Improved Energy Detector for Wideband Spectrum Sensing in Cognitive Radio Networks

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Abstract. *In this paper, an improved energy detector for wideband spectrum sensing is proposed. For a better detection of the spectrum holes the overall band is divided into equal non-overlapping sub-bands. The main objective is to determine the detection thresholds for each of these subbands jointly. By defining the problem as an optimization problem, we aim to find the maximum aggregated opportunistic throughput of cognitive radio networks. Introducing practical constraints to this optimization problem will change the problem into a convex and solvable one. The results of this paper show that the proposed improved energy detector will increase the aggregated throughput considerably.*

Keywords

Cognitive radio, improved energy detector, joint detection, spectrum sensing.

1. Introduction

One of the major challenges confronted in cognitive radio networks research literature is wideband spectrum sensing, for the determination of spectrum holes. The spectrum holes are used to allow opportunistic access to the users of cognitive radio networks in order to improve efficient spectrum utilization in these networks. However, allocating opportunistic access to the secondary users will require sophisticated hardware and software algorithms.

An existing approach is to use a tunable narrowband bandpass filter at the radio frequency (RF) front-end to search for only one band in the frequency spectrum over which existing narrowband spectrum sensing techniques can be applied [1]. Search over a wideband frequency spectrum requires a wideband RF front end architecture and this in turn requires a spectrum sensing algorithm that uses an estimate of the power spectral density (PSD) [2]. In order to improve the opportunistic spectrum access and to avoid interference by the primary network, the free and occupied bands should be distinguished in a short processing time.

In the literature of cognitive radio spectrum sensing, energy detectors are of primary interest because of their simple implementation and the lack of need for extra information about the primary users [3], [4]. However, the simple implementation of these detectors limits their performances [5], [6]. Therefore, to improve the performances of these detectors an improved energy detector has been proposed in [7]. With a modification of conventional energy detector, the new detector will preserve the simplicity of the structure but will improve the detection accuracy.

In this paper, a wideband spectrum sensing scheme using improved energy detectors is proposed. The aim is to maximize the aggregated opportunistic throughput of the cognitive radio network while keeping interference to the primary users minimal. This is done by subdividing the frequency band into equal and non-overlapping sub-bands [2]. Therefore, we need to find the energy thresholds simultaneously for each sub-band which allow us to determine whether primary users are active or not.

To achieve the aforementioned objectives, the energy thresholds should be optimized considering all the imposed constraints which will be explained later. In this paper we assume that there exists only one secondary user in the network. The following two parameters are needed, the first parameter is the aggregated data that can be sent through all the sub-bands which is called aggregated opportunistic throughput and the second parameter is the aggregated interferences from the secondary user to the primary user. The chosen thresholds will result in maximum aggregated opportunistic throughput. There exist two parameters regarding false alarm and miss-detection which should not exceed some given values.

The paper is structured as follows: In Section 2 a model for wideband spectrum sensing is described. This is followed by the description of improved energy detectors for wideband spectrum sensing in Section 3. Section 4 shows the analysis of the aggregated opportunistic throughput for a cognitive radio network. In Section 5 of this paper, the simulation results are presented and, finally, Section 6 concludes the paper.

2. System Model

The bandwidth W allocated to a primary network will be partially occupied by the primary users and this partial occupation of the bandwidth will allow for opportunistic access to secondary user, however, this requires detection of sub-bands. In order to detect the unused sub-bands of the spectrum, the given bandwidth is subdivided to K equal and non-overlapping sub-bands. The transmitted signal by the primary user is received by the secondary user and can be written as follows:

$$z(n) = \sum_{l=1}^{L-1} h(l)s(n-l) + v(n) \tag{1}$$

where $l = 1,2,\ldots,L$ and $h(l)$ is the impulse response of the channel between the primary transmitter and the secondary receiver, $s(n)$ is the transmitted signal from the primary user and $v(n)$ is the noise. It is assumed that noise and signal have Gaussian distribution with zero mean and σ_v^2, σ_s^2 variances, respectively. In a multipath fading environment, since the multipath delay spread is comparable to the transmitted signal duration, the wideband wireless channel exhibits frequency selectivity [8]. For each of the sub-bands the received signal and the channel frequency response can be represented in the frequency domain using the Discrete Fourier Transform (DFT):

$$\begin{aligned} Z_k &= \frac{1}{\sqrt{K}} \sum_{n=0}^{K-1} z(n) e^{-j2\pi nk/K} \\ &= H_k S_k + V_k \quad ,k = 1,\ldots,K \end{aligned} \tag{2}$$

where

$$H_k = \frac{1}{\sqrt{K}} \sum_{n=0}^{L-1} h(n) e^{-j2\pi nk/K} \quad ,k = 1,\ldots,K. \tag{3}$$

