Modeling the Flux-Charge Relation of Memristor with Neural Network of Smooth Hinge Functions

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Abstract. *The memristor was proposed to characterize the flux-charge relation. We propose the generalized flux-charge relation model of memristor with neural network of smooth hinge functions. There is effective identification algorithm for the neural network of smooth hinge functions. The representation capability of this model is theoretically guaranteed. Any functional flux-charge relation of a memristor can be approximated by the model. We also give application examples to show that the given model can approximate the flux-charge relation of existing piecewise linear memristor model, the window function memristor model, and a physical memristor device.*

Keywords

Memristor, mathematical model, smooth hinge function, neural network.

1. Introduction

Memristor was proposed to be the fourth basic circuit element, in order to characterize the relation between the flux φ and the charge q [1], [2]. The other three basic circuit elements are the resistor (current-voltage relation), the inductor (current-flux relation) and the capacitor (charge-voltage relation). According to [1], [3], the flux-charge relation of a charge-controlled memristor is expressed as

$$\varphi = \varphi(q) \tag{1}$$

and the current and voltage relation is described by (note that $\mathrm{d}q = i\mathrm{d}t$ and $\mathrm{d}\varphi = v\mathrm{d}t$)

$$v(t) = M(q(t))i(t) \tag{2}$$

where

$$M(q) = \mathrm{d}\varphi(q)/\mathrm{d}q. \tag{3}$$

Similarly, a flux-controlled memristor is expressed as:

$$q = q(\varphi) \tag{4}$$

and the current and voltage relation is described by

$$i(t) = W(\varphi(t))v(t) \tag{5}$$

where

$$W(\varphi(t)) = \mathrm{d}q(\varphi)/\mathrm{d}\varphi. \tag{6}$$

The charge-controlled memristor and the flux-controlled memristor are also referred as the ideal memristor [2], [3].

The concept of memristor was generalized to memristive systems [4], [5]. A current-controlled memristive system is described by

$$\dot{x} = f(x,i,t), \tag{7a}$$

$$v = R(x,i,t)i \tag{7b}$$

where x is the state variable of the memristive system. The ideal memristor is a special case of the memristive system (letting $x = q$, $f(x,i,t) = \dot{q} = i$ and $R(x,i,t) = M(q(t))$). Similarly, a voltage-controlled memristive system is described by

$$\dot{x} = f(x,v,t) \tag{8a}$$

$$i = G(x,v,t)v. \tag{8b}$$

The first physical memristor device was found by [6]. Then many other memristor devices with different physical mechanisms have been proposed by researchers, see [7], [8], [9] for examples. The mechanisms of these devices are very complex. The memristive system is widely used to model physical memristor devices [6], [10], [11], [12], since the ideal memristor (1) and (4) usually can not fully describe the behavior of the physical memristor devices. For example, voltage dependence is observed in many physical memristor devices [8], [10], [12]. The flux-controlled memristor (4) can not model the voltage dependence of the device. There is only one variable φ in (4), therefore the conductance $W(\varphi(t))$ in (5) can not represent the influence of the voltage $v(t)$ at a certain time t.

The memristive system is a generalization of the memristor in the perspective of current and voltage relation, as $v = M(q)i$ is generalized to $v = R(x,i,t)i$ and charge q is generalized to state variable x. To our best knowledge, the generalization of the memristor in the perspective of flux-charge relation is little studied. In this paper, we focus on the flux-charge relation of the memristor, considering that the memristor was originally proposed to characterize the missing relation between flux and charge [2]. We propose a generalized model to describe the flux-charge relation of the memristor based on the neural network of smooth hinge functions. Our model has good representation capability and thus is a suitable choice for modeling memristor devices. We will show that with theoretical analysis and three examples.

2. Generalized Flux-Charge Relation Model

Besides extending memristor to memristive system in the perspective of current and voltage relation, we believe that we can directly generalize the flux-charge relation of the memristor. Note that current $i(t)$ and voltage $v(t)$ are added as parametric variables into the memristive system to form a more general and complex system. We add the current $i(t)$ and voltage $v(t)$ into the flux-charge relation of the memristor as a generalization. The $\varphi = \varphi(q)$ relation of a charge-controlled memristor can be generalized to $\varphi = \varphi(q,i)$, and the $q = q(\varphi)$ relation of a flux-controlled memristor can be generalized to $q = q(\varphi, v)$. We use the neural network of smooth hinge functions to represent the generalized flux-charge relation of the memristor.

