# Low Complexity Noncoherent Iterative Detector for Continuous Phase Modulation Systems

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**Abstract.** *This paper focuses on the noncoherent iterative detection of continuous phase modulation. A class of simplified receivers based on Principal-Component-Analysis (PCA) and Exponential-Window (EW) is developed. The proposed receiver is evaluated in terms of minimum achievable Euclidean distance, simulated bit error rate and achievable capacity. The performance of the proposed receiver is discussed in the context of mismatched receiver and the equivalent Euclidean distance is derived. Analysis and numerical results reveal that the proposed algorithm can approach the coherent performance and outperforms existing algorithm in terms of complexity and performance. It is shown that the proposed receiver can significantly reduce the detection complexity while the performance is comparable with existing algorithms.*

## Keywords

Continuous phase modulation, noncoherent iterative detection, time-varying phase noise, serially concatenated systems, minimum achievable distance, principal component analysis, mismatched receiver, wireless communication.

## 1. Introduction

Continuous-Phase-Modulation (CPM) is a class of coded modulations with good power and bandwidth efficiency [1], which has been widely used in wireless communication systems. As shown in some previous works [2–5] that CPM can offer near capacity performance in various scenarios such as satellite communication, deep space communication, optical communication, digital video broadcasting (DVB) [6–9] and the discussion has generalized to CPM-based multiuser systems very recently [10–14], which show that CPM-based system can significantly improve the spectral-efficiency.

However, previous discussion presumed coherent detection which requires perfect acquisition of channel state information at the receiver side. This is usually unavailable in practice. On the other hand, noncoherent detection makes itself an attractive strategy due to the fact that no explicit phase estimation is required. It is firstly shown in [15] that noncoherent receiver using multiple-symbol differential detection can perform close to the coherent receiver with exponential complexity. As a matter of fact, it was analytically explained in [1] that the performance of the maximum-likelihood noncoherent detection is actually identical to coherent receiver in terms of Euclidean distance. Inspired by this work, some simplified noncoherent detectors were later developed in [16–20]. It is shown that near coherent performance is obtained with significantly reduced complexity.

Though achieving near coherent performance, existing algorithms aforementioned exhibit considerable complexity for CPMs of large alphabet size, i.e. $M \geq 4$. Therefore, in this paper we develop a reduced complexity noncoherent iterative receiver for uncoded and coded CPM system. The proposed receiver is built upon two modules: a low-dimensional front-end and a trellis-based detector. The front-end is based upon Principal-Component-Analysis (PCA) [21] which is in particular suited for partial response system based on the technique reported in [22, 23]. The trellis-based detector followed employs Exponential-Window (EW) [24] to further reduce the complexity of detection complexity.

Another concern is to evaluate the performance of the proposed receiver subject to time-varying phase noise, which could be introduced by the channel or inaccurate estimated carrier frequency. To achieve a better performance, we employ a factor to weight the soft metric delivered from the outer convolutional decoder to the inner CPM detector. The optimum value of this factor is obtained through exhaustive simulations. It turns out that the performance of the proposed receiver offers better performance tackling the time-varying phase noise if the factor is properly chosen.

This paper is organized as follows. The system model is presented in Section 2, Section 3 presents the simplified receivers and derives the equivalent Euclidean distance. The proposed receiver is compared to some existing ones in terms of complexity. Section 4 gives the numerical and simulated results and Section 5 concludes the paper.

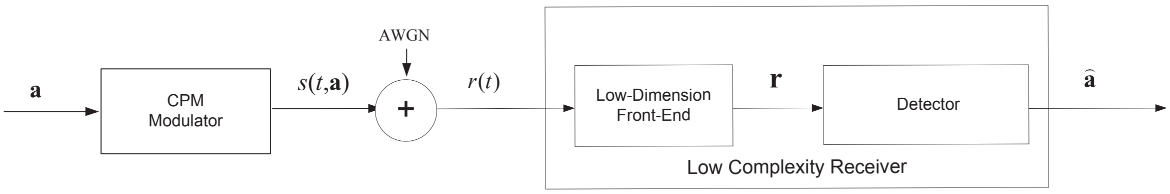


**Fig. 1.** System model.

## 2. System Model

The system model given in Fig. 1. The modulator generates the transmitted signal $s(t,\mathbf{a})$ given the $M$-ary information sequence $\mathbf{a}$=$\{c_0,c_1,\ \dots\ ,c_{N-1}\}$, and the complex baseband signal of CPM in the time interval $nT < t < (n+1)T$ is defined as [1]

$$s(t,\mathbf{a}) = \sqrt{\frac{2E_s}{T}}\exp\{j2\pi h\sum_{i=0}^{n} a_i q(t-iT)\} \tag{1}$$