Since the Fourier Transform is a linear transformation, the signal will preserve its statistical characteristics from the time domain in the frequency domain. We also assume that the transmitted signal of the primary user, the channel gain and AWGN noise are all independent of each others.

3. Wideband Spectrum Sensing

In order to determine whether k^{th} sub-band is busy or not, we will perform the following hypotheses test:

$$\begin{aligned} \mathcal{H}_{0,k} &: Z_k = V_k, \\ \mathcal{H}_{1,k} &: Z_k = H_k S_k + V_k \end{aligned} \tag{4}$$

where Z_k is the secondary received signal, S_k is the primary transmitted signal, V_k is the Gaussian noise, and H_k is the channel gain between the primary transmitter and the secondary receiver.

After determining the channel status, we need to realize we have detected just noise or a noisy signal. To do this, we will first sample the received signal and take M samples and then find their p^{th} norm as follows:

$$Y_k = \sum_{n=1}^{M} |Z_k(n)|^p \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \lambda_k \tag{5}$$

where λ_k is the decision threshold for the k^{th} sub-band. Fig. 1 shows the joint detection of sub-bands using improved energy detector.

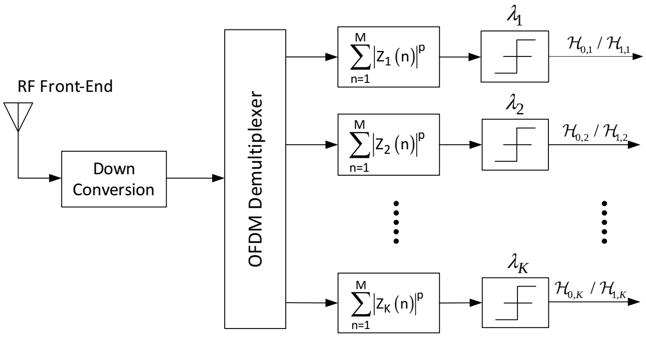

Fig. 1. Joint detection of sub-bands using improved energy detectors.

According to the Central Limit Theorem (CLT) [9], if the number of samples taken from the signal is large enough, we can assume Gaussian distribution for the samples [10]. Therefore, if either of the hypotheses $\mathcal{H}_{0,k}$ and $\mathcal{H}_{1,k}$ is true, then Y_k will have the following means and variances.

$$E[Y_k|\mathcal{H}_{0,k}] = M \frac{2^{\frac{p}{2}} \sigma_v^p}{\sqrt{\pi}} \Gamma\left(\frac{p+1}{2}\right), \tag{6}$$

$$Var[Y_k|\mathcal{H}_{0,k}] = M\sigma_v^{2p} \left(\frac{2^p \Gamma(\frac{2p+1}{2})}{\sqrt{\pi}} - \frac{2^p}{\pi} \Gamma^2\left(\frac{p+1}{2}\right) \right),$$

$$E[Y_k|\mathcal{H}_{1,k}] = M \frac{2^{\frac{p}{2}}}{\sqrt{\pi}} \left(\sigma_v^2 + |H_k|^2 \sigma_s^2\right)^{\frac{p}{2}} \Gamma\left(\frac{p+1}{2}\right), \tag{7}$$

$$Var[Y_k|\mathcal{H}_{1,k}] = M 2^p (\sigma_v^2 + |H_k|^2 \sigma_s^2)^p \left(\frac{\Gamma(\frac{2p+1}{2})}{\sqrt{\pi}} - \frac{\Gamma^2(\frac{p+1}{2})}{\pi} \right).$$

