Bounds on Minimum Energy per Bit for Optical Wireless Relay Channels

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Abstract. *An optical wireless relay channel (OWRC) is a classical three node network consisting of source, relay and destination nodes with optical wireless connectivity. The channel law is assumed Gaussian. This paper studies the bounds on minimum energy per bit required for reliable communication over an OWRC. It is shown that capacity of an OWRC is concave and energy per bit is monotonically increasing in square of the peak optical signal power, and consequently the minimum energy per bit is inversely proportional to the square root of asymptotic capacity at low signal to noise ratio. This has been used to develop upper and lower bound on energy per bit as a function of peak signal power, mean to peak power ratio, and variance of channel noise. The upper and lower bounds on minimum energy per bit derived in this paper correspond respectively to the decode and forward lower bound and the min-max cut upper bound on OWRC capacity.*

Keywords

Optical wireless (OW), network information theory (NIT), optical wireless relay channel (OWRC), channel capacity, energy per bit E_b.

1. Introduction

Information theory provides the scientific and theoretical foundation for the development of today's most beloved computers, smart phones and the Internet. Channel capacity is the central concept within information theory and draws the boundary between the physically possible and impossible in terms of reliable data rates. "A mathematical theory of communication" [1] laid the foundations of information theory that focused on determining and achieving the capacity of single input single output (SISO) channel.

"Two-way communication channel" [2] initiated another new field of study, the network information theory (NIT). NIT is a field that has been evolving to answer the questions that are not directly answerable by the link based classical information theory. NIT shifted the focus to studying the capacity of networks comprising multiple transmitters and receivers competing and cooperating for the capacity of underlying SISO channels to communicate to one another simultaneously. The problem though simple to formulate has defied a general solution till date. However, a lot of work has been done to find out capacity regions for fundamental network structures like broadcast channel, multiple access channel, relay channel, multiple input multiple output (MIMO) channel amongst many others [3].

Due to higher achievable bit rate and absence of regulatory controls and cost optical wireless is attracting attention for use in access network. This is despite of handicap of short coverage distance and constraint on peak signal power due to concern for safety of human eye. Capacity of optical wireless SISO is studied by a number of researchers [4–6]. Research in OW systems and in particular terrestrial OW links has for a long time attempted at increasing the availability and the reliability of the links, but recently it has been realized that probably, the better way to design systems is to attempt throughput maximization [7]. To overcome the degradation of OW channel due to scintillation Chatzidiamantis et al. [8] proposed using relay channels. Presently the study of OW network structures like relay and MIMO channels have been attracting attention [9–11].

Whereas capacity has been the dominant measure for a channel or network performance, minimum energy per bit needed for reliable communication has evolved into an alternative metric [12–14]. This metric becomes specially relevant in case of sensor relay networks where battery life is a critical design factor.

This paper studies minimum energy per bit requirement for reliable communication over a Gaussian optical wireless relay channel (OWRC). OWRC is a network comprising three nodes, source, relay and destination, connected through optical wireless links. The channel law for OWRC is assumed to be Gaussian. This study finds its relevance in view of the increasing use of wireless relay networks that could possibly be optical. The energy per bit bounds developed in this paper correspond to the bounds on the capacity of an OWRC in [15]. These bounds on the OWRC capacity have been briefly discussed in Sections 2.1.1 and 2.1.2.

The paper is organized as follows. Section 2 defines the optical wireless relay channel and its channel capacity and other preliminaries related to the study of energy per bit. Sections 3 and 4 provide original results on energy per bit for the optical wireless relay channel (OWRC). Upper and lower bounds on the energy per bit are derived.

2. Optical Wireless Relay Channel

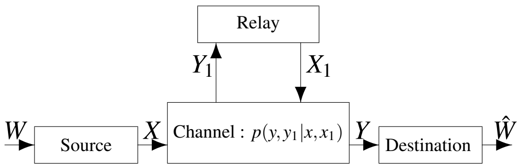

Fig. 1. The relay channel.

