Performance of Cross-layer Design with Multiple Outdated Estimates in Multiuser MIMO System

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Abstract. *By combining adaptive modulation (AM) and automatic repeat request (ARQ) protocol as well as user scheduling, the cross-layer design scheme of multiuser MIMO system with imperfect feedback is presented, and multiple outdated estimates method is proposed to improve the system performance. Based on this method and imperfect feedback information, the closed-form expressions of spectral efficiency (SE) and packet error rate (PER) of the system subject to the target PER constraint are respectively derived. With these expressions, the system performance can be effectively evaluated. To mitigate the effect of delayed feedback, the variable thresholds (VTs) are also derived by means of the maximum a posteriori method, and these VTs include the conventional fixed thresholds (FTs) as special cases. Simulation results show that the theoretical SE and PER are in good agreement with the corresponding simulation. The proposed CLD scheme with multiple estimates can obtain higher SE than the existing CLD scheme with single estimate, especially for large delay. Moreover, the CLD scheme with VTs outperforms that with conventional FTs.*

Keywords

Multiuser MIMO, cross-layer design, imperfect feedback, multiple estimates, variable thresholds.

1. Introduction

The ultimate objectives of wireless communication systems are to satisfy the quality of service (QoS) and rate requirements. Therefore, in order to ensure high data rates, low latency and increased link throughput, the advanced technique schemes are often employed, such as cross-layer design (CLD) and multiple-input multiple-output (MIMO) are adopted to obtain high spectral efficiency (SE) and capacity [1]. Moreover, CLD combining adaptive modulation (AM) and automatic repeat request (ARQ) is widely accepted as an efficient means to improve the overall performance of transmission in fading channels [2-6]. In [2], a cross-layer design combined adaptive modulation and coding at the physical layer (PHY) and ARQ protocol at the data link layer (DLL) over Nakagami fading channel is developed. By employing different space-time coding schemes, the performance of MIMO system with CLD is studied in [3]. The performance of CLD with STBC is analyzed in optical MIMO system in [4]. Considering that prefect feedback is hard to achieve, [5] and [6] study the performance of CLD with STBC under delayed feedback and imperfect estimation information, respectively. However, the above CLD schemes are designed for single-user MIMO environment, and thus they will be not suitable for multiuser scenario in practice. For this, the performance of AM scheme for multiuser system with imperfect feedback is studied in [7], but the analysis is only suitable for single antenna system. Under the feedback constraint, a CLD scheme is presented for multiuser MIMO systems, and the corresponding performance is investigated in Rayleigh fading channel [8]. Unfortunately, these CLD or AM schemes for imperfect feedback basically consider single outdated estimate only, and thus the performance improvement is limited. Although [9] presents an AM scheme based on multiple channel estimations to improve SE, the scheme is only designed for continuous-rate modulation. Thus, the practicability is not strong because the modulation is often based on discrete rate in practice.

According to the analysis above, the system performances of single user MIMO with CLD scheme are well studied. Moreover, most of CLD schemes are based on perfect CSI, but the perfect CSI is difficult to obtain due to channel estimation error or feedback delay. Although some schemes are designed for imperfect feedback, they are limited in single outdated estimate (SOE) information and fixed switching thresholds, and thus the system performance is hard to be improved effectively. Motivated by the reason above, we will develop a CLD scheme for multiuser MIMO system with imperfect feedback information by combining the AM at the PHY and ARQ as well as user scheduling at the DLL, where multiple outdated estimates (MOE) method is presented to improve the system performance. Based on this MOE method and performance analysis under imperfect channel state information (CSI), the probability density function (PDF) of effective signal-to-noise-ratio (SNR) and the fading gain switching thresholds for AM are respectively derived. According to these results, the closed-form expressions of average packet error rate (PER) and SE of the system are obtained. These expressions include the ones under conventional SOE or under perfect CSI as special cases. With these expressions, the system performance under imperfect CSI can be effectively assessed. Besides, subject to target packet loss rate (PLR) constraint, the variable switching thresholds are derived by using the maximum a posteriori (MAP) method. Simulation results show that the proposed CLD scheme with MOE can obtain much higher SE than the conventional CLD scheme with SOE because of more available outdated information. Moreover, the derived variable thresholds (VTs) can further reduce the effect of imperfect CSI on the system performance. Namely, the system performance with VTs is superior to that with conventional fixed thresholds (FTs).

The notations throughout this paper are as follows. Bold upper case and lower case letters denote matrices and column vectors, respectively. The superscripts $(\cdot)^H$, $(\cdot)^T$ and $(\cdot)^*$ denote the Hermitian transposition, transposition and complex conjugation, respectively.

2. System Model

We address downlink transmission in a multiuser MIMO system employing antenna selection (AS) shared by K users, and the system operates in a flat Rayleigh fading channel and homogenous case. There are n_T transmit antennas at the base station (BS) and n_R receive antennas at each user side. For a homogeneous case, the statistics of users are the same and their SNRs are assumed to be independent, identically distributed (*i.i.d*) random variables [10], [7]. The scheduler of the BS will select the user with the maximum absolute SNR, that is, the absolute SNR-based scheduling scheme (greedy scheduling) is used, which means maximizing the throughput of a multiuser system [7].