After determining the statistical characteristics of Y_k, the probability of a false alarm and detection for each sub-band is calculated as follows:

$$\begin{aligned} P_f^k(\lambda_k) &= P(Y_k \geq \lambda_k|\mathcal{H}_{0,k}) \\ &= Q\left(\frac{\lambda_k - E[Y_k|\mathcal{H}_{0,k}]}{\sqrt{Var[Y_k|\mathcal{H}_{0,k}]}} \right) \end{aligned} \tag{8}$$

and

$$\begin{aligned} P_d^k(\lambda_k) &= P(Y_k \geq \lambda_k|\mathcal{H}_{1,k}) \\ &= Q\left(\frac{\lambda_k - E[Y_k|\mathcal{H}_{1,k}]}{\sqrt{Var[Y_k|\mathcal{H}_{1,k}]}} \right). \end{aligned} \tag{9}$$

As it is shown by the above derivations and formulae, the proper value for λ_k is critical for finding correct probabilities. In other words, a large value of λ_k will result in small probabilities and vice versa. Therefore, λ_k should be chosen in a way to yield optimum probabilities.

4. Joint Detection of Sub-Bands

In this Section a method for joint detection of wideband spectrum is developed by the improved energy detector. As can be seen from Fig. 1, the aim is to find a vector of optimum thresholds $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \ldots, \lambda_K]^T$ for all the sub-bands simultaneously. This vector should be chosen in a way to allow the cognitive radio network to utilize free sub-bands with the least interferences to the primary users. For an optimum threshold vector, the probabilities of false alarm, detection and miss detection can be defined as

$$\begin{aligned} \boldsymbol{P}_f(\boldsymbol{\lambda}) &= [P_f^1(\lambda_1), P_f^2(\lambda_2), ..., P_f^K(\lambda_K)]^T, \\ \boldsymbol{P}_d(\boldsymbol{\lambda}) &= [P_d^1(\lambda_1), P_d^2(\lambda_2), ..., P_d^K(\lambda_K)]^T, \\ \boldsymbol{P}_m(\boldsymbol{\lambda}) &= [P_m^1(\lambda_1), P_m^2(\lambda_2), ..., P_m^K(\lambda_K)]^T. \end{aligned} \tag{10}$$

If r_k is defined as the achievable throughput for the secondary user in the k^{th} sub-band, then $\boldsymbol{r} = [r_1, r_2, \ldots, r_K]^T$ is the vector of throughput for all sub-bands. If the transmitted power and the channel gains between secondary users are known, $\boldsymbol{r}$ can be estimated using the Shannon capacity formula [8]. The probability of opportunistic access to the k^{th} sub-band is $1 - P_f^k(\lambda_k)$, therefore the aggregated achievable throughput for cognitive radio network can be defined as [2]:

$$\mathrm{R}(\boldsymbol{\lambda}) = \boldsymbol{r}^T\left[\mathbf{1} - \boldsymbol{P}_f(\boldsymbol{\lambda})\right]. \tag{11}$$

According to the above equation, the lower the value of $\boldsymbol{P}_f(\boldsymbol{\lambda})$ is, the higher achievable throughput would be. However, there exists a trade-off between $\boldsymbol{P}_f(\boldsymbol{\lambda})$ and $\boldsymbol{P}_m(\boldsymbol{\lambda})$, decreasing $\boldsymbol{P}_f(\boldsymbol{\lambda})$ will increase $\boldsymbol{P}_m(\boldsymbol{\lambda})$ which in turn will cause more interferences, also maximizing the $\mathrm{R}(\boldsymbol{\lambda})$ will increase $\boldsymbol{P}_m(\boldsymbol{\lambda})$ which will result in higher interference to the primarily network. Therefore, the interference from the secondary network should be capped. The interference from the secondary network for each active primary user of the network can be considered with constant coefficients. If we take $\boldsymbol{c} = [c_1, c_2, \ldots, c_K]^T$ as the interference vector caused by the secondary user, then the total interference to the primary user is

$$\sum_{i=1}^{K} c_i P_m^i(\lambda_i). \tag{12}$$

The aim is to find an optimum threshold for each sub-band so that the aggregated throughput is maximized conditioned on that the total interferences to the primary user, probabilities of miss-detection and false alarm are smaller than a given threshold. Therefore, the optimization problem for the aggregated throughput in a system with one primary user is