Specifically, a generalized charge-controlled memristor is given by

$$\begin{aligned}\varphi(q,i) &= a_0 q(t) + b_0 i(t) + c_0 + \\ &\quad \Sigma_{k=1}^{m} \eta_k \ln(1+\exp(a_k q(t) + b_k i(t) + c_k)).\end{aligned} \tag{9}$$

In (9), $\ln(1+\exp(a_k q(t) + b_k i(t) + c_k)$ is the base function of the neural network of smooth hinge functions [13], m is the number of base functions, a_k, b_k, c_k, η_k are parameters. Similar to $v(t) = M(q(t))i(t)$, $M(q) = \mathrm{d}\varphi(q)/\mathrm{d}q$ of (1), the current and voltage relation of (9) is given by $v(t) = \frac{\mathrm{d}\varphi(q,i)}{\mathrm{d}t} = \frac{\partial\varphi(q,i)}{\partial q} i(t) + \frac{\partial\varphi(q,i)}{\partial i}\frac{\mathrm{d}i(t)}{\mathrm{d}t}$. Similarly, a generalized flux-controlled memristor is given by

$$\begin{aligned}q(\varphi,v) &= a_0 \varphi(t) + b_0 v(t) + c_0 + \\ &\quad \Sigma_{k=1}^{m} \eta_k \ln(1+\exp(a_k \varphi(t) + b_k v(t) + c_k)).\end{aligned} \tag{10}$$

The representation capability of our model is theoretically guaranteed by using the neural network of smooth hinge functions. Any continuous function can be approximated by the neural network of smooth hinge functions to arbitrary precision with a sufficient number of base functions [13]. The memristor was proposed to characterize the relation between flux and charge. Therefore as long as there is a function relation between φ and q (in the form of $\varphi = \varphi(q,i)$ or $q = q(\varphi, v)$) of the memristor device, our model can approximate such flux-charge relation well. As far as we know, no existing memristor model can guarantee such representation capability. Due to the good approximation capability of the neural network of smooth hinge functions [13], the generalized flux-charge memristor model can properly fit the experimental data of the memristor device. The smooth hinge function $\ln(1+\exp(a_k q(t) + b_k i(t) + c_k)$ is differentiable in the domain. This advantage may make further analysis of the memristor easier, compared with the existing piecewise linear model, as will be shown in the following example.

Our generalized flux-charge memristor model also has good extensibility. Parametric Variables other than i or v, such as power p [8], can be easily added into the neural network of smooth hinge functions, in order to get a more precise description of the memristor device. At this situation, the base function becomes $\ln(1+\exp(a_k q(t) + b_k i(t) + c_k p(t) + d_k)$. Adding new variables will not change the aforementioned representation capability and smooth characteristics of the model [13]. For possible multi valued flux-charge relation, using the masked input technique given in [14], our model may still be able to describe such flux-charge relation.

3. Application Examples of Generalized Flux-Charge Relation Model

As analyzed in the last section, the representation capability of our generalized flux-charge relation model is theoretically guaranteed by the property of the neural network of smooth hinge functions. In this section we give three examples to show the representation capability of our model. We use the model to approximate two existing memristor models, a piecewise linear flux-charge relation model and a memristive system model. We also use our model to fit the voltage dependent flux-charge relation of physical memristor device based on experimental data.

In [1], [3], [15], the following piecewise linear flux-charge relation is used to characterize a memristor

$$\varphi(q) = bq + 0.5(a-b)(|q+1| - |q-1|). \tag{11}$$

a, b are the parameters of the model, and we can suppose $a \neq b$ for general cases [1]. We show that our smooth model can properly approximate the piecewise linear flux-charge relation given by (11). First, we equivalently represent (11) with hinge functions [16]

$$\varphi(q) = aq + (b-a)\max\{0, q-1\} - (b-a)\max\{0, -q-1\}. \tag{12}$$

Then according to [13], (12) can be approximated by

$$\begin{aligned}\varphi_(q) &= aq + (b-a)\ln(1+\exp\alpha(q-1))/\alpha \\ &\quad -(b-a)\ln(1+\exp\alpha(-q-1))/\alpha,\end{aligned} \tag{13}$$

as shown in Fig. 1. In (13), increasing parameter α can reduce the differences between a hinge function $\max\{0,x\}$ and a smooth hinge function $\ln(1+\exp\alpha x)/\alpha$ around the hinge $x = 0$ [13]. The pinched hysteresis loops of both models are also shown in Fig. 1. It can be seen that our model can properly approximate the existing piecewise linear model.

There are limitations of the existing piecewise linear model. One is that (12) is not differentiable at $q = \pm 1$, which will cause a sudden memristance change at $q = \pm 1$ (i.e., $M(q) = \mathrm{d}\varphi(q)/\mathrm{d}q$ in (2),(3) will discontinuously change between a and b). Our model does not have such limitation since we use the neural network of smooth hinge functions. A smooth hinge function $\ln(1+\exp x)$ has a continuous derivative $1/1+\exp(-x)$, and therefore (13) has a continuous derivative (i.e., $M(q)$) over the domain including $q = \pm 1$. Another limitation of the existing piecewise linear flux-charge relation model is that it may not properly represent the flux-charge relation of physical memristor device (we will show that in the following example).

