On Amplify-and-Forward Relaying over Hyper-Rayleigh Fading Channels

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Abstract. *Relayed transmission holds promise for the next generation of wireless communication systems due to the performance gains it can provide over non-cooperative systems. Recently hyper-Rayleigh fading, which represents fading conditions more severe than Rayleigh fading, has received attention in the context of many practical communication scenarios. Though power allocation for Amplify-and-Forward (AF) relaying networks has been studied in the literature, a theoretical analysis of the power allocation problem for hyper-Rayleigh fading channels is a novel contribution of this work. We develop an optimal power allocation (OPA) strategy for a dual-hop AF relaying network in which the relay-destination link experiences hyper-Rayleigh fading. A new closed-form expression for the average signal-to-noise ratio (SNR) at destination is derived and it is shown to provide a new upper-bound on the average SNR at destination, which outperforms a previously proposed upper-bound based on the well-known harmonic-geometric mean inequality. An OPA across the source and relay nodes, subject to a sum-power constraint, is proposed and it is shown to provide measurable performance gains in average SNR and SNR outage at the destination relative to the case of equal power allocation.*

Keywords

Relayed communications, hyper-Rayleigh fading, amplify-and-forward, power-allocation.

1. Introduction

Relayed transmission strategies are integral to the next generation of wireless communication systems due to the performance gains such as coverage extension and robustness to fading that these techniques can provide relative to non-cooperative systems [1], [2]. Amplify-and-Forward (AF) relaying, whereby the relay amplifies the received signal before re-transmitting it to the destination, has received considerable attention in the literature due to its low complexity of deployment, see for example [3], [4]. In variable gain AF schemes the instantaneous channel state information (CSI) of the previous hop is used to control the relay gain whereas in fixed/semi-blind AF relaying the relay gain is determined by channel statistics of the previous hop. Although the former scheme generally provides better diversity performance, the latter scheme trades-off the diversity performance with a lower complexity in the CSI estimation part and is therefore more attractive due to practical considerations [5].

When modeling land-mobile wireless communication channels, Rayleigh distributed fading is often considered to be the worst-case fading scenario [6], [7]. However, in recent years many published measurement campaigns have reported channels with fading more severe than Rayleigh or so-called hyper-Rayleigh fading; see for example [8] - [11] and references therein. These severe fading conditions have been observed in various scenarios of practical significance; in [8] the authors observed worse-than-Rayleigh fading in outdoor suburban measurements conducted at 1.5 GHz and they proposed to statistically model such fading with the Nakagami-*m* distribution with its fading severity parameter m taking values in the range $0.5 \leq m < 1$. The same authors also observed that the Weibull distribution with its shape factor less than 2 also provided an empirically best-fit in some cases. In [9] the authors performed indoor measurements at 2.4 GHz for two-way radio-frequency identification applications and used both the Nakagami-*m* and lognormal distributions to model the severe small-scale fading. In [10] measurements at 2.4 GHz within aircraft bodies were analyzed and the two-wave with diffuse power model, a physical wave model rather than a statistical approach, was proposed to model the observed hyper-Rayleigh fading. Finally, in [11] the authors reported vehicle-to-vehicle channel measurements in the 5 GHz band and observed that the weibull distribution provided the best-fit to worse-than-Rayleigh fading channel amplitudes. Despite this practical significance of hyper-Rayleigh fading scenarios a theoretical analysis of the power allocation problem for hyper-Rayleigh fading channels has not been performed previously even though power allocation for fading AF relaying networks has been widely studied in the literature, see for example [12], [13] and references therein. Through this work we aim to address this issue. In modeling hyper-Rayleigh fading we adopt the statistical approach of [8] and [9], which is to model the fading channel coefficient h as Nakagami-*m* distributed with its fading severity parameter m taking values in the range

$0.5 \leq m < 1$. The distribution of h under hyper-Rayleigh fading is then expressed as

$$f_h(h) = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m h^{2m-1} e^{-\frac{mh^2}{\Omega}}, \; 0.5 \leq m < 1 \tag{1}$$

where $\Gamma(\cdot)$ is the Gamma function [14, Eq. (8.310)], $\Omega = E[h^2]$, and $E[\cdot]$ denotes the statistical expectation operator. Our modeling approach is also motivated in part by the fact that the Nakagami-m distribution is widely considered for modeling AF relay channels, albeit with $m \geq 1$, and we aim to extend this work to include the practically significant case where $m < 1$, i.e., worse than Rayleigh fading.