where $E_s$ is the energy per symbol, $T$ is the symbol interval, $h = k/p$ is the modulation index ($k$ and $p$ are relatively prime integers), and the symbols $a_i$ are assumed independent and takes on values from the $M$-ary alphabet $\{\pm1,\pm3,\ \dots\ ,\pm(M-1)\}$. The function $q(t)$ is the phase response and its derivative is the frequency pulse, assumed of duration $L$. The information bearing phase $\theta(t,\mathbf{a})$ is defined accordingly as [1]

$$\begin{aligned}\theta(t,\mathbf{a}) &= 2\pi h\sum_{n=0}^{k} a_n q(t-nT)\\ &= \pi h\sum_{n=0}^{k-L} a_n + 2\pi h\sum_{k=N-L+1}^{k} a_n q(t-nT)\\ &= \theta_n + \theta(t),\end{aligned} \tag{2}$$

$$\theta_n \triangleq [\pi h\sum_{i=0}^{n-L} a_i]_{mod\ 2\pi}, \tag{3}$$

$$\theta(t) \triangleq 2\pi h\sum_{i=n-L+1}^{n} a_i q(t-iT) \tag{4}$$

where $\theta_n$ is the *accumulated phase*, and $\theta(t)$ is the *incremental phase* within one interval. At the receiver side, coherent trellis defined accordingly as shown in [1].

In this paper, we focus on the Additive-White-Gaussian-Noise (AWGN) channel wherein the received signal $r(t)$ reads

$$r(t) = s(t,\mathbf{a})e^{j\varphi_n} + n(t) \tag{5}$$

where $\varphi_{n \bmod 2\pi} \in [0,2\pi]$ is the phase noise assumed random and $n(t)$ is a zero-mean circularly symmetric white Gaussian noise process of two-sided power spectral density $N_0/2$. The phase noise is modeled as a discrete time random walk (Wiener) process defined as [24]:

$$\varphi_n = \varphi_{n-1} + \sigma_n \tag{6}$$

where $\varphi_n$ are assumed to be i.i.d Gaussian random variable. The normalized Euclidean distance is defined as

$$d^2(\mathbf{a},\mathbf{b}) = \frac{1}{2E_b}\int_0^{NT} |s(t,\mathbf{a}) - s(t,\mathbf{b})|^2\,dt, \tag{7a}$$

$$d^2_{min} = \min_{all\ \mathbf{a}\neq\mathbf{b}} d^2(\mathbf{a},\mathbf{b}) \tag{7b}$$

where $E_b$ is the average energy per information bit.

## 3. Low Complexity Iterative Receiver

The proposed receiver consists of two modules: front-end and detector. The front-end adopted is in fact a mismatched filter. The main idea is using an alternative signal space $s_R(t,\mathbf{a})$, whose size is much smaller than the original signal space $s(t,\mathbf{a})$. These filters should be optimized first such that the minimum achievable distance is maximized. The detector followed is also defined over $s_R(t,\mathbf{a})$ which reduces the search effort for optimum detection. As we shall see latter, the proposed receiver can reduce the complexity significantly while results in little performance loss.

### 3.1 Generalized PCA

Similar to [14, 21], the method presented here is based on eigenvalue analysis. The main idea is utilizing an alternative low-dimensional signal space $s_R(t,\mathbf{a})$ by shortening $L$ to $L_R$ at the receiver side. Comparing to the conventional method, the dimensionality is reduced from $D = M^L$ to $K(<< D)$. The details are presented below.

1. Calculate the correlation matrix $R_{D\times D}$ which is defined as
   $R_{D\times D} = \langle s_R(t,\mathbf{a}), s_R(t,\mathbf{a}))\rangle$,
   $(s_R(t,,\mathbf{a}) = \left[s_{R_0}(t,\mathbf{a}),\dots,s_{R_{D-1}}(t,\mathbf{a})\right])$;
2. Using eigenvalue decomposition we have
   $R_{D\times D} = Q_1 diag\left[\lambda_0,\dots,\lambda_{D-1}\right] Q_1^H$;

3. The orthogonal basis are obtained as $\beta(t) = diag[\lambda_0, \ldots, \lambda_{D-1}] Q_1 s(t)$;
4. The constellations are $\mathbf{s} = \langle s(t, \mathbf{a}), \varphi(t) \rangle$.

Only those basis having positive eigenvalue are considered effective (of which the number is $D$) to construct the low-dimensional front-end for CPMs.