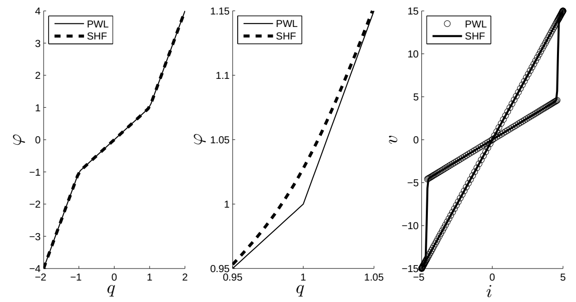

Fig. 1. Approximating existing piecewise linear (PWL) model (12) with smooth hinge function (SHF) model (13). $a = 1$, $b = 3$, $\alpha = 50$. The left figure shows the flux-charge relations of both models. The middle figure shows the local details of both flux-charge relations around $q = 1$: (12) is not differentiable at $q = 1$, while (13) is smooth. The right figure shows the pinched hysteresis loops of both models with input $i(t) = 5sin(\pi t)$.

Next we use our model to approximate an existing memristive system model. The following window function model is given by [17] to describe the first memristor device found by [6]:

$$v(t) = (R_{ON}x(t) + R_{OFF}(1 - x(t)))i(t), \tag{14a}$$

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = k(1 - (2x(t) - 1)^{2p})i(t) \tag{14b}$$

where $x(t)$ is the state variable, k and p are the parameters. This memristive system is a simplified model of the device and widely used. According to [18], there exists a function relation of φ and q of (14). More discussion on window function model can be found in [14], [19], [20], [21]. Here we plot the $\varphi - q$ curve of (14) through simulation, and use our model to approximate this $\varphi - q$ curve, as shown in Fig. 2. Specifically, our model is given by

$$\varphi(q) = a_0 q + b_0 - \ln(1 + \exp\alpha(a_1 q + b_1))/\alpha. \tag{15}$$

Note that for a small q, $\varphi(q) \approx a_0 q + b_0$, for a large q, $\varphi(q) \approx (a_0 - a_1)q + b_0 - b_1$ and $v/i = \mathrm{d}\varphi(q)/\mathrm{d}q$. The parameters are chosen such that $a_0 = R_{ON}$, $a_0 - a_1 = R_{OFF}$, $a_0 q + b_0$ is the approximating line for (q, φ) data point with a small q and $(a_0 - a_1)q + b_0 - b_1$ is the approximating line for (q, φ) data point with a large q. α can adjust the degree of bending of (14) between line $a_0 q + b_0$ and line $(a_0 - a_1)q + b_0 - b_1$.

It is easy to see that the existing piecewise linear model (12) may not properly approximate $\varphi - q$ curve of (14) (Circle points in Fig. 2). Because circle points in Fig. 2 do not have symmetry as (12). Our model can properly approximate the circle points. The pinched hysteresis loops show that our model is a good approximation of memristive system model (14). In this example, only 1 base function of the neural network of smooth hinge functions is used. Using other kinds of neural network instead of the neural network of smooth hinge functions may increase the complexity of the corresponding approximation model, as more base functions and parameters may be needed.

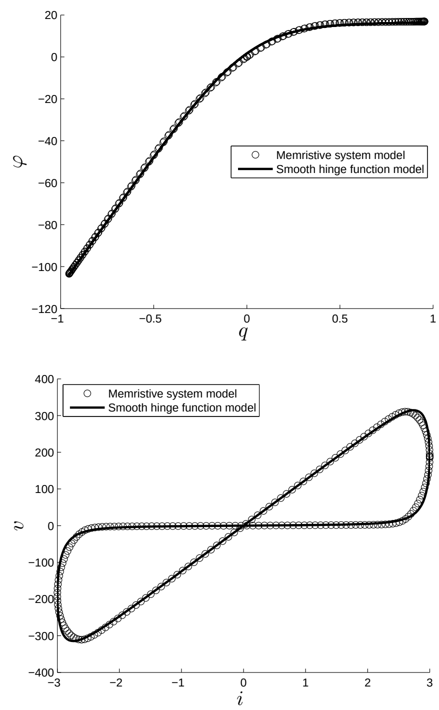

Fig. 2. Approximating existing memristive system model (14) with smooth hinge function model (15). $R_{ON} = 1$, $R_{OFF} = 125$, $k = 1$, $p = 5$, $\alpha = 0.05$, $a_0 = 1$, $a_1 = -124$, $b_0 = 15.5$, $b_1 = 0$. The top figure shows the flux-charge relations of both models. The bottom figure shows the pinched hysteresis loops of both models with input $i(t) = 3\sin(\pi t + \pi/2)$.