Cooperative communications over Nakagami-m fading links has been extensively studied in the literature, see [15] - [18] and references therein. In [15] the authors have analyzed the outage probability and other statistical parameters for the relay network without diversity. In [16] and [17] the end-to-end SNR and the average symbol error probability have been analyzed for multi-hop communications for various fading distributions of the links. Furthermore, performance bounds for the multi-hop scenario are also proposed in the latter references. These works assume equal power allocation between the source and relay nodes. Optimal power allocation (OPA) for semi-blind AF relaying, subject to a sum-power constraint, has been studied in [19] for Rayleigh fading links, i.e., Nakagami-m fading with $m = 1$ and in [18] for Nakagami-m fading links with $m \geq 1$. This letter aims to extend these investigations to address OPA for dual-hop semi-blind AF relaying for the case where the relay-destination link experiences hyper-Rayleigh fading.

We develop a new closed form expression for the exact average SNR at the destination for a dual-hop semi-blind AF relaying system. The two hops are assumed to experience independent but not necessarily identically distributed Nakagami-m fading with arbitrary m values. We then develop an upper bound for the average SNR at the destination for the case where the relay-destination link experiences hyper-Rayleigh fading. As will be evident from the derivations presented in the sequel, our analysis does not impose any restriction on the m parameter of the Nakagami faded source-relay link, which may or may not experience hyper-Rayleigh fading, i.e., $m < 1$ or $m \geq 1$, respectively. The performance of the proposed upper bound is compared with a well-known upper bound proposed in [16], which is based on the harmonic-geometric mean inequality. We then propose an OPA strategy to increase the average SNR at destination by maximizing the proposed upper bound and compare its performance with OPA achieved by numerical maximization of the exact average SNR expression. An increase in the average SNR is desirable as it reduces the outage probability of the instantaneous SNR in a similar fashion to the outage reduction obtained by introducing a fading margin into the link budget [20]. The remainder of this paper is organized as follows. The system and channel model under consideration are introduced in Section 2. Section 3 provides analytical expressions for the exact average SNR at destination and its proposed upper bound. Section 4 contains analysis for the proposed power allocation. Section 5 contains the numerical and simulation results. Finally, concluding remarks are given in Section 6.

2. System and Channel Model

We consider a dual-hop network consisting of the source (s), relay (r), and destination (d) nodes. The s-d link is assumed to be non-existent or in a deep fade leading to a relayed network without cooperative diversity [1], [2], and [5]. Furthermore, the s, r nodes are assumed to transmit over orthogonal channels. Let h_{sr} and h_{rd} denote the independent flat-fading channel coefficients for the s-r and r-d links, respectively. The corresponding average channel power gains for these links are denoted by σ_{sr}^2 and σ_{rd}^2, respectively. We consider both h_{sr} and h_{rd} to be independent Nakagami-m fading with fading parameters m_1, and m_2, respectively. The instantaneous value of the equivalent SNR at the destination node can then be written as [21]

$$\gamma_{\mathrm{eq}} = \frac{|A|^2 |h_{sr}|^2 |h_{rd}|^2 \rho}{|A|^2 |h_{rd}|^2 + 1} \tag{2}$$

where A is the fixed amplification factor provided by the semi-blind relay, $\rho = \frac{E_{\mathrm{s}}}{N_o}$ is the SNR at the source node, E_{s} is the average transmit power at source, and N_o is the identical power spectral density of additive white Gaussian thermal noise present at all node inputs. Let the average transmit power at the relay, with power allocation parameter α, is $E_{\mathrm{r}} = \alpha E_{\mathrm{s}}$, then the relay's power gain can be expressed as [1]

$$|A|^2 = \frac{E_{\mathrm{r}}}{E_{\mathrm{s}}\sigma_{\mathrm{sr}}^2 + N_0} = \frac{\alpha\rho}{\rho\sigma_{\mathrm{sr}}^2 + 1}. \tag{3}$$