The channel between the transmitter and the kth user is characterized by a $n_R \times n_T$ matrix such that $\mathbf{H}^k = [h_{j,i}^k]_{n_R \times n_T} = [\mathbf{h}_1^k, ..., \mathbf{h}_i^k ..., \mathbf{h}_{n_T}^k]$, whose elements are *i.i.d* complex Gaussian random variables (*r.v.s*) with zero mean and unit variance. $\mathbf{h}_i^k = [h_{1,i}^k ..., h_{j,i}^k ..., h_{n_R,i}^k]^T$ is the channel vector of the kth user. $h_{j,i}^k$ denotes the channel gain from the ith transmit antenna to the jth receive antenna for the kth user. The channel is assumed to be perfectly known at the receiver, and is fed back to the transmitter with delay τ. Each receiver tracks its own instantaneous channel SNR and feeds back the CSI to the BS over the outdated channel. For the current channel $h_{j,i}^k(t)$, we use its single delayed version $h_{j,i}^k(t-\tau)$ as its estimate $\hat{h}_{j,i}^k(t)$. The power correlation coefficient between the channel $h_{j,i}^k(t)$ and its estimation $\hat{h}_{j,i}^k(t)$ is given by $\rho = J_0^2(2\pi f_d \tau)$ [11], where $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind, and f_d is the maximum Doppler frequency. According to the CSI feedback, the BS scheduler selects the user that has the best link quality among all the K active users based on a greedy scheduling scheme, then the BS adapts the transmission rate to the signal of the scheduled user, and after that, the BS selects the best transmit antenna which has the maximum feedback SNR to perform the data transmission. If the transmitter selects the ith antenna for data transmission, the $n_R \times 1$ received signal vector $\mathbf{x}^k$ of user k can be written as [10]

$$\mathbf{x}^k = \mathbf{h}_i^k s^k + \mathbf{w}^k \tag{1}$$

where s^k is transmitted data symbol of user k with average energy E_s, and $\mathbf{w}^k$ is a $n_R \times 1$ noise vector, whose elements are *i.i.d* complex Gaussian random variables with mean zero and variance σ^2. The average SNR per data symbol is $\bar{\gamma} = E_s / \sigma^2$ [10]. When maximal-ratio combining (MRC) is used at the receivers, the instantaneous SNR with ith selected transmit antenna for the kth user can be expressed as $\gamma_i^k = \bar{\gamma} \sum_{j=1}^{n_R} |h_{j,i}^k|^2$, where $\sum_{j=1}^{n_R} |h_{j,i}^k|^2$ is chi-square distributed with $2n_R$ degrees of freedom. Hence, the probability density function (PDF) of γ_i^k can be given by

$$f_i^k(\gamma) = \frac{1}{\bar{\gamma}\Gamma(n_R)} (\gamma / \bar{\gamma})^{n_R - 1} \exp(-\gamma / \bar{\gamma}) \tag{2}$$

where $\Gamma(\cdot)$ is the gamma function. This is also the PDF of the feedback SNR for user k, $\hat{\gamma}_i^k = \bar{\gamma} \sum_{j=1}^{n_R} |\hat{h}_{j,i}^k|^2$. Considering that $\hat{h}_{j,i}^k$ and $h_{j,i}^k$ are from the same random process, they have the same probability distribution. The transmitter allocates the radio resource to a user who can achieve the largest feedback SNR when selecting the "best" transmit antenna, and thus the effective SNR, $\hat{\gamma}$ is expressed as

$$\hat{\gamma} = \max_{i=1,2,...,n_T; k=1,2,...,K} \{\hat{\gamma}_i^k\}. \tag{3}$$

According to the order statistics [12] and (3), and using transformation of variables, the CDF and PDF of $\hat{\gamma}$ can be obtained as follows:

$$\begin{aligned} F(\hat{\gamma}) &= [\int_0^{\hat{\gamma}} f_i^k(t) dt]^L \\ &= [1\text{-}\exp(\text{-}\hat{\gamma} / \bar{\gamma}) \sum_{n=0}^{n_R - 1} \frac{1}{n!} (\hat{\gamma} / \bar{\gamma})]^L \end{aligned}, \quad \hat{\gamma} \ge 0 \tag{4}$$

and

$$\begin{aligned} f(\hat{\gamma}) &= \frac{L(\hat{\gamma})^{n_R - 1}}{(n_R - 1)!} . (\frac{1}{\bar{\gamma}})^{n_R} \exp(-\frac{\hat{\gamma}}{\bar{\gamma}}) \sum_{m=0}^{L-1} \binom{L-1}{m} \\ &\times (-1)^m \exp(-m\hat{\gamma} / \bar{\gamma}) \sum_{c=0}^{m(n_R - 1)} \omega_{c,m} (\hat{\gamma} / \bar{\gamma})^c \end{aligned} \tag{5}$$

where $L = n_T K$, and $\omega_{c,m}$ is the coefficient of $\left(\hat{\gamma}/\bar{\gamma}\right)^c$ in the expansion of $(\sum_{n=0}^{n_R-1}\left(\hat{\gamma}/\bar{\gamma}\right)^n/n!)^m$.

Based on the analysis above, only a single delay channel is used for predicting the real channel. Although this single estimation method has a pervasive application, the estimated channel is not accurate enough since other outdated channel information is not utilized. For this reason, multiple outdated estimates will be employed for accurate estimation in the following CLD.

3. Cross-layer Design with Multiple Outdated Estimates

Considering that existing CLD schemes with imperfect feedback are basically based on single outdated estimation method in Section 2, the performance improvement is limited. Based on this, we will present the CLD scheme for multiuser MIMO based on multiple outdated estimates method so that the SE and PER performance will be effectively improved.