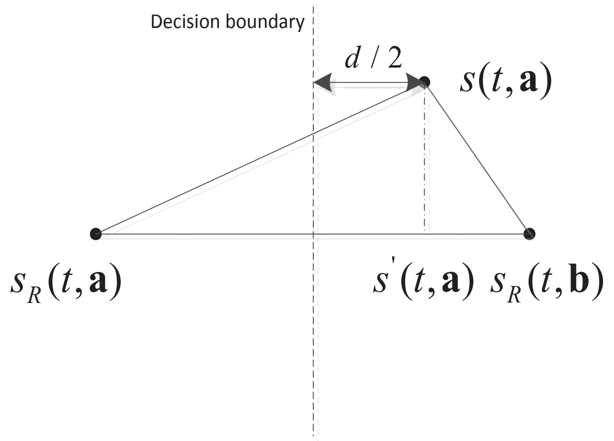


**Fig. 2.** Geometric interpretation of $d$.

Another module of the receiver is the mismatched detector. Now the performance of the proposed technique is evaluated in terms of equivalent minimum achievable Euclidean distance $d^2$. Define $\boldsymbol{r}, \boldsymbol{s_T}, \boldsymbol{n}, \boldsymbol{s}$ and $\boldsymbol{s'}$ the vector representation of $r(t)$, $n(t)$, $s(t,\mathbf{a})$, $s_R(t,\mathbf{a})$ and $s_R(t,\mathbf{b})$ given a set of orthogonal bases $\boldsymbol{\beta}(t) = \{\beta_1(t), \ldots, \beta_D(t)\}$, respectively. The detection problem is now formulated as [14]

$$|\boldsymbol{r} - \boldsymbol{s}|^2 \underset{\mathbf{a}}{\overset{\mathbf{b}}{\gtrless}} |\boldsymbol{r} - \boldsymbol{s'}|^2 \tag{8}$$

which is equivalent to

$$|\boldsymbol{s}|^2 - |\boldsymbol{s'}|^2 - 2\Re < \boldsymbol{r}, \boldsymbol{s} > + 2\Re < \boldsymbol{r}, \boldsymbol{s'} > \underset{\mathbf{a}}{\overset{\mathbf{b}}{\gtrless}} 0. \tag{9}$$

Let $y = |\boldsymbol{s}|^2 - |\boldsymbol{s'}|^2 - 2\Re < \boldsymbol{r}, \boldsymbol{s} > + 2\Re < \boldsymbol{r}, \boldsymbol{s'} >$, which is a Gaussian random variable by definition. The mean $m$ and variance $\sigma^2$ of $y$ are, respectively

$$m = |\boldsymbol{s}_T - \boldsymbol{s}|^2 - |\boldsymbol{s}_T - \boldsymbol{s'}|^2 \tag{10}$$

and

$$\sigma^2 = 2N_0 |\boldsymbol{s} - \boldsymbol{s'}|^2. \tag{11}$$

Therefore, the probability that the transmitted sequence $\mathbf{a}$ is wrongly detected as $\mathbf{b}$ is

$$\begin{aligned} \mathbf{P}(y > 0) &= Q\left(\sqrt{\frac{m^2}{\sigma^2}}\right) \\ &= Q\left(\sqrt{\frac{\left[|\boldsymbol{s}_T - \boldsymbol{s}|^2 - |\boldsymbol{s}_T - \boldsymbol{s'}|^2\right]^2}{2E_b |\boldsymbol{s} - \boldsymbol{s'}|^2} \cdot \frac{E_b}{N_0}}\right) \end{aligned} \tag{12}$$

where $E_b$ is the average transmitted energy per information bit. The equivalent Euclidean distance is readily recognized as

$$\begin{aligned} d^2 &= \frac{1}{2E_b} \frac{\left[|\boldsymbol{s}_T - \boldsymbol{s}|^2 - |\boldsymbol{s}_T - \boldsymbol{s'}|^2\right]^2}{|\boldsymbol{s} - \boldsymbol{s'}|^2} \\ &= \frac{1}{2E_b} \left[\frac{|s(t,\mathbf{a}) - s_R(t,\mathbf{b})|^2 - |s(t,\mathbf{a}) - s_R(t,\mathbf{a})|^2}{|s_R(t,\mathbf{a}) - s_R(t,\mathbf{b})|}\right]^2. \end{aligned} \tag{13}$$

As expected, (13) coincides with the result in [21, 25]. It is noticed that $d^2$ is positive by definition but is not additive. Therefore, no efficient method but an exhaustive search is employed to find this quantity in most cases. A geometric interpretation of $d$ is shown in Fig. 2. Except minimum achievable distance $d^2$, the performance of the PCA-based receiver is measured in terms of the average distance loss which defined as

$$\delta = \frac{1}{M^{2L}} \sum_{i=0}^{M^L} \sum_{j=0}^{M^L} d_{ij} - d'_{ij} \tag{14}$$

where $d_{ij}$ and $d'_{ij}$ denotes the Euclidean distance over one symbol interval between two signals, for the optimal (full-rank) receiver, and for the $D$ rank approximation, respectively. Another parameter indicating the performance loss is the average energy loss which is defined as

$$\varepsilon = \sum_{i=0}^{M^L} \lambda_i. \tag{15}$$

## 3.2 Achievable Capacity

The capacity of a communication system is defined as the maximum mutual information between the channel input and the channel output over all possible input distributions. Unfortunately, unlike the conventional memoryless modulation, it is impossible to obtain a closed-form expression for CPM based system. However, some recent results [14, 26] reveal that the capacity of such a system can be calculated numerically. Therefore, in this paper we generalize the discussion to noncoherent detection.