Van Der Meulen [16] introduced the relay channel that is a three node network: a source, a relay and a destination node; as shown in Fig. 1. Source node is transmit only while destination node is receive only node. The relay node receives the signal from the source node and transmits it to the destination node. A discrete memory-less relay channel is characterized by the triplet $(X \times X_1, p(Y,Y_1 \mid X,X_1), Y \times Y_1)$. There are four sets of alphabet, sets of input alphabet X and X_1 and output alphabet $Y$and Y_1, and a collection of probability distribution functions $p(\cdot,\cdot \mid x,x_1)$ on $Y \times Y_1$ space one for each $(x,x_1) \in X \times X_1$. The channel law $p(\cdot,\cdot \mid x,x_1)$ is assumed Gaussian.

$W = \{1,2,\cdots,2^{nR}\}$ is the set of messages {indices} to be sent to the destination by the source node, where R is the feasible rate and n is the number of bit in the code.

$X = (x^n(w))$ belongs to the code book $\{x^n(1), x^n(2),\cdots, X^n(2^{nR})\}$ at the source node containing n bit code words for $\forall w \in W$. At time k the source transmits $X_k = (x_k^n(w))$.

$Y_{1,k}$ is the output of the source-relay link at time k.

Relay function f_1^n such that $X_{1.k} = f_1^n(Y_{1,1}, Y_{1,2},\cdots, Y_{1,k-1})$ is the relay output at time k. However, generally block Markov coding is employed which implies $X_{1.k} = f_1^n(Y_{1,k-1})$. The relay transmits in time slot k depending only on what it received in time slot $k-1$.

Decoding rule d: $d(Y_k) = \hat{w} \in W$, where Y_k is the signal received by the destination node at time k.

Error occurs when $\hat{w} \neq w$. Average probability of error $P_e^{(n)}$ is defined as

$$P_e^{(n)} = \frac{\sum_{w=1}^{2^{n\times R}} P[\hat{w} \neq w]}{2^{n\times R}} \tag{1}$$

for $\forall w \in W$. R is the feasible rate, and capacity C is the supremum of the set of achievable rates.

The minimum energy per bit E_b is the infimum of the set of achievable energy per bit $E^{(n)}$ that is defined as

$$E^{(n)} = \frac{1}{nR_n}\big(\max_k E_s^{(n)}(k) + E_r^{(n)}\big) \tag{2}$$

where energy $E^{(n)}(k)$ for codeword k expended by the source node is

$$E_s^{(n)}(k) = \sum_{i=1}^{n} x_i(k) \tag{3}$$

and the energy spent by the relay $E_r^{(n)}(k)$ for the code word k is:

$$E_r^{(n)}(k) = \max_{y_1^n}\big(\sum_{i=1}^{n} x_{1i}\big). \tag{4}$$

The energy per bit $E^{(n)}$ is achievable if there exist a sequence of $(2^{nR}, n)$ codes such that probability of error $P_e^{(n)} \to 0$ as $n \to \infty$. The minimum energy per bit E_b is greater than $\limsup E^{(n)}$.

The OWRC input signals X and X_1 are inherently power signals and non-negative. They are further subject to both mean and peak power constraints dictated by the concerns of source power conservation and safety of human eye. These limitations translate to the following conditions on the optical intensity signal X and X_1.

$$X, X_1 \geq 0, \tag{5}$$

$$\mathrm{E}[X], \mathrm{E}[X_1] \leq \mathcal{E}, \tag{6}$$

$$\mathrm{Prob}[X > A], \mathrm{Prob}[X_1 > A] = 0. \tag{7}$$

A Gaussian OWRC is defined by the following equations:

$$Y_1 = g_1 x + Z_1, \tag{8}$$

$$Y = g_0 x + g_2 x_1 + Z \tag{9}$$

where g_0, g_1, g_2 are link gain as shown in Fig. 2. Z and Z_1 are zero mean Gaussian random variables with variance σ^2 depicting noise.

