Measurement and Calibration of a High-Sensitivity Microwave Power Sensor with an Attenuator

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Abstract. In this paper, measurement and calibration of a high-sensitivity microwave power sensor through an attenuator is performed using direct comparison transfer technique. To provide reliable results, a mathematical model previously derived using signal flow graphs together with non-touching loop rule analysis for the measurement estimate (i.e., calibration factor) and its uncertainty evaluation is comparatively investigated. The investigation is carried out through the analysis of physical measurement processes, and consistent mathematical model is observed. Later, an example of Type-N (up to 18 GHz) application is used to demonstrate its calibration and measurement capability.

Keywords
Direct comparison transfer, modeling, high-sensitivity, measurement uncertainty.

1. Introduction

Precise power measurements are essential for RF and microwave applications. Measurement instruments such as a microwave power sensor require accurate and traceable measurement capabilities, hence necessary calibrations. Direct comparison transfer technique has been widely accepted and implemented for calibrating a RF and microwave power sensor [1] - [5]. This method transfers the effective efficiency $\eta_{Std}$ and the calibration factor $K_{Std}$ of a reference standard to an uncalibrated power sensor which is the device under test (DUT), with the help of a power splitter [6] or a coupler which is used to minimize the source mismatch [7].

Sometimes, a DUT power sensor has an unmatched connector with the reference standards, and then an adaptor has to be used [3], [4], [5]. A generic model has been proposed in [5] to characterize the additional measurement error introduced by the adaptor, using signal flow graphs together with non-touching loop rule analysis [8]. Comparing to the general cases [1] - [4] where the DUT power sensor has a similar power range as the reference standard, there is another important application scenario where their power ranges are different (e.g., calibrating a high-sensitivity Agilent 8481D power sensor). An attenuator therefore has to be used, which acts as a 2-port adaptor in this case.

Fig. 1. Calibration of a high-sensitivity power sensor through an attenuator using direct comparison transfer technique.

However, in the literature, there is limited information reported for this calibration scenario with an attenuator. The mathematical model proposed for the calibrations using an adaptor in [5] could be a potential calibration model, although it still needs to be comparatively verified. Therefore in this paper, verification of the mathematical model for the calibration scenario with an adaptor and evaluation of its feasibility to calibrate a high-sensitivity power sensor with an attenuator as shown in Fig. 1 will be focused.

In the following, a brief description of the calibration system with an adaptor is given in Section 2 together with an introduction of the mathematical model derived using signal flow graphs together with non-touching loop rule analysis. In Section 3, through the analysis of physical measurement processes, the mathematical model for the calibration system with an adaptor in [5] is comparatively verified. The model is then implemented with a Type-N (up to 18 GHz) measurement system in Section 4 to evaluate its feasibility for calibrating a high-sensitivity power sensor with an attenuator as the 2-port adaptor. Finally, conclusions of this paper are given in Section 5.

2. Direct Comparison Transfer

2.1 A Brief Description

Direct comparison transfer technique for calibrating an RF and microwave power sensor through an adaptor/attenuator as shown in Fig. 1 consists of a signal gen-
erator and a 3-port power splitter which is used to minimize the source mismatch [7]. In this system, a monitoring sensor is connected to port 3 of the splitter. The effective efficiency $\eta_{DUT}$ and the calibration factor $K_{DUT}$ of a DUT are then estimated by alternately connecting a reference standard (with $\eta_{Std}$ and $K_{Std}$) and the DUT (through an adaptor/attenuator) to port 2 of the splitter. For simplicity in the rest of this paper, we focus on the developments of the mathematical model for $K_{DUT}$. The same methodology can be applied to $\eta_{DUT}$.

2.2 Mathematical Model

For the calibration system with an adaptor between the DUT sensor and the splitter, a mathematical model has been proposed in [5] using signal flow-graphs together with non-touching loop rules as

$$K_{DUT} = K_{Std} \times \frac{P_{DUT}}{P_{Std}} \times \frac{P_{Std}}{P_{Std}} \times \left| \frac{k_{Std}^{2}}{k_{DUT}^{2}} \right| \times \left| 1 - \Gamma_{DUT}S_{22A} - \Gamma_{DUT}S_{22A} \right|^{2}_{S_{21A}}(1 - \Gamma_{Std}^{2})^{2} \tag{1}$$

where

- $P_{DUT}$ and $P_{DStd}$ are the powers measured at port 2 using the DUT with an adaptor and that at port 3 using a monitoring sensor respectively,
- $P_{Std}$ and $P_{Std}$ are the powers measured at port 2 using a reference standard and that at port 3 using the same monitoring sensor as for measuring $P_{DStd}$,
- $k_{2Std}$ and $k_{2DUT}$ are some unknown terms related to the leakage of cable and connector, linearity and frequency error etc. when the reference standard and the DUT are connected to port 2.