At the $i$th epoch, designate the transmitted symbol, assuming the input $a_i$ is uniformly distributed, the average information rate $C$ is calculated as [14]

$$\begin{aligned} C &= \lim_{N\to\infty} \frac{1}{N} I\left(a_1^N, r_1^N\right) \\ &= \lim_{N\to\infty} \frac{1}{N} H\left(a_1^N\right) - H\left(a_1^N | r_1^N\right) \\ &= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} H(a_i) - \sum_{i=1}^{N} H(a_{i-1} | a_1^{i-1}, r_i^N) \\ &= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \log_2 M - \sum_{i=1}^{N} H(a_i | a_1^{i-1}, r_i^N) \\ &\leq \log_2 M \quad (SNR \to \infty) \end{aligned} \tag{16}$$

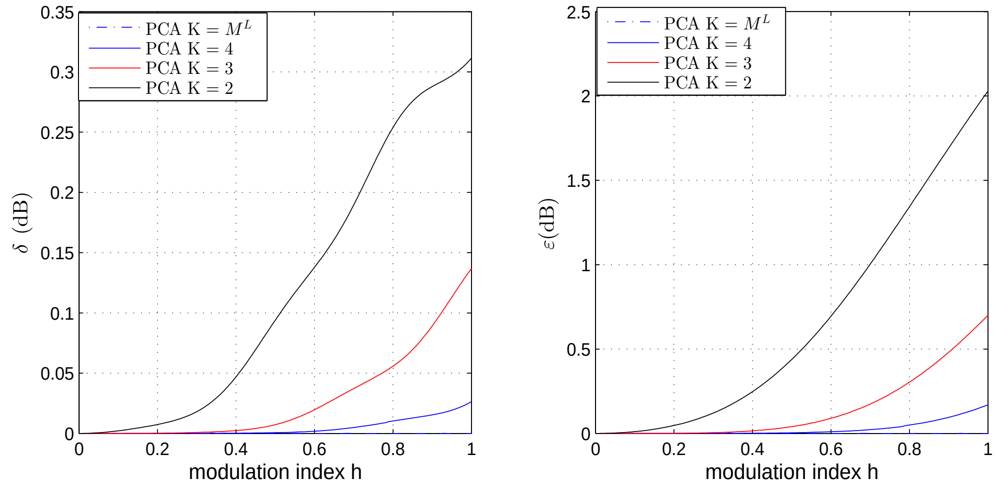


**Fig. 3.** The performance degradation by reducing the rank from $M^L = 16$ to $K = 2,3,4$.

where $M$ is the modulation level. The chain rule [26] and $H(a_i|a_1^N) = H(a_i) = \log_2 M$ are used. The upper bound $\log_2 M$ is achievable as the Signal-to-Noise-Ratio (SNR) is sufficiently large. The problem is now to calculate $H(a_i|a_1^N, r_i^N)$ which is rewritten as

$$H(a_i|a_1^{i-1}, r_i^N) = -E\left[\log_2 p\left(a_i|a_1^{i-1}, r_1^N\right)\right]. \qquad (17)$$

The quantity $H(a_i|a_1^{i-1}, r_i^N)$ is usually obtained through Monte-Carlo calculation [26] and the APP $p\left(a_i|a_1^{i-1}, r_1^N\right)$ is obtained employing the N-SISO algorithm proposed. Therefore, the performance of N-SISO can also be evaluated by the achievable capacity which will be shown later.

## 3.3 Serially Concatenated CPM

It is shown in [2–4] that serially concatenated CPM offers near capacity performance. A detailed discussion in [2–4] reveals that the performance is significantly improved due to the interleaver-gain. In this paper, both uncoded and coded CPM are considered. We show that the proposed noncoherent CPM receiver can successfully be applied to the concatenated system.