It is evident from the above relation that α can be used to optimize the link performance. Now substituting (3) into (2), the equivalent SNR for the relayed link can be expressed as

$$\gamma_{\mathrm{eq}} = \frac{\alpha\rho^2 |h_{\mathrm{sr}}|^2 |h_{\mathrm{rd}}|^2}{1 + \rho\sigma_{\mathrm{sr}}^2 + \alpha\rho |h_{\mathrm{rd}}|^2}, \tag{4}$$

which can be equivalently written as

$$\gamma_{\mathrm{eq}} = \frac{\rho |h_{\mathrm{sr}}|^2 |h_{\mathrm{rd}}|^2}{C + |h_{\mathrm{rd}}|^2} \tag{5}$$

where $C = \frac{1+\rho\sigma_{sr}^2}{\alpha\rho}$.

3. End-to-End SNR

The SNR at destination when averaged over the channel fading can be written as

$$E\left[\gamma_{\mathrm{eq}}\right] = E\left[\frac{\rho |h_{\mathrm{sr}}|^2 |h_{\mathrm{rd}}|^2}{C + |h_{\mathrm{rd}}|^2}\right]. \tag{6}$$

Given that the channel gains $h_{\mathrm{sr}}, h_{\mathrm{rd}}$ are Nakagami-m distributed, it follows that the respective channel power gains $|h_{\rm sr}|^2$ and $|h_{\rm rd}|^2$ are Gamma distributed random variables [21]. We define $X = |h_{\rm sr}|^2$ and $Y = |h_{\rm rd}|^2$ as independent not necessarily identically-distributed Gamma random variables with the probability density function (pdf) for X written as [21]

$$f_X(x) = \frac{x^{a_X-1}e^{-\frac{x}{b_X}}}{\Gamma(a_X)b_X^{a_X}}.$$

The pdf for Y is defined similarly. The Gamma pdf parameters (a_X, b_X) for X can be related with parameters of the parent Nakagami-m distribution as $a_X = m_1$ and $b_X = \frac{\sigma^2_{\rm sr}}{m_1}$, and similarly for Y we have $a_Y = m_2$ and $b_Y = \frac{\sigma^2_{\rm rd}}{m_2}$. Then the average SNR for the relayed link can be written as

$$\begin{aligned} E\left[\gamma_{\rm eq}\right] &= E\left[\frac{\rho XY}{C+Y}\right] \\ &= \rho\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x\frac{y}{C+y}f_X(x)f_Y(y)\mathrm{d}x\mathrm{d}y \\ &= \rho\int_0^{\infty} x\frac{x^{a_X-1}e^{-\frac{x}{b_X}}}{\Gamma(a_X)b_X^{a_X}}\mathrm{d}x \\ &\quad\times\int_0^{\infty}\frac{y}{C+y}\frac{y^{a_Y-1}e^{-\frac{y}{b_Y}}}{\Gamma(a_Y)b_Y^{a_Y}}\mathrm{d}y \\ &= \rho a_X b_X \underbrace{\int_0^{\infty}\frac{y}{C+y}\frac{y^{a_Y-1}e^{-\frac{y}{b_Y}}}{\Gamma(a_Y)b_Y^{a_Y}}\mathrm{d}y}_{I_1}. \end{aligned} \tag{7}$$