3.1 Multiple Outdated Estimates

In this subsection, we will give the MOE method. As analyzed in Section 2, $\hat{\gamma}$ is derived from the estimate channel. However, conventional estimation only considers a single delay channel $h_{j,i}^k(t-\tau)$. For this, we use multiple previous channel estimates, which are produced prior to $h_{j,i}^k(t-\tau)$, in combination to reduce the uncertainty in $\hat{h}_{j,i}^k(t)$. Assuming Z estimates, $h_{j,i}^k(t-\tau)$, $h_{j,i}^k(t-2\tau)$, …, $h_{j,i}^k(t-Z\tau)$ are available, then the correlation coefficient is given by

$$E\left\{h_{j,i}^k(t-u\tau)h_{j,i}^{k\,*}(t-v\tau)\right\} = J_0(2\pi f_d(v-u)\tau) \quad . (6)$$

Let the estimated channel vector be $\hat{\mathbf{h}}_{j,i}^k = [h_{j,i}^k(t-\tau), h_{j,i}^k(t-2\tau), \ldots, h_{j,i}^k(t-Z\tau)]^T$, then $\hat{\mathbf{h}}_{j,i}^k$ is Gaussian distributed with zero mean, and the covariance matrix is given by

$$E\left\{\begin{pmatrix} h_{j,i}^k(t) \\ \hat{\mathbf{h}}_{j,i}^k \end{pmatrix}\begin{pmatrix} h_{j,i}^{k\,*}(t) & (\hat{\mathbf{h}}_{j,i}^k)^H \end{pmatrix}\right\} = \begin{bmatrix} 1 & \mathbf{a}^H \\ \mathbf{a} & \mathbf{B} \end{bmatrix} \quad (7)$$

where $\mathbf{a} = E\left\{\hat{\mathbf{h}}_{j,i}^k h_{j,i}^{k\,*}(t)\right\}$, $\mathbf{B} = E\left\{\hat{\mathbf{h}}_{j,i}^k(\hat{\mathbf{h}}_{j,i}^k)^H\right\}$. $h_{j,i}^k(t)$ given $\hat{\mathbf{h}}_{j,i}^k$ is a Gaussian distributed with mean $\hat{h}_{j,i}^k(t) = \mathbf{a}^H\mathbf{B}^{-1}\hat{\mathbf{h}}_{j,i}^k$ and variance $\delta^2 = 1 - \mathbf{a}^H\mathbf{B}^{-1}\mathbf{a}$. $h_{j,i}^k(t)$ and $\hat{h}_{j,i}^k(t)$ are jointly complex Gaussian with power correlation coefficient $\rho = 1-\delta^2$ [9]. Since the multiple previous channel estimates are utilized, the correlation coefficient ρ will be large. As a result, $\hat{h}_{j,i}^k$ will be accurate enough to estimate the real $h_{j,i}^k$. Correspondingly, the effective SNR $\hat{\gamma}$ is very close to the real SNR γ, and the feedback information will be reliable. This multiple estimates method includes single outdated estimate (i.e., $Z = 1$) as a special case. Based on the correlation coefficient of multiple outdated estimates, the conditional PDF of γ given $\hat{\gamma}$ is expressed as [11]

$$p_{\gamma|\hat{\gamma}}\left(\gamma|\hat{\gamma}\right) = \frac{1}{(1-\rho)\bar{\gamma}} I_{n_R-1}\left(\frac{2\sqrt{\rho\hat{\gamma}\gamma}}{(1-\rho)\bar{\gamma}}\right) \times\left(\frac{\gamma}{\rho\hat{\gamma}}\right)^{\frac{n_R-1}{2}}\exp\left(-\frac{\gamma+\rho\hat{\gamma}}{(1-\rho)\bar{\gamma}}\right), \gamma\geq 0 \quad (8)$$

where $I_n(\cdot)$ is the n-order modified Bessel function of the first kind [13]. With (5) and (8), the joint pdf of $\hat{\gamma}$ and γ can be obtained as

$$\begin{aligned} p_{\gamma,\hat{\gamma}}\left(\gamma,\hat{\gamma}\right) = & \frac{L\bar{\gamma}^{-(n_R+1)}}{(1-\rho)(n_R-1)!}\left(\frac{\gamma\hat{\gamma}}{\rho}\right)^{\frac{n_R-1}{2}} I_{n_R-1}\left(\frac{2\sqrt{\rho\hat{\gamma}\gamma}}{(1-\rho)\bar{\gamma}}\right) \\ & \times\exp\left(-\frac{\gamma+\hat{\gamma}}{(1-\rho)\bar{\gamma}}\right)\sum_{m=0}^{L-1}\binom{L-1}{m}(-1)^m \\ & \times\exp\left(-\frac{m\hat{\gamma}}{\bar{\gamma}}\right)\sum_{c=0}^{m(n_R-1)}\omega_{c,m}\left(\frac{\hat{\gamma}}{\bar{\gamma}}\right)^c \end{aligned} \quad (9)$$

3.2 Cross-layer Design

By combining AM at the PHY and ARQ protocol as well as user scheduling at the DLL, we will give a cross-layer design scheme based on the MOE above for multiuser MIMO system with antenna selection, and MQAM is considered for modulation in the system.

At the transmitter, according to the delayed CSI feedback from the receiver, the BS scheduler selects the user that has the best link quality among all the active users, and then the BS selects the best transmit antenna which has the maximum feedback SNR. After that, the modulator performs adaptive modulation in terms of the feedback CSI, and subsequently, the modulated symbols are transmitted by the selected transmit antenna. At the receiver, the channel states are estimated and measured for controlling the modulation mode by using MOE method, and the estimated CSI is fed back to the transmitter via a feedback path. With the obtained CSI, the adaptive demodulation is performed and the resultant decoded bits are obtained. Then, these bit streams are mapped to packets, which are pushed upwards to the data link layer. According to [2], at the data link layer, the selective repeat ARQ protocol is implemented. If an error is detected in a packet, a retransmission request is generated by the ARQ generator, and is communicated to the ARQ controller at the transmitter. Otherwise, no retransmission request is sent.