2.1 Bounds on the OWRC Capacity

The capacity theorems for general relay channels have been established in [17]. Upper bound is based on the max-min cut capacity [17, Theorem 4]:

$$C = \max_{p(x,x_1)} \min(I(X,X_1;Y), I(X;Y,Y_1 \mid X_1)) \tag{10}$$

where $I(\cdot;\cdot)$ is the mutual information. $I(X,X_1;Y)$ is the mutual information of the cut at the destination node (multi-access cut); whereas $I(X;Y,Y_1 \mid X_1)$ is the mutual information of the cut at source node (broadcast cut). The inner bound given below is based on the concept of a degraded relay channel modifying mutual information of broadcast cut as $I(X:Y_1 \mid X_1)$ [17, Theorem 1];

$$C \leq \max_{p(x,x_1)} \min(I(X,X_1;Y), I(X;Y_1 \mid X_1)). \tag{11}$$

Upper and lower bounds on the capacity of a Gaussian OWRC have been derived [15]. The min-max cut upper bound is derived through evaluation of (10) using the concept of duality [6] considering gaussian measure on the input X and X_1 with mean $\mathcal{E}$ and variance $(1-\alpha)A^2$ [15] . The lower bounds are obtained by applying entropy power inequality [3, Chap. 16.7] to 11. The lower bounds are optimised by the choice of maximum entropy approaching probability measure on the input alphabet X and X_1 and decode and forward relay function. This relaying strategy is known to yield the maximal lower bound [13]. Separate bounds for $\alpha \in (0, \frac{1}{2})$ and $\alpha \in (\frac{1}{2}, 1]$ have been derived in view of different maxentropic measures applicable in these two ranges of α.

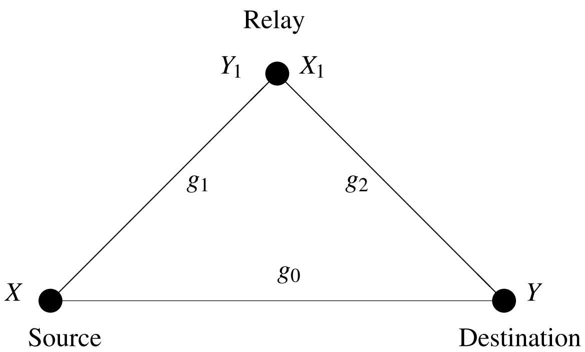

Fig. 2. Link gain coefficients for the relay channel.

2.1.1 Capacity Bounds for $0 < \alpha < \frac{1}{2}$

For $0 < \alpha < \frac{1}{2}$ the capacity of an OWRC, operating under peak power A and mean to peak power ratio α is upper bounded by [15]

$$C(A,\alpha A) \leq \begin{cases} \mathfrak{C}\Big(\frac{\left(g_1 g_2 + g_0\sqrt{g_0^2+g_1^2-g_2^2}\right)^2 (1-\alpha)A^2}{(g_0^2+g_1^2)\sigma^2}\Big) & \text{if } \frac{g_0^2+g_1^2}{g_2^2} > 1, \\ \mathfrak{C}\Big(\frac{(g_0^2+g_1^2)(1-\alpha)A^2}{\sigma^2}\Big) & \text{otherwise} \end{cases} \tag{12}$$

where $\mathfrak{C}(x) = \frac{1}{2}\log(1+x)$ and $C(A,\alpha A)$ is the capacity when peak signal is A and mean signal power is αA and it is lower bounded by

$$C(A,\alpha A) \leq \begin{cases} \mathfrak{C}\Big(\frac{\left(g_0\sqrt{g_1^2-g_2^2}+g_2\sqrt{g_1^2-g_0^2}\right)^2 e^{2\mu(1-\alpha)}A^2}{2\pi e\sigma^2 g_1^2(1-\mu\alpha)^2}\Big); \\ \text{if } g_0,\ g_2 > g_1 \\ \mathfrak{C}\Big(\frac{g_1^2 e^{2\mu(1-\alpha)}A^2}{2\pi e\sigma^2(1-\mu\alpha)^2}\Big);\ \text{otherwise} \end{cases} \tag{13}$$

where μ is the unique solution of the following equation,

$$\alpha = \frac{1}{\mu*} - \frac{e^{-\mu*}}{1-e^{-\mu*}}.$$

2.1.2 Capacity Bounds for $\frac{1}{2} \leq \alpha \leq 1$

When $\frac{1}{2} \leq \alpha \leq 1$ the capacity of an OWRC with peak signal A is upper bounded by [15],