The integral I_1 in (7) is further simplified by substitution and change of variables. After some manipulations, we obtain

$$I_1 = \frac{e^{C_1}}{\Gamma(a_Y)}\int_{C_1}^{\infty}\frac{(z-C_1)^{a_Y}e^{-z}}{z}dz \tag{8}$$

where the constant $C_1 = \frac{1+\rho\sigma^2_{\rm sr}}{\alpha\rho b_Y}$. Now using [14, Eq. (3.383.9)] and the identity [14, Eq. (8.331.1)],

$$\Gamma(x+1) = x\Gamma(x), \tag{9}$$

the integral I_1 can be expressed in closed form as

$$I_1 = a_Y e^{C_1} C_1^{a_Y}\Gamma(-a_Y, C_1) \tag{10}$$

where $\Gamma(a,x)$ is the upper incomplete Gamma function [14, Eq. (8.350.2)]. Substituting (10) into (7) we get

$$E\left[\gamma_{\rm eq}\right] = \rho a_X b_X a_Y e^{C_1} C_1^{a_Y}\Gamma(-a_Y, C_1). \tag{11}$$

The introduction of the incomplete Gamma function into (11) is novel in that it replaces the confluent hypergeometric function [14, Eq. (9.210.1)] conventionally used in SNR expressions for Nakagami-m faded relayed links, see for example [16], [18] and references therein. One may also observe from (11) that the average SNR at destination does not depend on m_1, which is canceled in the product $a_X b_X$. Therefore, our analysis is general in the sense that it is equally applicable whether only the r-d link is hyper-Rayleigh fading or additionally the s-r link is also hyper-Rayleigh fading. Now it can be shown that $\Gamma(-a,x)$ is upper-bounded as

$$\Gamma(-a,x) \le \frac{e^{-x}x^{-a}}{x+a}2^{1-a}\Gamma(1+a),\ \ x>0. \tag{12}$$

Proof. See Appendix. □

Substituting (12) and C_1 into (11) and expressing the Gamma distribution parameters (a_X, b_X) and (a_Y, b_Y) in (11) in terms of their parent Nakagami-m parameters, the average SNR for the relayed link can be upper-bounded as,

$$E\left[\gamma_{\rm eq}\right] \le \frac{\alpha\rho^2\sigma^2_{sr}\sigma^2_{rd}}{\rho\sigma^2_{sr}+\alpha\rho\sigma^2_{rd}+1}2^{1-m_2}\Gamma(1+m_2). \tag{13}$$

With a minor modification we obtain a more tractable upper-bound without significantly affecting the tightness as,

$$E\left[\gamma_{\rm eq}\right] < E\left[\gamma^{\rm UB}_{\rm eq}\right] \equiv \frac{\alpha\rho\sigma^2_{sr}\sigma^2_{rd}}{\sigma^2_{sr}+\alpha\sigma^2_{rd}}2^{1-m_2}\Gamma(1+m_2). \tag{14}$$

To compare the performance of the upper bound obtained in (14) we consider the upper-bound derived by [16] that is based on the well-known harmonic-geometric mean inequality of two positive numbers, which for the case under consideration, are the average SNRs of the two hops. The bound of [16] is valid for N-hop relayed communication over Nakagami-m fading links and, for the two-hop case $(N = 2)$ considered herein, leads to an alternative upper-bound expression for the average end-to-end SNR that can be expressed as,

$$E\left[\gamma^{\rm UB\text{-}KG}_{\rm eq}\right] = \frac{\Gamma\left(m_2+\frac{1}{2}\right)}{2\Gamma(m_2)\sqrt{m_2}}\times\rho\sigma^2_{sr}\sqrt{\frac{\alpha\rho\sigma^2_{rd}}{1+\rho\sigma^2_{sr}}}. \tag{15}$$

Proof. See Appendix. □

Now let $\rho_T = \frac{E_{\rm s}+E_{\rm r}}{N_o}$ denote the total transmit SNR. Then ρ can be written in terms of ρ_T and α as,

$$\rho = \frac{\rho_T}{(1+\alpha)}. \tag{16}$$

Substituting (16) into (14) allows the proposed upper bound for the average SNR to be written as

$$E\left[\gamma^{\rm UB}_{\rm eq}\right] = \frac{\rho_T}{1+\alpha}\left(\frac{\alpha\sigma^2_{\rm sr}\sigma^2_{\rm rd}}{\sigma^2_{\rm sr}+\alpha\sigma^2_{\rm rd}}\right)2^{1-m_2}\Gamma(1+m_2). \tag{17}$$