The details of the coded system are demonstrated in Fig. 4. The information sequence $\mathbf{u}$ is first coded by the outer encoder. The interleaved code $\mathbf{a}$ is modulated by CPM and transmitted to the AWGN channel. At the receiver side, the CPM receiver based on N-SISO generates the *a posterior probability* (APP) $p(a_i|\mathbf{r})$ of the code symbol $a_i$. The APPs are then deterleaved and passed to the outer decoder which manages to generate the prior information of the code symbol $p(a_i)$. The a priori information $p(a_i)$ is weighted by a factor before feeding it to the CPM receiver. This process repeats several times and finally obtains a decision of $\mathbf{u}$ designated as $\hat{\mathbf{u}}$. This process is visualized in Fig. 4. It should be pointed out that the performance of such a system relies heavily on the CPM receiver when noncoherent detection is considered. Therefore, our goal is to design a CPM receiver

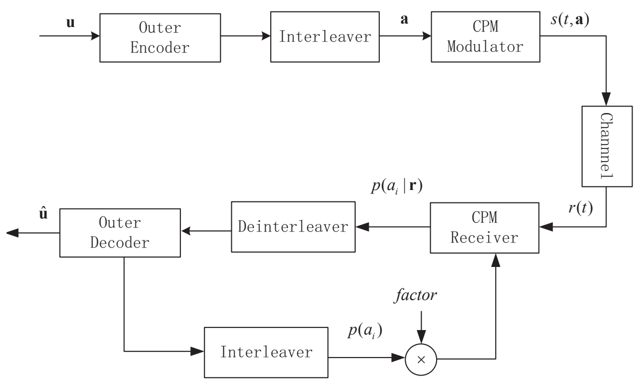


**Fig. 4.** The system model of the serially concatenated CPM.

which is able to successfully abstract the APPs noncoherently when subject to a time-varying phase noise.

## 3.4 Noncoherent Iterative Detector

The noncoherent soft-input-soft-output (N-SISO) detector presented in this paper is actually a bi-directional Viterbi algorithm employing forward and backward search to calculate the APP of each transmitted symbol. This quantity is later used to make hard decision (i.e., uncoded system) or fed to decoder for joint iterative detection (i.e., serially concatenated CPM). The details of this algorithm are presented below.

Let $s \triangleq (a_{n-Q}, a_{n-Q+1}, \ldots, a_{n-1})$ be the state of the noncoherent trellis at $k$-th epoch, wherein $Q$ is an integer incorporating the phase memory. The corresponding sufficient statistics from epoch 0 to epoch $n$ are denoted by $\mathbf{r}_0^n = (\mathbf{r}_0, \mathbf{r}_1, \ldots, \mathbf{r}_n)$. With $s'$ and $s$ being the start and end state, respectively. We have the following definitions of the branch metric [1, 24]

$$\Gamma_n(s',s) \propto \ln \frac{I_0\left(\left|\frac{2}{N_0}\sum_{i=0}^{n}\mathbf{r}_i\mathbf{s}_i^H\right|\right)}{I_0\left(\left|\frac{2}{N_0}\sum_{i=0}^{n-1}\mathbf{r}_i\mathbf{s}_i^H\right|\right)} \tag{18}$$

where $\mathbf{s}_n^H$ is the conjugate transpose of $\mathbf{s}_n$, and $I_0(|x|)$ is the zeroth-order modified Bessel function of the first kind, which can be calculated approximately by $I_0(|x|) \cong e^{|x|}$.

Therefore, the accumulated metric $A_{n+1}(s)$ is updated recursively as

$$A_{n+1}(s) = \max_{s'}(A_n(s') + \Gamma_n(s',s)). \tag{19}$$

Since $I_0(|x|)$ is nonlinear, it is difficult to calculate effectively over the entire received sequence. However, by introducing EW [24] into the branch metric calculation, define *beginning phase* for state $s$ at epoch $k$ as

$$\phi_n(s) \triangleq 2\pi h \sum_{i=0}^{n-L} \hat{a}_i \mod 2\pi \tag{20}$$

where $\{\hat{a}_i\}$ is the sequence associated to the survival state $s$, which is updated in a per-survivor processing (PSP) [25] approach. Then applying the EW into calculating branch metric, we obtain

$$\Gamma_n(s',s) = \frac{2}{N_0}(|q_{n-1}(s') + e^{j\phi_n(s)}\mathbf{r}_k\mathbf{s}_k^H| - |q_{n-1}(s')|) \tag{21}$$

where $I_0(|x|) \cong e^{|x|}$ is used, and $q_{n-1}(s')$ is designated as *phase reference symbol* which corresponds to an unlimited phase memory increasing with time. This quantity can be recursively updated as

$$q_n(s) \triangleq \eta \cdot q_{n-1}(s') + e^{j\phi_n(s)}\mathbf{r}_n\mathbf{s}_n^H \tag{22}$$

where $\eta \in (0,1)$ is the so-called *forgetting factor*. By adjusting the *forgetting factor*, the branch metric is actually flexible against time-varying phase noise. Generally speaking, the smaller $\eta$ is the more robust the algorithms is under time-varying channels. Moreover, we use the technique of reduced state sequence detection (RSSD) [27] to further reduce the complexity of N-SISO. The proposed was previously successfully used in high mobility systems, where the phase noise is time varying. The APP of $a_i$ could be evaluated recursively based on the N-SISO proposed below.