For discrete-rate MQAM, the constellation size M_n is defined as $\{M_0 = 0, M_n = 2^n, n = 1,\ldots,N\}$, where M_0 means no data transmission. The MQAM of constellation size M_n is used for modulation when the SNR $\hat{\gamma}$ falls in the nth region $[\gamma_n, \gamma_{n+1})$. According to [2], the PER of MQAM with two dimensional Gray code over additive white Gaussian noise (AWGN) channel for the received SNR $\hat{\gamma}$ and constellation size M_n is approximately given by

$$Per_n(\hat{\gamma}) \cong \begin{cases} 1, & if\ \hat{\gamma} < \gamma_{pn} \\ a_n \exp(-g_n\hat{\gamma}), & if\ \hat{\gamma} \geq \gamma_{pn} \end{cases} \quad (10)$$

where $\{a_n, g_n, \gamma_{pn}\}$ are constellation and packet-size dependent constants, and they can be obtained by fitting (10) to the exact PER.

We first define the target packet loss rate for the data link layer as P_{loss}. Since truncated ARQ is used at the data link layer, the packets in error may be retransmitted up to $N_r^{\max}$ (i.e., maximum number of retransmissions). Hence, the target PER is $P_o = P_{loss}^{1/(N_r^{\max}+1)}$ at the physical layer, which is generally limited as $P_o < 1$. The switching thresholds $\{\gamma_n\}$ can be set to be the required SNR to achieve the target PER, P_o, over an AWGN channel. By inverting the P_o in (10), we can obtain the switching threshold values as

$$\gamma_n = -\ln(P_o / a_n) / g_n, \quad n = 1,\ldots,N,$$
$$\gamma_0 = 0, \qquad \gamma_{N+1} = +\infty. \quad (11)$$

The above thresholds do not consider the impact of delayed feedback information, so they are referred as fixed thresholds (FTs). Because our CLD scheme is based on MOE, the resulting system performance will perform better than the conventional CLD scheme based on SOE only, which will be confirmed in the following numerical results.

4. Performance Analysis of CLD with Multiple Estimates

In this section, we will give the performance analysis of CLD with antenna selection (referred to as CLD-AS) for multiuser MIMO with imperfect feedback. Based on the switching thresholds described in (11), we can calculate the probability that the effective SNR $\hat{\gamma}$ falls in the n-th region $[\gamma_n, \gamma_{n+1})$, denoted by Pr_n, as

$$Pr_n = \frac{L}{(n_R-1)!}\sum_{m=0}^{L-1}\binom{L-1}{m}(-1)^m \sum_{c=0}^{m(n_R-1)} \frac{\omega_{c,m}}{(m+1)^{n_R+c}} \times[\Gamma(n_R+c, \frac{m+1}{\bar{\gamma}}\gamma_n) - \Gamma(n_R+c, \frac{m+1}{\bar{\gamma}}\gamma_{n+1})] \quad (12)$$

where $\Gamma(\cdot,\cdot)$ is incomplete Gamma function [13]. For discrete-rate adaptive scheme, the SE at the PHY is defined as the ensemble average of valid transmission rate. So the average SE of the system at the PHY can be given by

$$\overline{Se}_{phy} = \sum_{n=1}^{N} R_n Pr_n = \sum_{n=1}^{N} \frac{R_n L}{(n_R-1)!}\sum_{m=0}^{L-1}\binom{L-1}{m}(-1)^m \sum_{c=0}^{m(n_R-1)} \frac{\omega_{c,m}}{(m+1)^{n_R+c}} \times[\Gamma(n_R+c, \frac{m+1}{\bar{\gamma}}\gamma_n) - \Gamma(n_R+c, \frac{m+1}{\bar{\gamma}}\gamma_{n+1})] \quad (13)$$

We defined ensemble average PER at the PHY for multiuser MIMO system with CLD-AS and feedback delay as

$$\overline{Per} = (\textstyle\sum_{n=1}^{N} R_n \overline{Per}_n) \big/ (\textstyle\sum_{n=1}^{N} R_n Pr_n) \quad (14)$$

where $\overline{Per}_n$ denote the average PER for constellation size M_n, and it can be obtained as:

$$\overline{Per}_n = \int_{\gamma_n}^{\gamma_{n+1}} \left(\int_0^{\infty} Per_n(\gamma)\, p_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma})\, d\gamma \right) f(\hat{\gamma})\, d\hat{\gamma} \quad (15)$$

where

$$Per_n(\gamma) \cong \begin{cases} 1, & if\ \gamma < \gamma_{pn} \\ a_n \exp(-g_n\gamma), & if\ \gamma \geq \gamma_{pn} \end{cases} \quad (16)$$

Substituting (16), (5) and (8) into (15) gives

$$\overline{Per}_n = I1_n + I2_n - I3_n \quad (17)$$

where $I1_n$, $I2_n$, $I3_n$ are written as (18)-(20) at the bottom of the paper due to their long expressions. Equation (17) is a closed-form expression of the average PER for multiuser MIMO with CLD-AS and MOE under imperfect CSI. With (17), the average PLR at the data link layer with the maximum number of retransmissions $N_r^{\max}$ is

$$\overline{Plr} = \overline{Per}^{N_r^{\max}+1}. \quad (21)$$

Thus, the average number of transmissions per packet can be calculated as:

$$\bar{N} = 1 + \overline{Per} + \ldots + \overline{Per}^{N_r^{\max}} = (1 - \overline{Per}^{N_r^{\max}+1}) / (1 - \overline{Per}). \quad (22)$$

Using (22) and (14) as well as (13), the overall average SE of multiuser MIMO system with CLD-AS and multiple estimates under imperfect CSI can be obtained as:

$$\overline{Se} = \overline{Se}_{phy} \big/ \bar{N}. \quad (23)$$

When $N_r^{\max}$ is set to be zero, then $\bar{N} = 1$, and (23) is reduced to h_{phy}, which corresponds to the average SE at the physical layer only.