Similarly, inserting (16) into (15) leads to an alternative upper bound on the average SNR at destination that can be expressed as

$$E\left[\gamma^{\rm UB\text{-}KG}_{\rm eq}\right] = \frac{\rho_T}{1+\alpha}\left(G\sigma^2_{\rm sr}\sqrt{\frac{\alpha\rho_T\sigma^2_{\rm rd}}{1+\alpha+\rho_T\sigma^2_{\rm sr}}}\right) \tag{18}$$

where $G = \frac{\Gamma\left(m_2+\frac{1}{2}\right)}{2\Gamma(m_2)\sqrt{m_2}}$. Note that the distance-dependent pathloss effects are taken into account here by modeling the mean channel power gains as $\sigma^2_{\rm sr} = d^{-n}$, and $\sigma^2_{\rm rd} = (1-d)^{-n}$ for the s-r and r-d links, respectively, where n is the pathloss exponent.

4. Optimal Power Allocation

We now demonstrate that OPA, which maximizes the upper bound of the average SNR $\alpha = \alpha_1$, suffers no performance penalty relative to the OPA based on numerical maximization of the exact expression for average SNR $\alpha = \alpha_{\text{opt}}$. The former scheme has the advantage of being more tractable for mathematical analysis. Now from basic calculus the optimal value of α that maximizes (17) is found to be

$$\alpha_1 = \sqrt{\frac{\sigma_{sr}^2}{\sigma_{rd}^2}}. \tag{19}$$

It is apparent from the above formulation that this power allocation requires only the knowledge of the channel statistics. Substituting α_1 into (17), the upper bound of the average SNR can be written as

$$E\left[\gamma_{\text{eq}}^{\text{UB}}\right]_{\alpha=\alpha_1} = \rho_T \sigma_{\text{rd}}^2 \left(\frac{\sigma_{\text{sr}}^2}{\sigma_{\text{sr}}^2 + \sqrt{\sigma_{\text{sr}}^2 \sigma_{\text{rd}}^2}}\right)^2 2^{1-m_2}\Gamma(1+m_2). \tag{20}$$

4.1 Outage Analysis

The outage probability of the received SNR is one of the conventional metrics that is used to assess the performance of a wireless communication link. It is well-understood from the concept of fading-margin of a wireless link that increasing the average SNR value at destination reduces the outage probability of the instantaneous received SNR [20], [22]. In the previous section we have demonstrated the advantage of our proposed OPA strategy in providing an SNR gain at the destination; in this section we demonstrate that the same OPA strategy results in a reduced outage probability of the instantaneous received SNR, or conversely, a power saving when achieving a target outage. Given that the outage probability is inversely proportional to the average value of the received SNR, the proposed upper-bound maximizing power allocation strategy is shown to require smaller values of the total transmit SNR relative to equal power allocation to achieve the same outage probability. To this end, we begin by expressing the upper-bound from (14) as

$$E\left[\gamma_{\text{eq}}^{\text{UB}}\right] = \frac{\rho_T}{1+\alpha}\left(\frac{\alpha\sigma_{\text{sr}}^2}{\lambda+\alpha}\right) 2^{1-m_2}\Gamma(1+m_2) \tag{21}$$

where $\lambda = \sigma_{\text{sr}}^2/\sigma_{\text{rd}}^2$. Furthermore, let us denote the required total transmit SNR to achieve a target outage with optimal and equal power allocation as $\rho_T\mid_{\alpha=\alpha_1}$ and $\rho_T\mid_{\alpha=1}$, respectively. Then using (21) we can write the transmit SNR for optimal power allocation as

$$\rho_T\mid_{\alpha=\alpha_1} = \frac{(\lambda+\alpha_1)(1+\alpha_1)}{\alpha_1\sigma_{\text{sr}}^2 2^{1-m_2}\Gamma(1+m_2)} \times E\left[\gamma_{\text{eq}}^{\text{UB}}\right], \tag{22}$$

whereas for the equal power allocation case the transmit SNR can be expressed as

$$\rho_T\mid_{\alpha=1} = \frac{2(\lambda+1)}{\sigma_{\text{sr}}^2 2^{1-m_2}\Gamma(1+m_2)} \times E\left[\gamma_{\text{eq}}^{\text{UB}}\right]. \tag{23}$$