## 4. Numerical and Simulated Results

The performance of the proposed receiver is evaluated in this section in terms of complexity and simulated Bit-Error-Rate (BER). Firstly, the PCA-based receiver is evaluated using the measurements $\delta$ and $\varepsilon$ illustrated by quaternary CPM2RC in Fig. 3, which shows $K = 4$ is good enough to obtain near optimum performance, while the rank of signals is reduced significantly. The maximum loss no matter in terms of $\delta$ or $\varepsilon$ is marginal ($\ll 0.1$).

**Algorithm of Noncoherent Soft-Input Soft-Output (N-SISO)**

**Define**
$A_n(s) = \log(\alpha_n(s))$,
$B_n(s) = \log(\beta_n(s))$,
$\Gamma_n(s',s) = \log(\gamma_n(s',s))$.
**Initialization**
$A_n(0) = Constant \quad (Constant \gg 1)$,
$B_n(0) = Constant \quad (Constant \gg 1)$,
$\varphi_n(s) = 0 \quad (any \quad s \in S)$
in which $\varphi_n(s)$ is the *beginning phase* of state $s$ at time $kT$.
**Forward Recursion**
$A_n(s) = \max_{s'}(A_{n-1}(s') + \Gamma_{n-1}(s',s))$,

$\varphi_n(s) = \varphi_{n-1}(s') + 2\pi h \sum_{i=n-L+1}^{n} \hat{a}_i q(t - iT)$,

$\hat{a}_i$ is the tentative decision, *beginning phase* is updated according to the corresponding survival path. Note, $\Gamma_{n-1}(s',s)$ could not be obtained until forward recursion is completed. After forward recursion is finished, all $\Gamma_{n-1}(s',s)$ are stored for backward recursion.
**Backward Recursion**
$B_n(s') = \max_{s}(B_n(s)) + \Gamma_{n-1}(s',s))$,
**Output Soft Metric**
$\ln P(a_i = a|r_0^{N-1}) = \max_{(s',s)}(A_n(s') + \Gamma_n(s',s) + B_{n+1}(s))$.
This value is for hard-decision, if fed to outer decoder, it has to subtract $\ln P(a_i)$

We then evaluate the performance of the proposed receiver in terms of minimum achievable distance, which is shown in Fig. 5. It is readily seen that the performance loss using PCA-based receiver is negligible by shortening $L = 2$ to $L_R = 1$. The maximum loss is usually no more than 0.2 dB. When modulation index $h$ is approaching 1, both optimum and PCA receiver experience significant loss due to the fact that $h = 1$ is the so called *weak index* [1]. Therefore, we shall avoid this quantity to make sure the minimum achievable distance is maximized.

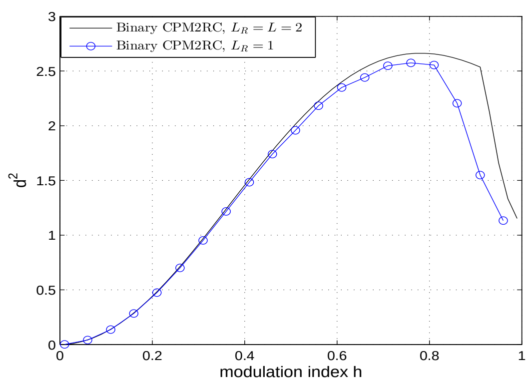


**Fig. 5.** The minimum achievable distance of optimum receiver and PCA receiver.

Then we proceed to the discussion of the noncoherent detection of both uncoded and Serially-Concatenated-CPM

(SCCPM) systems [2–4]. The outer coding scheme is $(7,5)_8$ convolutional code. The slow time-varying phase noise is considered in this paper. The optimal $\eta$ is 0.95, $Q$ is set to be 2 and RSSD is adopted.

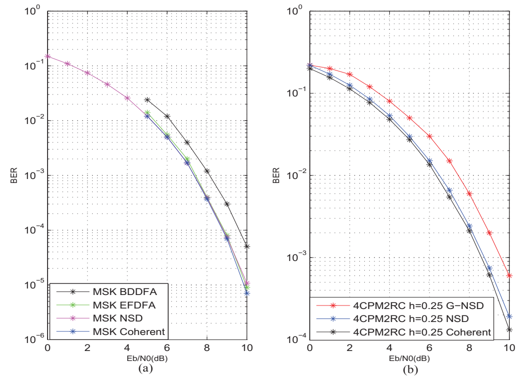


**Fig. 6.** BER comparison of the proposed N-SISO and some existing noncoherent detection algorithms for binary CPM1REC with $h = 0.5$ and quaternary CPM2RC with $h = 0.25$ [14]. The algorithms for comparison are from [16](BDDFA, EFDFA NSD) and [28] (G-NSD).

