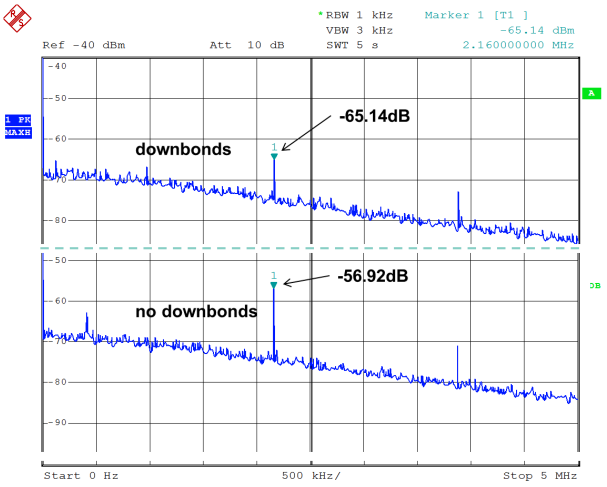

Fig. 9. Measurement result showing downbonds improve isolation.

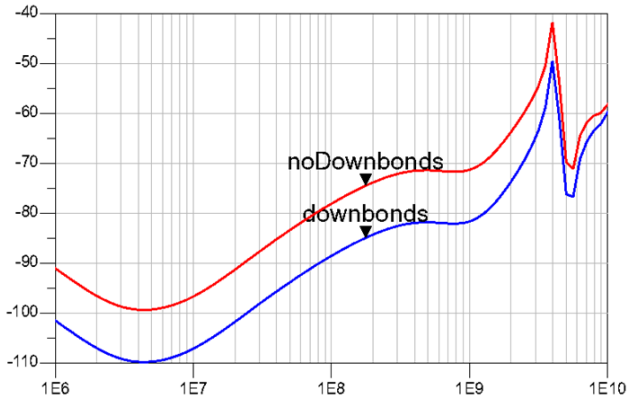

Fig. 10. Downbonds improve isolation shown by simulations.

The modeling methodology predicting RFIC interference issues allows investigation into various floorplan options and verification of isolation and grounding strategies. Application of this modeling methodology guides in making well quantified choices and trade-offs in the implementation of RF products, ultimately enabling achievement of single-pass design success, avoiding costly re-spins and loss of market opportunities. As a first verification of the methodology, measurements were performed of the impact of the downbonds on the coupling, showing indeed an isolation improvement of > 8 dB (see Fig. 9 and Fig. 10, which compare well).

7. Conclusions

Importance Sampling has very effectively been exploited in SRAM design and certainly will also be used in other IC-design processes. Recently, Eindhoven University of Technology, Fachhochschule Oberösterreich (Hagenberg im Mühlkreis, Austria) and Bergische Universität Wuppertal did start work on Importance Sampling for Communication Systems, where one aims for low Bit Error Rates (BER). First outcomes show promising results.

Uncertainty Quantification has led us to identify parameters that mostly contribute to the variation of an output quantity. In [14, 18] other concepts of 'dominant' parameters have been described. Reduced models should preserve the main statistical characteristics of the full model.

The modeling methodology predicting RFIC interference issues, presented here, allows investigation into various floor planning options and verification of different isolation- and grounding strategies. It shows a good agreement of model predictions and measurements. Application of this modeling methodology guides in making well quantified design choices and trade-offs.

Acknowledgements

The 1st, 2nd and 4th author did part of the work within the project ARTEMOS (Ref. 270683-2), *"Agile RF Transceivers and front-Ends for future smart Multi-standard cOmmunications applicationS"*, http://www.artemos.eu/ (ENIAC Joint Undertaking).

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