$$\begin{aligned} (P1) \qquad & \max \mathrm{R}(\boldsymbol{\lambda}), \\ & s.t. \sum_{k=1}^{K} c_k P_m^{(k)}(\lambda_\mathrm{i}) \leq \varepsilon_\mathrm{j}, \\ & \boldsymbol{P}_m(\boldsymbol{\lambda}) \preceq \boldsymbol{\alpha}, \\ & \boldsymbol{P}_f(\boldsymbol{\lambda}) \preceq \boldsymbol{\beta} \end{aligned} \tag{13}$$

where the constraint $\boldsymbol{P}_m(\lambda) \preceq \boldsymbol{\alpha}$ limits the interference in each sub-band with $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \ldots, \alpha_K]^T$, and the third constraint $\boldsymbol{P}_f(\lambda) \preceq \boldsymbol{\beta}$ sets a minimum opportunistic spectral utilization for each sub band given by $\boldsymbol{\beta} = [1-\beta_1, 1-\beta_2, \ldots, 1-\beta_K]^T$. For a sub-band with high throughput, a high threshold should be set to allow the secondary user to utilize the band for a longer period of time (small probability of false alarm). On the other hand, for a sub-band which is occupied by primary user with high priority, the threshold should be set lower to prevent the access of the secondary user to the sub-band (small probability of miss-detection). But, if the sub-band is occupied with low priority primary user, then a compromise can be made to allow for some opportunistic access of the secondary user. In this way the throughput will significantly improve. Therefore, in determining the threshold vector, there should be a balance among the channel condition, opportunistic access and the priorities for each sub-band.

Generally, the objective function and constraints in $P1$ are non-convex which makes it difficult to efficiently solve for the global optimum. Therefore, in this paper, practical constraints are imposed which change the problem into a convex problem. The $Q(.)$ function is a monotonically decreasing function. Therefore, it is possible to change the second and third constraints into linear constraints by the following modifications,

$$1 - P_d^k(\lambda_k) \leq \alpha_k, \;\; k = 1, 2, \ldots, K. \tag{14}$$

By replacing (9) in (14) we have

$$\lambda_k \leq \lambda_{max,k}, \;\; k = 1, 2, \ldots, K \tag{15}$$

where

$$\begin{aligned} \lambda_{\max,k} \;\; = \;\; & M\frac{2^{\frac{p}{2}}}{\sqrt{\pi}}(\sigma_v^2 + |\mathrm{H}_k|^2\sigma_s^2)^{\frac{p}{2}}\Gamma(\frac{p+1}{2}) \\ & + \mathrm{Q}^{-1}(1-\alpha_k)[2^p M(\sigma_v^2 + |\mathrm{H}_k|^2\sigma_s^2)^p \\ & -(\frac{\Gamma(\frac{2\mathrm{p}+1}{2})}{\sqrt{\pi}} - \frac{\Gamma^2(\frac{\mathrm{p}+1}{2})}{\pi})]^{\frac{1}{2}}. \end{aligned} \tag{16}$$

Similarly for the third constraint we have

$$\lambda_k \geq \lambda_{\min,k}, \;\; k = 1, 2, \ldots, K \tag{17}$$

where

$$\lambda_{min,k} = M\frac{2^{\frac{p}{2}}\sigma_v^p}{\sqrt{\pi}}\Gamma(\frac{p+1}{2}) + 2^{\frac{p}{2}}\sigma_v^p\sqrt{M(\frac{\Gamma(\frac{2p+1}{2})}{\sqrt{\pi}} - \frac{\Gamma^2(\frac{p+1}{2})}{\pi})}\mathrm{Q}^{-1}(\beta_k). \quad (18)$$

Therefore, $P1$ can be rewritten as

$$(P2) \quad \min_{\lambda} \sum_{k=1}^{K} r_k P_f^k(\lambda_k), \quad s.t. \sum_{i=1}^{K} c_i P_m^{(i)}(\lambda_i) \leq \varepsilon, \quad \lambda_{min,k} \leq \lambda_k \leq \lambda_{max,k}, \; k = 1,2,\ldots,K. \quad (19)$$

Although the second constraint in $P2$ is linear, the problem is still non-convex. However, as shown in [2], the function $P_f^k(\lambda_k)$ is convex in λ_k if $P_f^k(\lambda_k) \leq 1/2$ and similarly the function $P_m^k(\lambda_k)$ is convex if $P_m^k(\lambda_k) \leq 1/2$. By this assumption for $P_f^k(\lambda_k)$ and $P_m^k(\lambda_k)$, $P2$ changes to a convex and solvable problem.