In the third example, we show that our generalized model is suitable to model physical memristor devices. Specifically, we model the voltage dependent flux-charge relation of physical AgInSbTe memristor device [9]. The co-existence of extrinsic electrochemical metallization effect and intrinsic memristive characteristics was confirmed in the AgInSbTe memristor [9]. In the gradual resistance tuning of the device, pulses with different voltage amplitudes and 5 μs width were applied to the AgInSbTe memristor device [9]. From the experiment data, we calculate the (φ, q) data for each pulse of different voltage amplitudes. Voltage dependent flux-charge relation is observed, as shown in Fig. 3.

The flux-charge relation of the device varies with different voltage amplitudes. $q = q(\varphi)$ can not describe such voltage dependent flux-charge relation. Our generalized $q = q(\varphi, v)$ model is needed. We use our generalized model in the form of (10) to approximate such voltage dependent flux-charge relation. From Fig. 3 we can see that our model fits the data well. The number of smooth hinge functions m and the parameters $a_i,\ b_i,\ c_i,\ i = 1,\dots,m$ in (10) are artificially selected. Then parameters $a_0,\ b_0,\ c_0$ and $\eta_i,\ i = 1,\dots,m$ are calculated by least squares method based on the experimental data.

The behavior of physical AgInSbTe memristor device is complicated. Besides the amplitude of the pulse, the pulse width also affects the gradual resistance tuning of the AgInSbTe memristor [9]. Pulses with $-1V$ amplitude and different widths were applied to the AgInSbTe memristor [9]. As shown in Fig. 4, the flux-charge relation of the device varies with the pulse widths. We can analogously model such flux-charge relation by replacing the variable v in (10) with pulse width Δ. Then the base function of $q = q(\varphi, \Delta)$ is in the form of $\ln(1 + \exp(a_k\varphi(t) + b_k\Delta + c_k))$. Fig. 4 shows that our model fits the data properly.

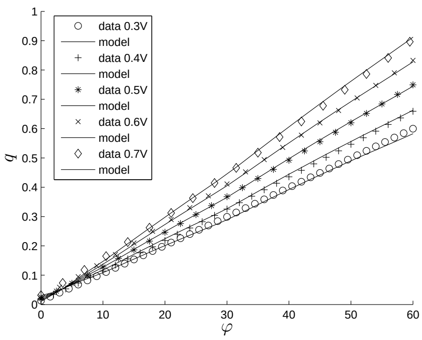

Fig. 3. Approximate the voltage dependent flux-charge relation of the AgInSbTe memristor. The $\varphi(t)$ at the nth pulse is calculated by $\varphi(t) = nA_v\Delta$, where A_v is the amplitude and Δ is the width of the pulse. Similarly, $q(t)$ at the nth pulse is calculated by $q(t) = \Sigma_{k=1}^{n} i_k\Delta$, where i_k is the current measured at the kth pulse. 10 smooth hinge base functions are used in the model.

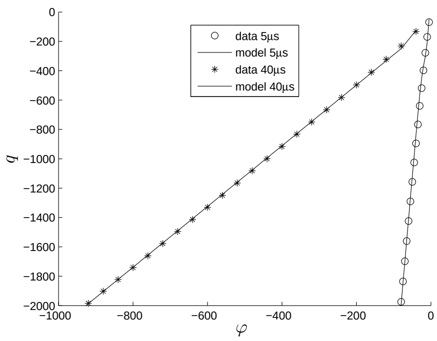

Fig. 4. Approximate the flux-charge relation of the AgInSbTe memristor with different pulse widths. The $\varphi(t)$ at the nth pulse is calculated by $\varphi(t) = nA_v\Delta$, where A_v is the amplitude and Δ is the width of the pulse. Similarly, $q(t)$ at the nth pulse is calculated by $q(t) = \Sigma_{k=1}^{n} i_k\Delta$, where i_k is the current measured at the kth pulse. 4 smooth hinge base functions are used in the model.

4. Conclusions

In this paper, we propose the generalized flux-charge relation model of memristor considering that the memristor was originally proposed to characterize the flux-charge relation. Such generalization is little studied, but the example of voltage dependent flux-charge relation of the AgInSbTe memristor indicates that such generalization is necessary.

The usage of neural network of smooth hinge functions theoretically guarantees the representation capability of the model. Any functional flux-charge relation, even with multiple parametric variables, can be approximated by our model. With examples, we show that our model is capable of representing the existing memristor models, and approximating voltage dependent flux-charge relation of physical memristor device.

Besides the three examples given in this paper, our model can be applied to model other types of memristors. Because the representation capability of the model is theoretically guaranteed and there is effective identification algorithm for the neural network of smooth hinge functions [13]. Given the (φ, q) data of the specific memristor, our model can be applied to model the memristor.

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