5. Variable Thresholds for CLD with Multiple Estimates

Considering that the switching thresholds in (11) can not adapt to the change of delayed feedback information, the performance loss will happen when feedback has delay. For this, we will present a variable threshold method for imperfect feedback by employing the Bayes' theorem [14] and MAP criterion [14], [5]. With the variable thresholds, the system performance will be effectively improved, and outperforms that with fixed thresholds (11).

In what follows, we employ the estimated instantaneous PER, $\hat{Per}_n$, to calculate VTs. $\hat{Per}_n$ can be obtained by maximizing the conditional PDF of $p_{Per_n|\hat{\gamma}}(Per_n \mid \hat{\gamma}) = p_{Per_n,\hat{\gamma}}(Per_n,\hat{\gamma}) / p_{\hat{\gamma}}(\hat{\gamma})$. From the functional relationship between Per_n and γ in (16), using the analysis method in [14], the joint PDF of Per_n and $\hat{\gamma}$ is derived as,

$$
\begin{aligned}
& p_{Per_n,\hat{\gamma}}\left(Per_n,\hat{\gamma}\right) = \left|\frac{\partial \gamma}{\partial Per_n}\right| f_{\gamma,\hat{\gamma}}\left(Per_n^{-1},\hat{\gamma}\right) \\
& = \sum_{m=0}^{L-1}\binom{L-1}{m}(-1)^m \sum_{c=0}^{m(n_R-1)} \frac{L\omega_{c,m} e^{\varepsilon^2}\hat{\gamma}^c}{g_n a_n \bar{\gamma}^{n_R+c+1}(1-\rho)\Gamma(n_R)} \\
& \times \left(\frac{\hat{\gamma}}{g_n\rho}\right)^{\frac{n_R-1}{2}} \exp\left(-\frac{\left(m-m\rho+1\right)\hat{\gamma}}{(1-\rho)\bar{\gamma}} - \frac{\varepsilon^2}{g_n(1-\rho)\bar{\gamma}}\right) \\
& \times \varepsilon^{n_R-1} I_{n_R-1}\left(\frac{2\varepsilon\sqrt{g_n\rho\hat{\gamma}}}{g_n(1-\rho)\bar{\gamma}}\right) \\
& \approx \sum_{m=0}^{L-1}\binom{L-1}{m}(-1)^m \sum_{c=0}^{m(n_R-1)} \frac{L\omega_{c,m} D \varepsilon^{n_R-1.5} e^{\beta(\varepsilon)}}{2a_n\sqrt{\pi b_n\sqrt{g_n\rho\hat{\gamma}}}}
\end{aligned}
\tag{24}
$$

where

$b_n = [g_n(1-\rho)\bar{\gamma}]^{-1}$, $\varepsilon = \sqrt{\ln(a_n / Per_n)}$,

$\beta(\varepsilon) = \left(1-b_n\right)\varepsilon^2 + 2\varepsilon b_n\sqrt{\rho\hat{\gamma} g_n}$, and

$$D = \frac{\left(1/\bar{\gamma}\right)^{n_R+c+1}\hat{\gamma}^c}{g_n\left(1-\rho\right)\Gamma\left(n_R\right)}\left(\frac{\hat{\gamma}}{g_n\rho}\right)^{(n_R-1)/2} \exp\left(-\frac{\left(m-m\rho+1\right)\hat{\gamma}}{\left(1-\rho\right)\bar{\gamma}}\right).$$

According to (24), by omitting the terms not related to ε, the MAP function can be defined as

$$I_{MAP} = \varepsilon^{n_R-1.5} e^{\beta(\varepsilon)} \ . \tag{25}$$

By solving the equation $\frac{\partial I_{MAP}}{\partial Per_n} = \frac{\partial I_{MAP}}{\partial \varepsilon} \cdot \frac{\partial \varepsilon}{\partial Per_n} = 0$, we can obtain the approximated expression of the estimated PER as

$$\hat{Per}_n \approx a_n \exp\left(-n_R^2\rho g_n\hat{\gamma} / \left(n_R - g_n\left(1-\rho\right)\right)^2\right) \tag{26}$$

Let $\hat{Per}_n = P_o$, then with (26) we can achieve the VTs dependent on the feedback delay as

$$\gamma_{An} = \ln\left(\frac{a_n}{P_o}\right) \cdot \left(1 - \frac{g_n\left(1-\rho\right)}{n_R}\right)^2 \Big/ \left(\rho g_n\right). \tag{27}$$

By submitting the obtained VTs into (14) and (23) in Section 4, we can calculate the average PER and SE, and obtain the corresponding closed-form expressions of multiuser MIMO system with CLD-AS and MOE under imperfect CSI. Unlike the FTs in (11), the VTs in (27) consider the effect of delayed feedback. Thus, with (27), the impact of feedback delay may be reduced, and reliable PER performance will be realized. When the feedback is perfect (i.e. $\rho = 1$), (27) is reduced to (11), and thus the derived VTs include the FTs as special cases.