5. Simulation Results

In this section we outline a numerical example of the proposed solution. If 48 MHz of bandwidth is allocated to the network with one primary user, we can then divide the spectrum into 8 equal and non-overlapping sub-bands. The coefficients r_k, c_k and the channel gain for each of the sub-bands is tabulated in Tab. 1.

k	1	2	3	4	5	6	7	8
$\lvert H_k\rvert^2$	0.38	0.29	0.23	0.26	0.35	0.39	0.33	0.27
$r(kbps)$	806	755	356	327	68	720	15	972
c	5.59	3.91	0.71	4.21	0.44	2.03	0.58	2.85

Tab. 1. Simulation parameters.

For the simulations, we have changed p from 1 to 4 with the step size of 0.01. The value of p that maximizes the aggregated opportunistic throughput is chosen as the optimum value. The maximum probabilities of false alarm and miss-detection are set to 0.5 and 0.2 respectively. For simplicity, we assume that the transmitted signal in each sub-band has unit power and the noise power $\sigma_v^2 = 0.7$.

The problem $P2$ is analyzed based on the conventional and improved energy detectors. Fig. 2 shows the aggregated throughput for the secondary user in terms of aggregated interferences to the primary user. It can be observed from this figure that using improved energy detector can improve the aggregated throughput considerably compared to the conventional detectors. This result shows that using improved energy detector allows for an efficient use of the spectrum.

In Fig. 3, the aggregated opportunistic access in terms of number of samples and for $\varepsilon = 0.16$ is shown. The result confirms the superiority of improved energy detectors.

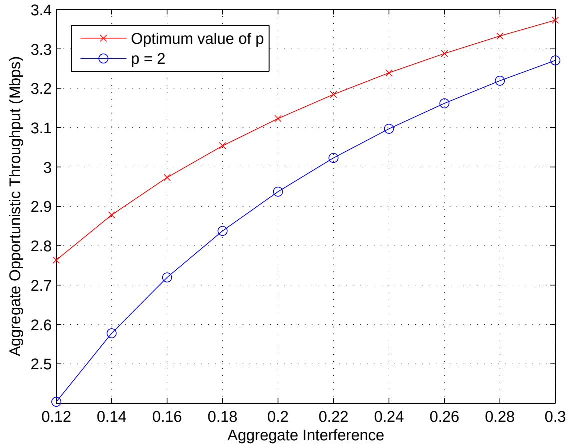

Fig. 2. The aggregated opportunistic throughput vs. aggregated interference for $M = 70$.

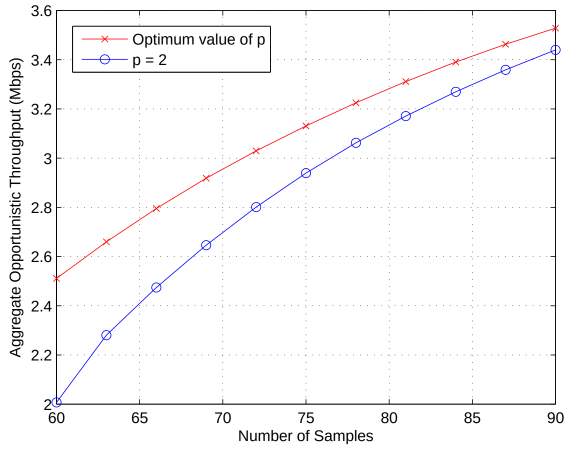

Fig. 3. The aggregated opportunistic throughput vs. Number of samples for $\varepsilon = 0.16$.

6. Conclusions

In this paper a joint detection technique for spectrum sensing of a wideband secondary network using improved energy detector was investigated. For better accuracy and finding the empty holes of the spectrum, the overall band was divided into a number of sub-bands and the improved energy detector was implemented for each individual sub-band. The main objective of this technique was to simultaneously find the thresholds for each sub-band which would result in an accurate spectrum sensing and would improve the opportunistic access significantly. In this paper, the joint detection of sub-bands was formulated as a set of optimization problems. The opportunistic access for the secondary user was improved by the deployment of the improved energy detectors, and optimum solutions were found and compared to the results of conventional methods. The results showed a significant improvement in aggregated opportunistic throughput.

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