By rearranging (23) the upper bound average SNR can be written in terms of λ and $\rho_T\mid_{\alpha=1}$ as

$$E\left[\gamma_{\text{eq}}^{\text{UB}}\right] = \frac{\sigma_{\text{sr}}^2 2^{1-m_2}\Gamma(1+m_2)}{2(\lambda+1)}\rho_T\mid_{\alpha=1}. \tag{24}$$

Substituting (24) into (22), we finally get

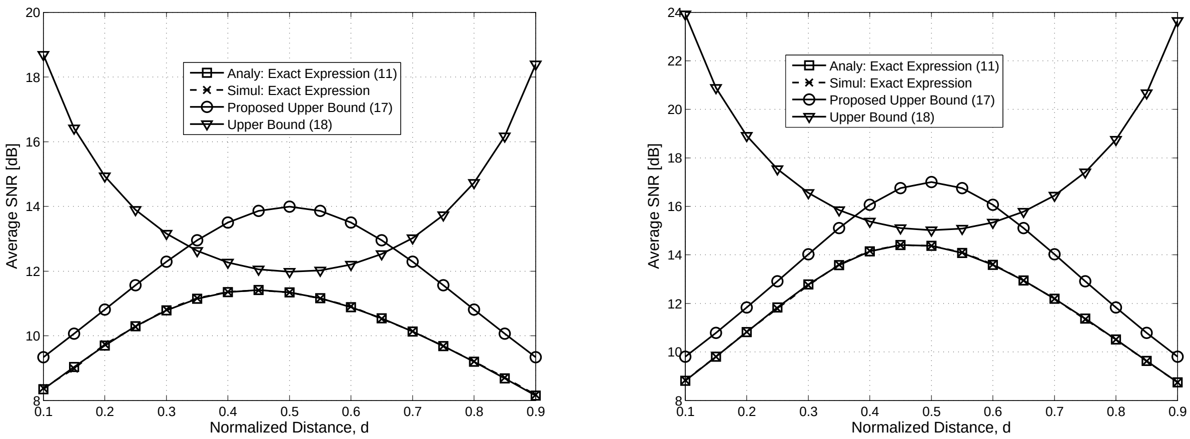
$$\rho_T\mid_{\alpha=\alpha_1} = \frac{(\lambda+\alpha_1)(1+\alpha_1)}{2\alpha_1(\lambda+1)}\rho_T\mid_{\alpha=1}. \tag{25}$$

5. Numerical and Simulation Results

In this section we provide some numerical and simulation results in order to validate the accuracy of the proposed analytical results. Though the results of our theoretical analysis are valid for all practical values of the pathloss exponent n, for illustrative purposes values larger than 2, i.e., $n = 3$ and 4 are used. This choice is based on the heuristic reasoning that a cluttered environment, which results in small scale fading more severe than Rayleigh fading, is expected to exhibit a corresponding pathloss exponent larger than the free-space pathloss exponent of $n = 2$. For example in [9] the value $n = 4$ was reported for the 2-way channel and in general $n = 3,4$ or similar values are expected to be observed. For the results presented in this section, all simulations were carried out using the Matlab computational software, whereas the analytical plots were generated using both Matlab and Mathematica.