In this model (1),

$$\Gamma_{DUT} = S_{21A} + \Gamma_{DUT}S_{21A}S_{12A} - \Gamma_{DUT}S_{22A}S_{11A} \tag{2}$$

where $\Gamma_{DUT}$ is the reflection coefficient of the DUT, and $S_{lma}$ is the scattering parameter (S-parameter) of the adaptor with $l, m = 1$ or 2. $\Gamma_{Std}$ is the reflection coefficient of the reference standard. It is noted that $\Gamma_{c2}$ is the equivalent source reflection coefficient at port 2 of the splitter and equal to

$$\Gamma_{c2} = S_{22} - \frac{S_{21A}S_{12A}}{S_{31}} \tag{3}$$

Here $S_{pq}$ is the $S$-parameter of the 3-port power splitter with $p, q = 1, 2$ or 3.

However, this model (1) has not been comparatively validated due to limited information reported in the literature. Therefore in the following, a different interpreting way from [5] for deriving the mathematical model is focused, which is performed through the analysis of its corresponding physical measurement processes.

3. Physical Measurement Processes

![Fig. 2. Flow of the microwave power to a DUT power sensor with an adaptor.](image)

![Fig. 3. A simplified flow-graph for an adaptor before a microwave power sensor corresponding to Fig. 2.](image)

From the definition [9], the calibration factor $K_{2}$ of the DUT power sensor is

$$K_{2} = \frac{P_{m}}{P_{r-2}} \tag{4}$$

With Fig. 3, it is obtained that

$$P_{r-2} = P_{r-1} \left| S_{21A} \right|^{2} \tag{5}$$

That is,

$$K_{2} = \frac{P_{m}}{P_{r-1}} \left| \frac{1 - \Gamma_{2}S_{22A}}{S_{21A}} \right|^{2}. \tag{6}$$

As the calibration factor $K_{1}$ for the DUT power sensor integrated with an adaptor is defined to be $P_{m}/P_{r-1}$, it is then derived,

$$K_{2} = K_{1} \left| \frac{1 - \Gamma_{2}S_{22A}}{S_{21A}} \right|^{2}. \tag{7}$$

Moreover, the calibration factor $K_{1}$ at the interface with the microwave splitter (interface no. 1 as shown in Fig. 2) has been well-studied. Consistent conclusions have been achieved comparing the reported works in [5] with the method [1]. $K_{1}$ can be expressed as

$$K_{1} = K_{Std} \times \frac{P_{DUT}}{P_{DStd}} \times \frac{P_{Std}}{P_{Std}} \times \left| \frac{k_{Std}^{2}}{k_{DUT}^{2}} \right| \times \left| 1 - \Gamma_{1} \Gamma_{2} \right|^{2}. \tag{8}$$
That is, for calibrating a DUT through an adaptor shown in Fig. 1, $K_{DUT}$ of the stand-alone DUT can be obtained from (7) and (8) as

$$K_{DUT} = K_2 = K_{Std} \times \frac{P_{DUT}}{P_{Std}} \times \frac{P_{Std}}{P_{DUT}} \times \frac{k_{Std}}{k_{2DUT}} \left(1 - \Gamma_1 \Gamma_{e2} \right)^2 \times \left(1 - \Gamma_2 \Gamma_{e2} \right)^2 \left(1 - \frac{\Gamma_2 S_{22A}}{S_{21A}} \right)^2. \quad (9)$$

For the item $|1 - \Gamma_1 \Gamma_{e2}|$ in (9), $\Gamma_1$ can be derived in terms of $S$-parameters of the adaptor and the reflection coefficient $\Gamma_2$ ($\Gamma_2 = \Gamma_{DUT}$) of the DUT power sensor as

$$\Gamma_1 = S_{11A} + \frac{S_{21A} \Gamma_2 S_{12A}}{1 - \Gamma_2 S_{22A}}. \quad (10)$$

Then,

$$|1 - \Gamma_1 \Gamma_{e2}| = \left|1 - \left\{ \frac{S_{11A} + S_{21A} \Gamma_2 S_{12A}}{1 - \Gamma_2 S_{22A}} \right\} \Gamma_{e2} \right|$$

$$= \left|1 - \Gamma_2 S_{22A} - \{ S_{11A} - S_{11A} \Gamma_2 S_{22A} + S_{21A} \Gamma_2 S_{12A} \} \Gamma_{e2} \right|$$

$$= \left|1 - \Gamma_2 S_{22A} - \{ S_{11A} - S_{11A} \Gamma_2 S_{22A} + S_{21A} \Gamma_2 S_{12A} \} \Gamma_{e2} \right|$$

$$= \left|1 - \Gamma_2 S_{22A} - \Gamma_{A-DUT} \Gamma_{e2} \right| \Gamma_{e2} \quad (11)$$

where $\Gamma_{A-DUT} = S_{11A} + \Gamma_{DUT} S_{21A} S_{12A} - \Gamma_{DUT} S_{22A} S_{11A}$ and $\Gamma_2 = \Gamma_{DUT}$ as indicated in Fig. 2.