The noncoherent detection of uncoded CPMs such as binary CPM of 1REC (i.e., MSK) and quaternary of 2RC (i.e., 4CPM2RC) is shown in Fig. 6. It is observed that all algorithms obtain the near coherent performance eventually. However, the proposed algorithm performs slightly better than other existing algorithms [16, 17, 28]. Though N-SISO and G-NSD receiver require 16 and 64 states, respectively, it can be seen that N-SISO performs about 0.5 dB better than G-NSD. Based on the results in Fig. 6, it can be concluded that N-SISO not only reduces the complexity of branch metric but has a better performance even with fewer state number, due to the fact that N-SISO can fully exploit the Markovity/memory of CPM by using EW.

The discussion is now generalized to coded CPMs. The APPs obtained by detector are first weighted by a factor and then passed to the decoder. For serially concatenated MSK (SCMSK), an improvement of about 5 dB is obtained by increasing the number of iterations to 5. For 10 iterations, performance loss compared to coherent detection is narrowed down to 1 dB. It is also seen that the improvement due to the refined factor is quite obvious. However, in most cases this quantity can not be predetermined but obtained through exhaustive simulations.

In Fig. 8, a serially concatenated quaternary CPM (SCQCPM) is evaluated. First, a 8-state N-SISO is tested, which is not good enough for N-SISO even with more iterations. As a contrast, a 16-state N-SISO with 5 iterations is a good trade-off between the performance and complexity. This result reveals that RSSD affects more on the '*quality*' of the reliability.

An interesting phenomenon is that for full response CPM (SCMSK), doubling the number of iterations could improve the performance by about 0.5 dB, but for partial response system (SCQCPM) only 0.1 dB is obtained. Thus, if extra improvement of SCQCPM is expected, state number has to be further increased. But in practice, 5 iterations may meet the requirement, more iterations gain a little but bring more time delay and complexity.

Finally, we compare the proposed detector with some previously developed detectors in terms of complexity and simulated bit error rate. In this case, the time-vary phase noise is considered. The stand derivation of the phase noise $\phi_n$ is 5° or 15°, corresponding to moderate and strong phase noise, respectively. The optimal *forgetting factor* is obtained by simulations. It is seen from Fig. 9 that the proposed receiver actually is more robust than existing detectors such as Tikhonov [17], dp-BCJR [17], GA [20] and A-SISO [18] proposed before.

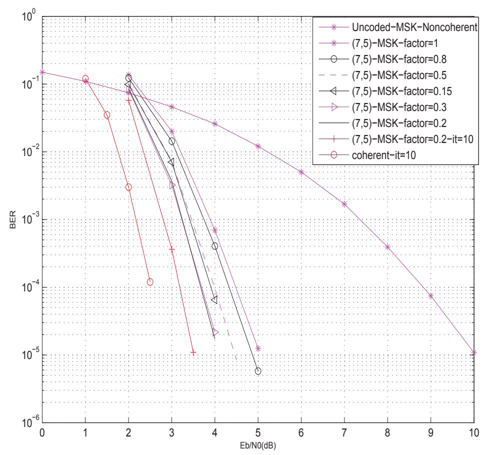


**Fig. 7.** Noncoherent iterative detection of SCMSK, $N = 1024$, $S$-Random interleaver. In this figure, 'it' stands for the number of iterations.