$$C(A,\alpha A) \leq \begin{cases} \mathfrak{C}\Big(\frac{\left(g_1 g_2 + g_0\sqrt{g_0^2+g_1^2-g_2^2}\right)^2 A^2}{4(g_0^2+g_1^2)\sigma^2}\Big) & \text{if } \frac{g_1^2}{g_0^2+g_2^2} > 1, \\ \mathfrak{C}\Big(\frac{(g_0^2+g_1^2)A^2}{4\sigma^2}\Big) & \text{otherwise,} \end{cases} \tag{14}$$

is lower bounded by

$$C(A,\alpha A) \leq \begin{cases} \mathfrak{C}\Big(\frac{g_1^2 A^2}{2\pi e\sigma^2}\Big); & \text{if } g_0,\ g_2 \leq g_1, \\ \mathfrak{C}\Big(\frac{\left(g_0\sqrt{g_1^2-g_2^2}+g_2\sqrt{g_1^2-g_0^2}\right)^2 A^2}{2\pi e\sigma^2 g_1^2}\Big); & \text{otherwise.} \end{cases}$$

From the above expressions for the capacity of an OWRC in equations (12), (13), (14), (15) it is obvious that in general the capacity can be expressed as

$$C(A,\alpha A) = \mathfrak{C}\Big(\frac{\beta A^2}{\sigma^2}\Big) \geq \mathfrak{C}\Big(\frac{\alpha^2 A^2}{\sigma^2}\Big) \tag{15}$$

where $\beta > 0$ is the the coefficient of A^2. β is a function of link gains g_0, g_1, g_2 and mean to peak power ratio α. We will use these capacity bounds to work out energy per bit requirements in the next section.

3. Energy per Bit for OWRC

For bounding the energy per bit E_b we need to establish it is a non decreasing function of A^2. To do this it is required to be shown that the capacity of an ORWC is concave in A^2. Before we proceed to prove concavity of capacity of an OWRC let us define it in general terms as.

Definition 1 (Capacity of OWRC)**.** With peak power A and mean $\mathcal{E}$ to peak power ratio $\alpha = \frac{\mathcal{E}}{A}$ at the source and relay node, capacity of the OWRC is

$$C_k(A,\ \alpha A) = \frac{1}{k} \sup_{\substack{E(X)\leq\mathcal{E} \\ E(X_1)\leq\mathcal{E} \\ P(X>A=0) \\ P(X_1>A=0)}} I(X^k;Y^k), \tag{16}$$

$$C(A,\ \alpha A) = \sup_k C_k(A,\ \alpha A) \tag{17}$$

$$= \lim_{k\to\infty} C_k(A,\ \alpha A). \tag{18}$$

Lemma 2 (Concavity of Capacity of OWRC)**.** The capacity of an OWRC under average and peak power constraints (5-7) satisfies the following:

1. $C(A,\ \alpha A) \geq 0$ if $A > 0$ and tends to ∞ as $A \to \infty$.
2. $C(A,\ \alpha A) \to 0$ as $A \to 0$.
3. $C(A,\ \alpha A)$ is concave and strictly increasing in A^2.
4. $C^2(A,\ \alpha A)$ is concave and strictly increasing in A^2.
5. $\frac{A^2}{C^2(A,\ \alpha A)}$ is non decreasing in A, $\forall A > 0$.

Proof. 1. Since $C(A,\ \alpha A)$ is greater than or equal to $\mathfrak{C}(\frac{\alpha^2 A^2}{\sigma^2})$ which is strictly larger than zero for $\forall A > 0$ and approaches ∞ as $A \to \infty$.

2. Since the upper and lower bounds [15] on $C(A,\ \alpha A)$ go to zero as $A \to 0$.

3. $C_k(A,\alpha A)$ is concave in A^2. Therefore, $C(A,\alpha A) = \sup_k C_k(A,\alpha A)$ is concave being a supremum of a concave function [18, Theorem D, p. 16]. Because of concavity and prepositions (1) and (2) of this lemma, that is $C(A,\alpha A) = 0$ at $A = 0$ and $C(A,\alpha A) \to \infty$ when $A \to \infty$, $C(A,\alpha A)$ is monotonically non decreasing function in A.