Substituting $|1 - \Gamma_1 \Gamma_{e2}|$ in (9) with (11), it is found that the derived mathematical model (9) through the analysis of physical measurement processes as described above, is exactly the same as the one developed using the signal flow graphs together with non-touching loop rule analysis in [5] which is shown as (1) in this paper. That is, the mathematical model ((9) or (1)) has been comparatively validated and is suitable for the microwave power sensor calibration with an adaptor between the DUT sensor and the power splitter.

Moreover, our recent works [10] indicated that a high-sensitivity microwave power sensor (e.g., Agilent 8481D power sensor) could be a potential reference standard for calibrating a thermal voltage converter at a higher operating frequency. Accurate calibration of such a high-sensitivity microwave power sensor then becomes important. However, this type of sensor usually has a different power range comparing to the primary RF and microwave power standard. Therefore, an attenuator has to be used. In the following, the feasibility of the mathematical model (9) or (1)) to calibrate such a high-sensitivity microwave power sensor with an attenuator as the 2-port adaptor will be evaluated and focused.

4. Feasibility Study and Analysis

4.1 Practical Calibration System

Feasibility study of the mathematical model (9) (or (1)) for the calibration scenario shown in Fig. 1, is physically realized using a Type-N microwave power sensor calibration system where a 30 dB attenuator acts as the 2-port adaptor between a DUT power sensor and a splitter (similar to [11]). The practical calibration system is presented in Fig. 4.

As shown in Fig. 4, an Agilent 8481D power sensor (power range: 100 pW – 10 µW and frequency range: 10 MHz – 18 GHz) is the DUT sensor. The reference standard is a thermistor mount which is fitted with a Type-N connector and calibrated in term of the effective efficiency at 1 mW directly by means of a microwave micro-calorimeter. The microwave micro-calorimeter is a primary power standard with fixed output power which converts the absorbed microwave energy into the heat (i.e., thermalize the microwave energy).

Key referenced parameters (i.e., $\eta_{Std}$ and $K_{Std}$) of the thermistor mount at 1 mW is then transferred to the Agilent 8481D sensor (with a smaller power range of [100 pW, 10µW]) through a 30 dB attenuator. It is noted that for the power leveling and monitoring at port 3 of the power splitter shown in Fig. 4, an Agilent 8481A sensor (power range: 1 µW – 100 mW and frequency range: 10 MHz – 18 GHz) is used.
4.2 Performance Evaluation and Analysis

The measurement estimate $K_{DUT}$ is calculated accordingly using the mathematical model (9) (or (1)), while its associated measurement uncertainty is evaluated following an internationally recommended guideline, *Guide to the Expression of Uncertainty in Measurement* (GUM) [12] which has been widely accepted. For simplicity in the demonstration of uncertainty evaluation, the following relationship is used to represent the mathematical model,

$$y = f(x_1, x_2, x_3, ..., x_N).$$

(12)

Here, $y$ is the estimate $K_{DUT}$, and $x_1, x_2, x_3, ..., x_N$ represent the influencing quantities $K_{Std}, \Gamma_{DUT}, \Gamma_{\varepsilon_2}, \Gamma_{\varepsilon_3}, S_{21A}$ etc.

According to the *Law of Propagation of Uncertainty* in the GUM [12], the combined standard uncertainty $u_c$ associated with $y$ (i.e., $u_c(y)$) can be obtained from the standard uncertainties of $x_1, x_2, x_3, ..., x_N$ through

$$u_c(y) = \sqrt{\sum_{i=1}^{N} \left( \frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) + 2 \sum_{i=1}^{N} \sum_{j>i}^{N} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j)}$$

(13)

where $u(x_i)$ is the associated standard uncertainty for $i^{th}$ influencing quantity $x_i$. $u(x_i, x_j)$ is the covariance between $x_i$ and $x_j$, and equal to

$$u(x_i, x_j) = r(x_i, x_j)u(x_i)u(x_j).$$

(14)

Here, $r(x_i, x_j)$ is the correlation coefficient between $x_i$ and $x_j$. It is noted that the correlation coefficient $r$ between the influencing quantities is relatively small in this calibration, and it is also inherently unreliable due to small sample size in practical calibrations as reported in [13]. Therefore in this study, $u_c(y)$ is evaluated with the assumption of zero correlation ($r = 0$) between the influencing quantities.