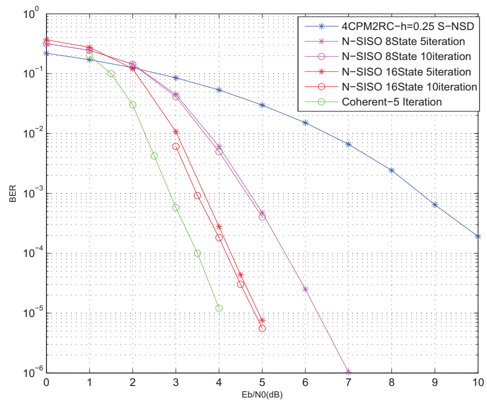


**Fig. 8.** Noncoherent iterative detection of SCQCPM with symbol interleaver. $N = 1024$, $S$-Random interleaver.

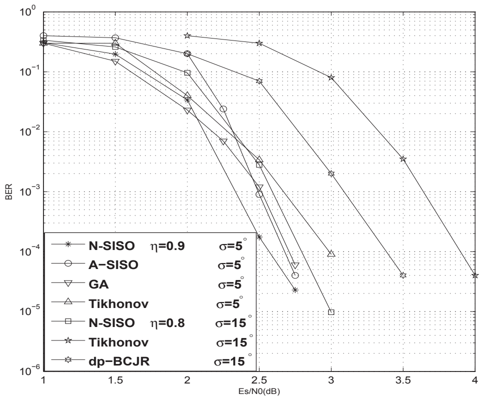


**Fig. 9.** Noncoherent iterative detection of SCQCPM under strong phase noise.

| Proposed | Tikhonov [17] | dp-BCJR [17] | GA [20] |
|---|---|---|---|
| $M\|S\|(6M+10)$ | $p(16pM+6M+14)$ | $61MD$ | $7D+56$ |

**Tab. 1.** Computational complexity of proposed and existing algorithms.

The complexity of the proposed algorithm and the algorithms developed in [17, 18, 20] is compared in Tab. 1, wherein $D$ denotes the discretization levels [17] and $|S|$ is the number of states required by the detector. The numerical results were previously partially reported in [17]. The computational complexity is evaluated in terms of number of operations per symbol including additions and multiplications between two real arguments. For dp-BCJR, it requires at least $D = 16$ discretization levels to obtain a reliable phase estimation and thus has a higher complexity than Tikhonov. It can be seen that the computational complexity of N-SISO and other algorithms varies with the parameters of CPM. However, for most binary CPMs, N-SISO has lower complexity. As the $M$ increases, the proposed algorithm would have a comparable complexity with Tikhonov, which is still lower than dp-BCJR.

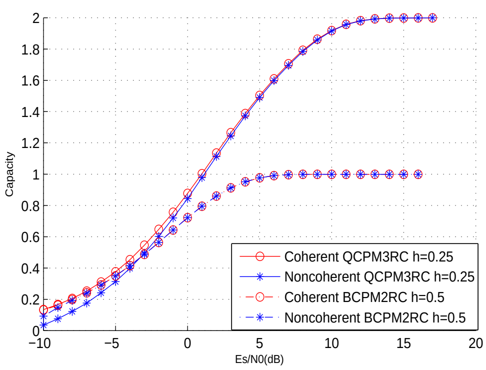


**Fig. 10.** The capacity of noncoherent detection vs. coherent detection for binary and quaternary CPMs [14].

The aforementioned results is explained in Fig. 10 where the capacities (bits/channel use) employing coherent and noncoherent detection are demonstrated. It is observed that the coherent detection and noncoherent detection (i.e., N-SISO) actually have the same capacity as $SNR \to \infty$. When $SNR$ is low, the noncoherent detection is worse than coherent detection, but this gap is marginal. This reflects the fact that the minimum achievable distance of coherent and noncherent detection is identical [1].

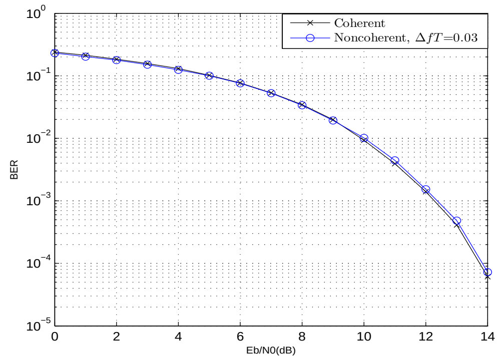


**Fig. 11.** The performance of an octal CPM2RC employing N-SISO when the carrier frequency is mi-estimated.

The performance of N-SISO subject to mis-estimated carrier frequency is also considered. Here, $\Delta fT = (f_c - \hat{f}_c)T$, wherein $f_c$ and $\hat{f}_c$ are the true carrier and the estimated carrier, respectively. Due to the existence of $\Delta fT$, there is always a time-varying and unknown phase noise $2\pi\Delta ft$ each interval. It is observed in Fig. 11 that the proposed algorithm successfully suppresses the phase noise and obtains a performance approaching coherent detection.

# 5. Conclusions

In this paper, we proposed a simplified noncoherent iterative receiver for CPM systems. The techniques of exponential window, principal components analysis, singular value decomposition and reduced-state sequence detection are generalized from coherent detection to noncoherent detection. Numerical and analytical results reveal that the proposed receiver can approach coherent performance. In the case of tackling time-varying phase noise, the proposed receiver has a better performance than some existing detectors. The proposed receiver can be generalized to other CPM-based systems such as satellite communication system, deep space communication system, optical communication system, digital video broadcasting (DVB), and multiuser system [10–13] to build simplified receivers.

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