4. As $C(A,\alpha A)$ is non-negative, increasing and concave function in A^2 so $C^2(A,\alpha A)$ is also concave in A^2 [18, Theorem C, p. 16].

5. It follows from the concavity of $C^2(A,\alpha A)$ that for any $0 < A_1 < A_2$

$$\frac{A_1^2}{A_2^2}C^2(A_2,\alpha A_2) + \frac{A_2^2 - A_1^2}{A_2^2}C^2(0,0) \le C^2(A_1,\alpha A_1).$$

Because $C(0,0) = 0$, the above relation translates to

$$\begin{aligned} \frac{A_1^2}{A_2^2}C^2(A_2,\alpha A_2) &\le C^2(A_12,\alpha A_12), \\ \frac{A_1^2}{C^2(A_12,\alpha A_1)} &\le \frac{A_2^2}{C^2(A_2,\alpha A_2)}. \end{aligned} \tag{19}$$

Equation (19) shows that $\frac{A^2}{C^2(A,\alpha A)}$ is a non decreasing function in A^2.

□

Lemma 3 (Minimum Energy per Bit for OWRC)**.** When the source and relay nodes have same peak power A and mean power $\mathcal{E}$ constraints and $A \ge 0$ and $0 < \alpha \le 1$, the minimum energy per bit E_b for OWRC is given by

$$E_b^2 = \lim_{A \to 0} \frac{2\alpha^2 A^2}{C^2(A,\alpha A)}. \tag{20}$$

Proof. The achievability and weak converse can be established by showing that

$$E_b^2 = \inf_{A>0} \frac{2\alpha^2 A^2}{C^2(A,\alpha A)}. \tag{21}$$

The proposition (5) of Lemma 2 allows replacement of inf by lim.

Achievability: There exists $E' > 0$ and $\varepsilon > 0$ such that

$$\begin{aligned} E &> \sqrt{\frac{2\alpha^2 A'^2}{C^2(A',\alpha A')}} \\ &= \inf_{A>0} \frac{2\alpha A}{C(A,\alpha A)} + \varepsilon. \end{aligned} \tag{22}$$

Thus there exists $R < C(A',\alpha A')$ that can be achieved using random coding with average and peak power constraints. This proves achievability of E.

Weak Converse: We need to prove that for any sequence $(2^{nR_n}, n)$ of codes with $P_e^{(n)} \to 0$

$$\begin{aligned} \liminf E^{(n)^2} &\ge E_b^2 \\ &= \inf_{A>0} \frac{2\alpha^2 A^2}{C^2(A,\alpha A)}. \end{aligned}$$

Fano's inequality yields

$$R_n \le C(A_n,\alpha A_n) + \frac{1}{n}(P_e^{(n)}) + R_n P_e^{(n)}. \tag{23}$$

Therefore,

$$R_n \ge \frac{C(A_n,\alpha A_n) + \frac{1}{n}(P_e^{(n)})}{(1 - P_e^{(n)})}.$$

Now by applying definition of energy per bit (2)

$$\begin{aligned} E^{(n)^2} &\ge \frac{2\alpha^2 A^2}{R_n^2} \\ &\ge \frac{2\alpha^2 A^2\ (1 - P_e^{(n)})^2}{(C(A_n,\alpha A_n) + \frac{1}{n}H(P_e^{(n)}))^2} \\ &= \frac{2\alpha^2 A^2}{C^2(A_n,\alpha A_n)} \times \frac{(1 - P^{(n)})^2}{(1 + \frac{\frac{1}{n}H(P_e^{(n)})}{C(A_n,\alpha A_n)})^2} \\ &\ge E_b \frac{(1 - P^{(n)})^2}{(1 + \frac{\frac{1}{n}H(P_e^{(n)})}{C(A_n,\alpha A_n)})^2}, \end{aligned} \tag{24}$$

$P_e^{(n)} \to 0$, $C(A_n,\alpha A_n) > 0$ and $H(P_e^{(n)}) > 0$ yields $\liminf E^{(n)} \ge E_b$.