For the standard uncertainty $u(x_i)$ for $x_i$, it can be evaluated using either Type A or Type B method according to the GUM. For the Type A method, $u(x_i)$ is evaluated by the statistical analysis of series of observations; while for the Type B method, $u(x_i)$ is obtained from other information including previous measurement data, specifications from manufacturers, data provided in calibration and other certificates, and uncertainties assigned to reference data taken from handbooks, etc. Moreover for the complex-valued microwave quantities such as $S$-parameters and reflection coefficients, their standard uncertainties are evaluated with the assumption of zero correlation between their real and imaginary parts as we discussed in [14].

4.2.1 Evaluations Using the GUM and MCM Methods

In this paper, uncertainty evaluation using the GUM method is focused. This is because the uncertainty evaluation using the GUM method has been accepted and used in most of current routine calibration works. When evaluating $u(x_i)$ following the Type B method, probability distribution for $x_i$ needs to be prior-determined. Normally with the assumptions of Gaussian distributions for all the influencing quantities (ordinary cases), the measurement uncertainty of $K_{DUT}$ can be evaluated with the mathematical model (9) (or (1)) accordingly.

To validate the assumed probability distributions, Monte Carlo method (MCM) as recommended in [15] is chosen for a comparison. In the Monte Carlo simulations, the characteristics of assumed Gaussian distributions for all the influencing quantities are directly from the measurement estimates.

Fig. 5 shows an example of the simulated results using the MCM method. From Fig. 5, it is found that the representative distribution for $K_{DUT}$ estimated using the MCM method also approximates to be Gaussian distributed. This is because the recommended guideline [15] is the *Law of Propagation of Distributions* essentially, which propagates the assigned probability distributions to the influencing quantities to the desired parameter ($K_{DUT}$ in this study) as we discussed in [16]. For the MCM method, the measurement estimate and its associated uncertainty are determined from the experimental distribution as shown in Fig. 5.

The measurement estimate $K_{DUT}$ and its associated combined standard uncertainty $u_c(K_{DUT})$ for the DUT power sensor (i.e., Agilent 8418D sensor) using the GUM and MCM methods are shown in Fig. 6 respectively. From Fig. 6, it is found that both the methods generate very close results especially at the lower frequencies. This indicates that the assumptions of Gaussian distribution for all the influencing quantities in (9) are suitable.

In the next subsection, feasibility of the mathematical model (9) (or (1)) to calibrate a high-sensitivity power sensor with an attenuator is further evaluated and compared to the calibration data from manufacturer. The uncertainty evaluated using the GUM method is used in performance comparison as it is implemented in most of current routine calibration works.
selected for analysis. It is observed from Fig. 8 that generally the calibration results from our method with uncertainty evaluation using the GUM method show very good agreements (all the \( |E_n| \leq 0.5 \) ) with the calibration data from manufacturer. These observations therefore demonstrate a good measurement and calibration capability of the mathematical model (9) (or (1)) to calibrate a high-sensitivity power sensor with an attenuator.

Moreover it is found from Fig. 8 that extremely excellent agreements are achieved when the operating frequency \( f \in [1, 10] \) GHz (i.e. \([10^0, 10^1]\) shown in Fig. 8) as all the \( |E_n| \) in this region is \( \leq 0.1 \). For other regions \( (f \leq 1 \) GHz and \( f \geq 10 \) GHz), the discrepancies between the results from the GUM method and from the manufacturer become larger. The main reasons for larger discrepancies at lower frequencies \( (f \leq 1 \) GHz) is due to the performance limitation of the microwave micro-calorimeter used in this study, while the larger discrepancies at higher frequencies \( (f \geq 10 \) GHz) might be because the reflection due to the impedance mismatch of additional attenuator with the splitter or the DUT sensor becomes significant at higher frequency and then affects our calibration results.

### 5. Conclusions

In this paper, we presented a feasibility study to calibrate a high-sensitivity microwave power sensor through an attenuator using direct comparison transfer technique. Our previously derived mathematical model using signal flow graphs together with non-touching loop rule analysis has been further investigated and validated with the analysis of physical measurement processes.

Performance of the mathematical model (9) (or (1)) was evaluated using the GUM and MCM Methods through a Type-N (up to 18 GHz) measurement system first, and then compared to the calibration data from manufacturer. Good agreements (all the \( |E_n| \leq 0.5 \) ) with the data from manufacturer have been achieved which demonstrates a good calibration and measurement capability of the mathematical model (9) (or (1)) to calibrate a high-sensitivity microwave power sensor with an attenuator.
References


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