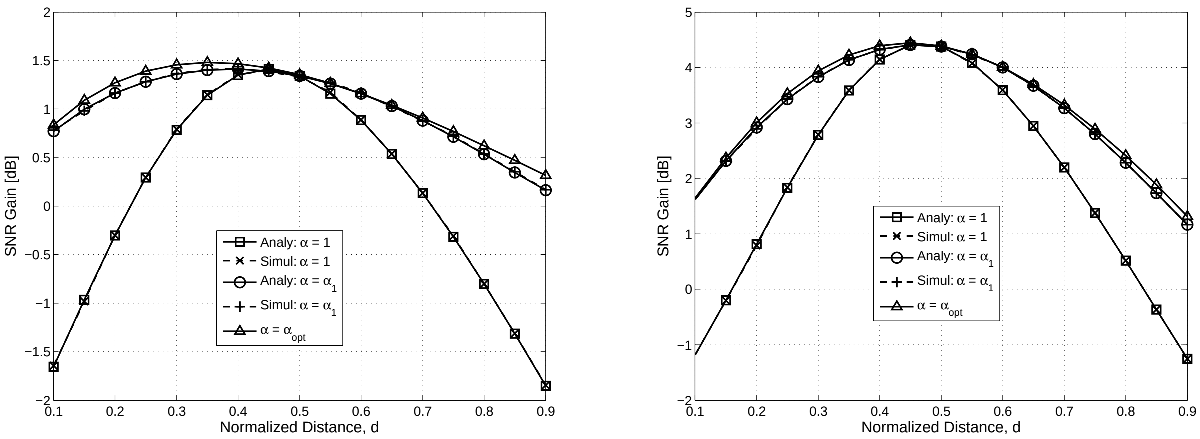
In Fig. 1, we plot the upper bound proposed in (17) against the normalized s-r link distance, $d = d_{\text{sr}}/d_{\text{sd}}$, where d_{ij} is the distance between nodes i and j. The exact SNR expression from (11) is also graphed in the same figure along with Monte Carlo simulations, using results of [23], which verify the correctness of our analysis. The upper bound from (18) is also plotted for comparison. From Fig. 1 one can observe that our proposed bound is significantly tighter than the upper bound of [16] for the case where the relay location approaches the source or destination location, i.e., the power imbalance between the average SNRs of the s-r and r-d links increases. When the relay is midway between these two nodes then our proposed upper-bound performs not significantly different from the bound of [16]. The looseness of the latter bound towards the link edges, as observed in the figure, can be attributed to the harmonic-geometric mean inequality, which loosens as the power imbalance between the average SNRs of the two hops increases.

In Fig. 2, the performance of the proposed power allocation strategy is demonstrated by plotting the SNR gain against the normalized distance d, where the SNR gain is defined as the quotient $E\left[\gamma_{\text{eq}}\right]/\rho_T$. Monte Carlo simulations are also shown in the figure to verify the correctness of the derived analytical expressions. One may observe from Fig. 2 that substituting $\alpha = \alpha_1$ into (11) provides the same SNR gain as that achieved by numerical optimization of (11). The crossovers seen between the two curves in the figure are



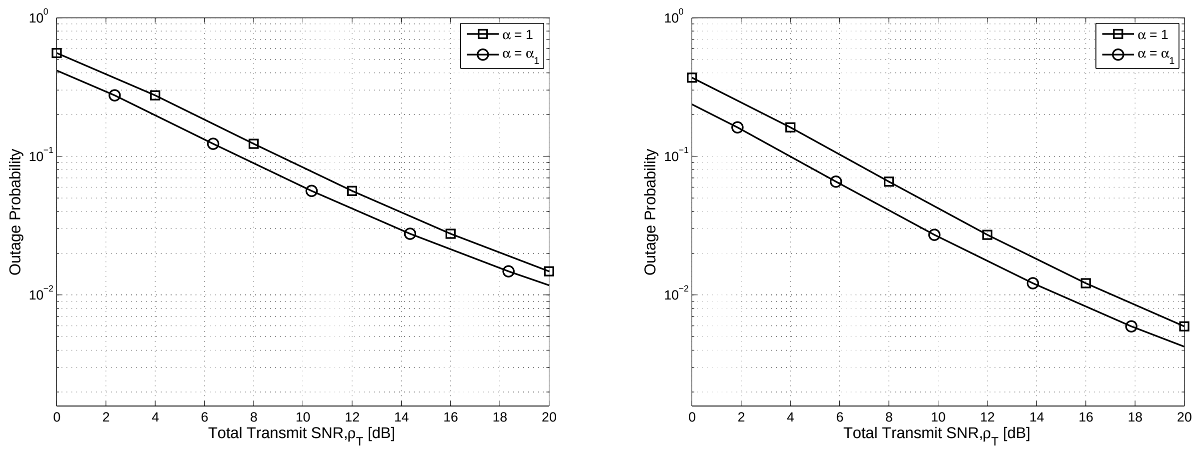
(a) pathloss exponent, n = 3. (b) pathloss exponent, n = 4.