□

4. Bounds on Energy per Bit

4.1 Energy Per Bit

From (20), energy per nat is

$$E_b = \sqrt{\lim_{A^2 \to 0} \frac{2\alpha^2 A^2}{C^2(A,\alpha A)}} \tag{25}$$

and $C(A,\alpha A)$ can be expressed in the generic form as

$$C(A,\alpha A) = \frac{1}{2}\log(1 + \beta A^2). \tag{26}$$

This yields the energy per nat E_{nat} as

$$E_{nat} = \frac{2\alpha}{\sqrt{\beta}} \tag{27}$$

and energy per bit E_b is

$$E_b = \frac{2\alpha}{\sqrt{\beta}} \log 2. \tag{28}$$

4.2 Lower Bound on Energy per Bit

Energy per bit for the upper bound on capacity of OWRC (12), (14) will yield the lower bound on energy per bit. The bounds are as follows:

Proposition 4. For $0 < \alpha < \frac{1}{2}$ the lower bound on E_b is

$$E_b \geq \frac{2\,\alpha\,\sigma \log 2}{1-\alpha} \frac{\sqrt{g_0^2+g_1^2}}{g_1 g_2 + g_0\sqrt{g_0^2+g_1^2}}. \tag{29}$$

Proof. Applying (28) to upper bound on OWRC capacity (12) we have

$$\begin{aligned} E_b \quad \geq \quad & \frac{\alpha\,\sigma \log 2}{(1-\alpha)} \min\Bigg(\min_{g_o^2+g_1^2>g_2^2} \underbrace{\frac{\sqrt{g_0^2+g_1^2}}{g_1\; g_2 + g_0\sqrt{g_0^2+g_1^2-g_2^2}}}_{k_1}, \\ & \min_{g_o^2+g_1^2\leq g_2^2} \underbrace{\frac{1}{\sqrt{g_0^2+g_1^2}}}_{k_2} \Bigg). \end{aligned} \tag{30}$$

Assuming $g_o^2+g_1^2 \leq g_2^2$,

$$\begin{aligned} k_1 &= \frac{\sqrt{g_0^2+g_1^2}}{g_1\; g_2 + g_0\sqrt{g_0^2+g_1^2-g_2^2}} \\ &\leq^f \frac{\sqrt{g_0^2+g_1^2}}{g_1\;\sqrt{g_0^2+g_1^2} + g_0\sqrt{g_0^2+g_1^2}} \quad (31) \\ &= \frac{1}{\sqrt{g_0^2+g_1^2}} \quad = k_2 \quad (32) \end{aligned}$$

where (f) stems from i) replacement of g_2 by $\sqrt{g_o^2+g_1^2}$, and ii) dropping of g_2^2 from $\sqrt{g_0^2+g_1^2-g_2^2}$ in the denominator. □

Proposition 5. For $\frac{1}{2} < \alpha \leq 1$ the lower bound on E_b is

$$E_b \geq \frac{2\,\sigma \log 2\;\; \sqrt{g_0^2+g_1^2}}{g_1 g_2 + g_0\sqrt{g_0^2+g_1^2}}. \tag{33}$$

Proof. This is obtained using (28) and (14) in a fashion similar to that of proposition 4. □

4.3 Upper Bound on Energy per Bit

This bound corresponds to decode and forward lower bound on capacity of OWRC (13) and (15).

Proposition 6. For $0 < \alpha < \frac{1}{2}$ the upper bound on E_b is

$$E_b \leq 2\alpha\,\sigma\,\lambda(\alpha) \log 2\, \min\left(\frac{1}{g_0+g_2}, \frac{1}{g_1} \right) \tag{34}$$

where $\lambda(\alpha) = \frac{\sqrt{2\pi e}(1-\mu\alpha)}{e^{\mu(1-\mu\alpha)}}$.