6. Numerical Results and Analysis

In this section, we will use the derived formulae to assess the performance of average PER and SE of multiuser MIMO system with CLD and multiple outdated estimates over Rayleigh fading channel. The channel is assumed to be flat fading. The Gray code is employed to map the data bits to MQAM constellations, and 10^6 Monte-Carlo simulations are employed for the performance evaluation. The set of MQAM constellations is $\{M_n\}_{n=0,1,\ldots,7}=\{0, 2, 4, 8, 16, 32, 64, 128\}$. The target packet loss rate at the data link layer, $P_{loss} = 10^{-3}$, and the maximum number of ARQ retransmissions $N_r^{\max} = 2$. Thus, the target PER, P_o, is equal to 0.1. In the simulation results below, xTyRzU denotes the multiuser MIMO system with x transmit antennas and y receive antennas and z users. Unless other mentioned, when the impact of feedback delay on the performance is evaluated, the average SNR, $\bar{\gamma}$, is set to 20 dB.

Fig. 1 shows the average SE versus the normalized time delay ($f_d\tau$) for 2T2R3U system with CLD under imperfect CSI, where MOE and SOE are used for comparison.

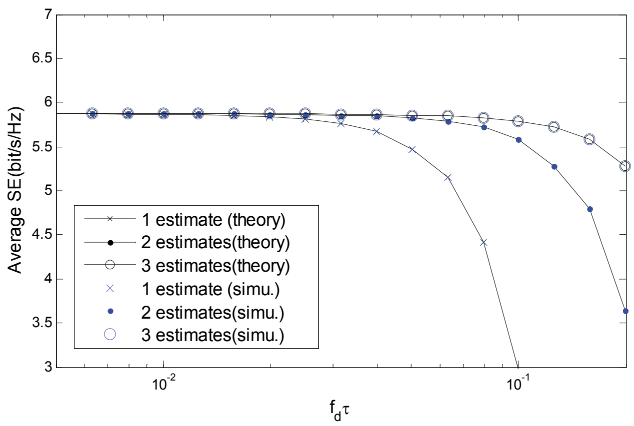

Fig. 1. Average SE for 2T2R3U system with multiple outdated estimates.

The theoretical average SE is calculated by using (23) with the switching thresholds defined in (11). It is observed that the theoretical SE is in good agreement with the simulation result, and thus the derived theoretical expression (23) is valid. It is shown that multiple outdated estimates method can obviously reduce the effect of the time delay. This is because the former makes fully use of multiple outdated channel information to decrease the uncertainty of feedback information. As a result, the system with MOE can obtain much higher SE than that with the conventional SOE, especially for large time delay.

In Fig. 2, we give the average PER versus $f_d\tau$ for 2T2R3U system with CLD under imperfect CSI, where MOE and SOE are used for comparison. The theoretical average PER is calculated by using (17) with the switching thresholds defined in (11). From this figure, we can see that multiple outdated estimates method enables the system to tolerate larger delay. It is shown that the CLD with MOE method can tolerate the normalized time delay up to about 0.01 with a slight degradation in the average PER. But when $f_d\tau$ increases beyond 0.01, the PER performance will degrade increasingly. The CLD with single estimate fails to meet the target PER ($P_0 = 0.1$) at $f_d\tau = 0.062$, the CLD with two estimates fails at $f_d\tau = 0.12$, while the CLD with three estimates fails at $f_d\tau = 0.21$, which means that three-estimate method can tolerate larger delay than the former two due to relatively more feedback information. Besides, the theoretical average PER agrees with the simulation for different time delays, and thus the derived average PER expressions of CLD with imperfect feedback are also valid.

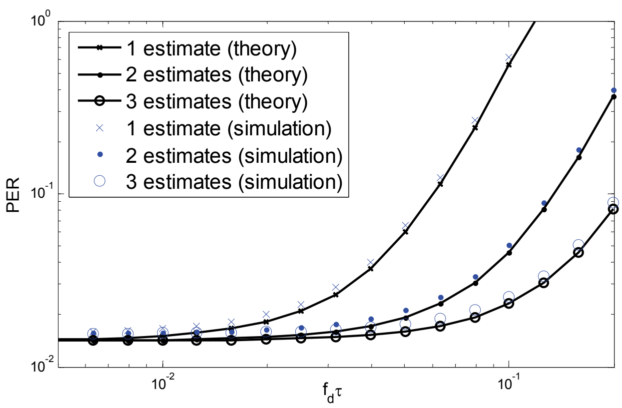

Fig. 2. Average PER for 2T2R3U system with multiple outdated estimates.

In Fig. 3, we plot the average SE versus average SNR $\bar{\gamma}$ for multiuser MIMO system with CLD and MOE under imperfect CSI, where 2T1R1U and 2T1R3U systems are considered for comparison, and $f_d\tau = 0.1$. The average SE is calculated by using (23) with the switching thresholds defined in (11). It is shown that CLD is able to increase SE with SNR, and multiuser 2T1R3U system can obtain higher SE than the corresponding single user 2T1R1U system due to the available multiuser diversity. From this figure, we can see that the systems with MOE have higher SE than those with the conventional SOE due to the reason analyzed in Fig. 1. The system SE with 3 estimates is higher than that with 2 estimates, and the system SE with 2 estimates is higher than that with 1 estimate. Moreover, with the outdated channel information increase, the obtained SE increment will decrease. This is because the added outdated channel information is too old to provide more reliable channel information for the system.

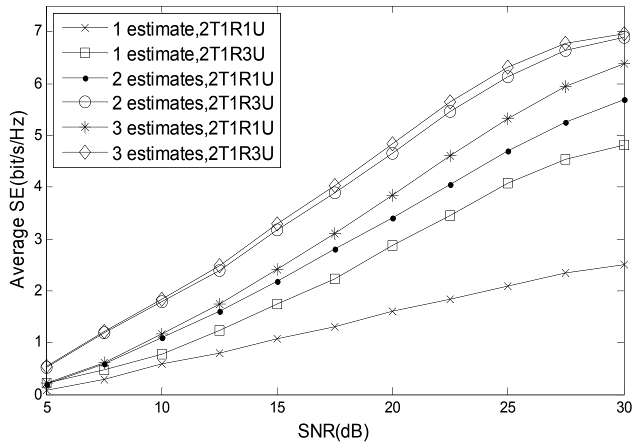

Fig. 3. Average SE for multiuser MIMO system with multiple outdated estimates.

In Fig. 4, we give the average PER versus average SNR $\bar{\gamma}$ for multiuser MIMO system with CLD and MOE under imperfect CSI, the system configurations are the same as Fig. 3, and $f_d\tau = 0.1$. The average PER is calculated by using (17) with the switching thresholds defined in (11). Some results similar to Fig. 3 can be found. Namely, multiuser 2T1R3U system has lower PER than the single user 2T1R1U system, and the systems with MOE have lower PER than those with the conventional SOE. Moreover, from Figs. 1-4, it is found that two outdated estimates can obtain obvious performance superiority over single outdated estimate, and has less complexity than other multiple outdated estimates (more than two). Thus, it provides a tradeoff between the performance and complexity. Based on this, in the following figures, we use it for the performance evaluation and comparison.

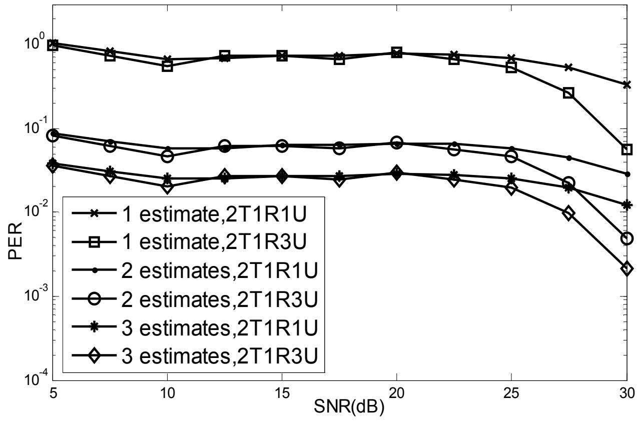

Fig. 4. Average PER for multiuser MIMO system with multiple outdated estimates.

Considering the influence of imperfect CSI on the switching thresholds, we use the VTs instead of conventional FTs to improve the system performance further. Fig. 5 and Fig. 6 show the average SE and PER of 3T1R2U with CLD under imperfect feedback, where FTs and VTs are used for the performance evaluation. In Fig. 5, we plot the average SE vs. the normalized time delay ($f_d\tau$), the average SE is calculated by using (23) with VTs defined in (27). It is found that the SE with VTs is higher than that with FTs, especially for large time delay, the SE increment is obvious. This is because the application of VTs can lower the system PER (as shown Fig. 6), which will bring about the decrease of the average number of transmissions $\bar{N}$ according to (22). As a result, the increase of overall average SE is obtained. Moreover, the derived theoretical SE and the corresponding simulation are very close for both VTs and FTs. Besides, the SE with two estimates is obviously higher than that with conventional single estimate as expected.

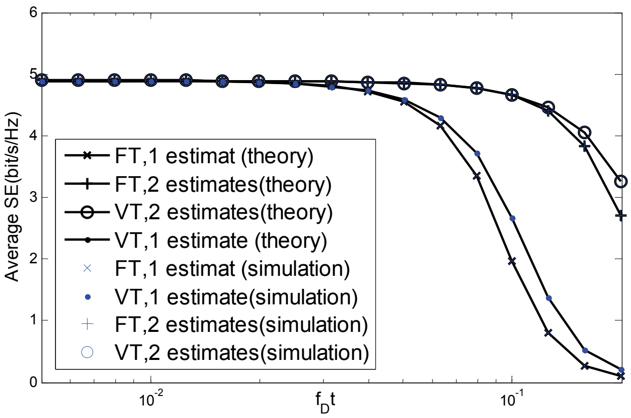

Fig. 5. Average SE for multiuser MIMO system with FTs and VTs for different estimates.

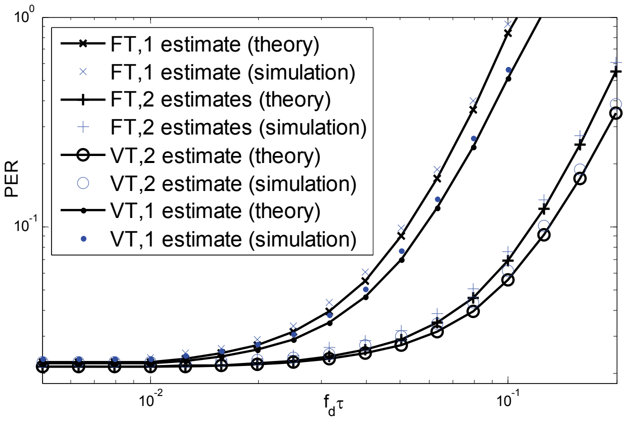

Fig. 6. Average PER for multiuser MIMO system with FTs and VTs for different estimates.

In Fig. 6, we plot the average PER vs. the normalized time delay ($f_d\tau$) for 3T1R2U with CLD, where VTs are from (27). It is found that the PER with VTs is lower than that with FTs, especially for large time delay, the performance superiority becomes obvious. Moreover, the derived theoretical PER can match the corresponding simulation well for both VTs and FTs, which testifies that the derived theoretical expression is valid for PER performance evaluation. Besides, the average PER with two estimates is obviously lower than that with single estimate, which indicates the two outdated estimates is a practical method in view of performance and complexity.