Proof. Using the decode forward lower bound on capacity (13) and (28) we get

$$\begin{aligned} E_b \quad \leq \quad & 2\alpha\,\sigma\,\lambda(\alpha) \log 2\, \min\Bigg(\min_{g_0,g_2\geq g_1} \frac{1}{g_1}, \\ & \min_{g_0,g_2<g_1} \underbrace{\frac{g_1}{g_0\sqrt{g_1^2-g_2^2} + g_2\sqrt{g_1^2-g_0^2}}}_{k3} \Bigg). \end{aligned} \tag{35}$$

Now if k_3 is closely observed in the light of above condition and assume lowest possible values of g_0 and g_2 that is negligibly small compared to g_1 we get

$$\begin{aligned} k_3 &= \frac{g_1}{g_0\sqrt{g_1^2-g_2^2} + g_2\sqrt{g_1^2-g_0^2}} \\ &< \frac{g_1}{g_0 g_1 + g_2 g_1} \\ &= \frac{1}{g_0+g_2}. \end{aligned} \tag{36}$$

That results in the upper bound on E_b given in this proposition. □

Proposition 7. For $0 < \alpha < \frac{1}{2}$ the upper bound on E_b is

$$E_b \leq \sqrt{2\pi e}\;\sigma\; \log 2\, \min\left(\frac{1}{g_0+g_2}, \frac{1}{g_1} \right). \tag{37}$$

Proof. This can be proved on the same lines as the proof of proposition 6. □

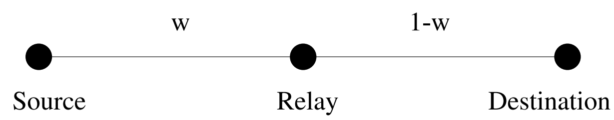

Fig. 3. Setup of relay channel for simulation.

Energy per bit as a function of source-relay distance for the relay channel set up in Fig. 3 is shown in Fig. 4. The source-destination distance is set equal to 1. The relay node is positioned anywhere between the source and destination. Let the source-relay distance be w, $0 \leq w \leq 1$. The link gain $g_i, (i = 0, 1, 2)$ is inversely proportional to the square of link distance as per free space path loss principle. Without loss of generality assuming $g_0 = 1$, the normalised source-relay and relay-destination link gains are $g_1 = w^{-2}$ and $g_2 = (1-w)^{-2}$. With these assumptions bounds on minimum energy per bit (29), (33), (34), (37) as function of relay location w are plotted. Minimum energy per bit E_B has been normalised to the standard deviation of noise σ in the plot. The upper and lower bounds on E_b diverge if relay node is in the vicinity of source node. The bounds are convergent when the relay node is near the destination. If the relay is placed midway between the source and destination nodes normalised minimum energy per bit $\frac{E_b}{\sigma}$ for reliable communication under decode and forward strategy is -2.9 dB and 0.7 dB for $\alpha = 0.3$ and $0.5 \leq \alpha \leq 1$ respectively.

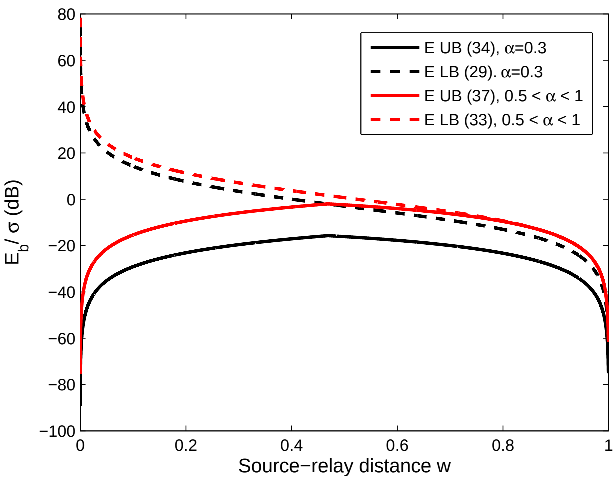

Fig. 4. Lower and upper bounds on energy per bit for an OWRC for $\alpha = 0.3$ (29, 34) and $0.5 \leq \alpha < 1$ (33, 37).

5. Conclusion

It has been proven that the capacity of a Gaussian OWRC is a monotonically increasing concave function in the square of peak signal A^2. It is also shown that energy per bit is a non decreasing function in A^2. Using these results upper and lower bounds on minimum energy per bit (29), (33), (34), (37) required for reliable communication over an OWRC are derived. Minimum energy per bit being an important performance metric will help develop better theoretical understanding of optical wireless relay channels and networks.

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