7. Conclusions

Based on multiple outdated channel information, we have studied multiuser MIMO system with CLD-AS in Rayleigh fading channel. By the performance analysis, the average PER and SE of the system subject to target PLR have been derived. As a result, closed-form expressions of average PER and SE are achieved. They include the existing PER and SE expressions employing one outdated estimate as special cases, and can match the corresponding simulations very well. Thus, with these expressions, CLD performance in multiuser MIMO system can be effectively assessed, and the impact of delayed CSI on the system performance can be analyzed well. Moreover, multiple estimates method can bring about the obvious performance improvement of the system when compared to conventional single estimate method. By means of the maximum a posteriori method and approximate PER expression, we have derived the variable switching thresholds as well. These VTs include the conventional FTs as special cases, can adapt to the delayed feedback information, and reduce the effect of feedback delay. Simulation results show the presented CLD with MOE can obtain better SE and PER performance than the existing CLD with SOE because multiple outdated channel information is fully utilized. It is shown that two outdated estimates can implement effective tradeoff between the performance and complexity. Furthermore, the proposed variable thresholds method can also improve the system performance effectively, and it performs better than conventional fixed thresholds method, especially for large time delay.

In this paper, we mainly address the performance study of multiuser CLD with AS under homogeneous case. In practice, however, the statistics of the SNRs of users may not be identical since individual users may locate at different distances from the BS. So the users may have different path loss and average SNRs, which is referred to as heterogeneous case. For this reason, in the future work, we will further study the performance of the multiuser CLD system under heterogeneous case where both path loss and small-scale fading (Rayleigh fading) are considered. It is expected that some practical results can be achieved.

$$I1_n = \frac{L}{\Gamma(n_R)}\sum_{m=0}^{L-1}\binom{L-1}{m}(-1)^m \sum_{c=0}^{m(n_R-1)} \omega_{c,m}\sum_{\xi=0}^{\infty}\rho^{\xi}\left(1-\rho\right)^{n_R+c}\left[1-\frac{\Gamma\left(\xi+n_R,\gamma_{pn}/\left(\left(1-\rho\right)\bar{\gamma}\right)\right)}{\Gamma\left(i+n_R\right)}\right]$$
$$\times\frac{\left[\Gamma\left(n_R+c+\xi,\gamma_n\left(\frac{m+1}{\bar{\gamma}}+\frac{\rho}{\left(1-\rho\right)\bar{\gamma}}\right)\right)-\Gamma\left(n_R+c+\xi,\gamma_{n+1}\left(\frac{m+1}{\bar{\gamma}}+\frac{\rho}{\left(1-\rho\right)\bar{\gamma}}\right)\right)\right]}{\Gamma\left(\xi+1\right)\left[\left(1-\rho\right)\left(m+1\right)+\rho\right]^{n_R+c+\xi}} \quad (18)$$

$$I2_n = \frac{L}{\Gamma(n_R)\bar{\gamma}^{n_R}}\frac{a_n}{\left[g_n\left(1-\rho\right)\bar{\gamma}+1\right]^{n_R}}\sum_{m=0}^{L-1}\binom{L-1}{m}(-1)^m\sum_{c=0}^{m(n_R-1)}\frac{\omega_{c,m}}{\bar{\gamma}^c}\left(m+1+\frac{\rho g_n\bar{\gamma}}{g_n\left(1-\rho\right)\bar{\gamma}+1}\right)^{-(n_R+c)}$$
$$\times\left[\Gamma\left(n_R+c,\gamma_n\left(\frac{m+1}{\bar{\gamma}}+\frac{\rho g_n}{g_n\left(1-\rho\right)\bar{\gamma}+1}\right)\right)-\Gamma\left(n_R+c,\gamma_{n+1}\left(\frac{m+1}{\bar{\gamma}}+\frac{\rho g_n}{g_n\left(1-\rho\right)\bar{\gamma}+1}\right)\right)\right] \quad (19)$$

$$I3_n = \frac{La_n}{\Gamma(n_R)}\sum_{m=0}^{L-1}\binom{L-1}{m}(-1)^m\sum_{c=0}^{m(n_R-1)}\omega_{c,m}\sum_{\xi=0}^{\infty}\left[g_n\left(1-\rho\right)\bar{\gamma}+1\right]^{\xi+n_R}\left[1-\frac{\Gamma\left(\xi+n_R,\gamma_{pn}/\left(\left(1-\rho\right)\bar{\gamma}\right)\right)}{\Gamma\left(\xi+n_R\right)}\right]$$
$$\times\frac{\rho^{\xi}\left(1-\rho\right)^{n_R+c}\left[\Gamma\left(n_R+c+\xi,\gamma_n\left(\frac{m+1}{\bar{\gamma}}+\frac{\rho}{\left(1-\rho\right)\bar{\gamma}}\right)\right)-\Gamma\left(n_R+c+\xi,\gamma_{n+1}\left(\frac{m+1}{\bar{\gamma}}+\frac{\rho}{\left(1-\rho\right)\bar{\gamma}}\right)\right)\right]}{\Gamma\left(\xi+1\right)\left[\left(1-\rho\right)\left(m+1\right)+\rho\right]^{n_R+c+\xi}} \quad (20)$$

Acknowledgements

The authors would like to thank the anonymous reviewers for their valuable comments and suggestions which improve the quality of this paper greatly. This work is supported in part by Open Research Fund of National Mobile Communications Research Laboratory of Southeast University (2012D17), Aeronautical Science Foundation of China (20120152001), National Natural Science Foundation of China (61172077), and China Postdoctoral Science Foundation (2013M541661).

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