Fig. 1. Comparison of exact average SNR with its upperbounds. Considered parameter values are: $(m_2, \alpha, \rho_{T,dB}) = (0.5, 1, 10)$.



(a) pathloss exponent, n = 3. (b) pathloss exponent, n = 4.

Fig. 2. SNR gain as a function of normalized distance. Considered parameter values are: $(m_2, \rho_{T,dB}) = (0.5, 10)$.



(a) pathloss exponent, n = 3. (b) pathloss exponent, n = 4.

Fig. 3. Outage probability as a function of total transmit SNR. Considered parameter values are: $(m_1, m_2, d, \gamma_{TH,dB}) = (1.2, 0.5, 0.75, 0)$.

due to the fact that we have optimized the upper bound rather than the exact average SNR expression, a fact which has also been observed in [19] for the case of Rayleigh fading.

In Fig. 3, the SNR outage performance of the proposed OPA scheme is shown in relation with the outage for equal power allocation, i.e., $\alpha = 1$. For the latter case, the SNR outage is plotted as a function of total transmit SNR by using the exact outage expression derived in [24, Eq. (20)] for dual-hop semi-blind AF relaying. This somewhat extensive outage expression, though straightforward to evaluate with computational software, is not reproduced here in the interest of brevity. Now to evaluate the outage performance for the $\alpha = \alpha_1$ case, (25) is used to compute the required total transmit SNR to achieve the same outage values as those previously achieved for the $\alpha = 1$ case. The threshold SNR for both outage curves is set to $\gamma_{\mathrm{TH}} = 0$ dB without loss of generality. From Fig. 3 one may observe that the OPA strategy provides an SNR saving of around 2 dB over the equal power allocation case. Such savings in transmit power can be significant when taking into account the fact that the communication channel under consideration is subject to worse than Rayleigh fading. Furthermore, a similar range of values of the power savings have also been reported in [19] for power allocation based on the outage expression for the case of Rayleigh fading.

6. Conclusion

We have derived a new exact expression for the average SNR of a dual-hop semi-blind AF relaying network subject to Nakagami-m fading with arbitrary m. Additionally for the case when the r-d link is subject to worse-than-Rayleigh fading, we have proposed a new upper bound on the average SNR at destination, which has been shown to outperform a previously proposed bound that is based on the well-known harmonic-geometric mean inequality. We have also demonstrated that a power allocation strategy based on maximizing the proposed upper bound rather than the exact average SNR expression has no significant performance loss compared with the latter. The proposed OPA strategy has also been shown to reduce the outage probability relative to the case of equal power allocation. These results hold significance for relayed communication in all practical scenarios of interest, where hyper-Rayleigh fading can occur.

Appendix A: Proof of (12)

Using [14, Eq. (8.353.3)], $\Gamma(-a,x)$ can be expressed as

$$\Gamma(-a,x) = \frac{e^{-x}x^{-a}}{\Gamma(1+a)} \times \int_0^\infty \frac{e^{-t}t^a}{x+t}\mathrm{d}t,\ Re(a) > -1, x > 0. \tag{26}$$

Multiplying (26) on the right by $(x+a)/(x+a)$ we obtain,

$$\Gamma(-a,x) = \frac{e^{-x}x^{-a}}{\Gamma(1+a)\,(x+a)} \int_0^\infty \frac{x+a}{x+t} e^{-t}t^a \mathrm{d}t. \tag{27}$$

Then using the gamma function inequality [25],

$$2^{a-1} \le \Gamma(1+a) \le 1,\ 0 < a \le 1. \tag{28}$$

$\Gamma(-a,x)$ can be upper-bounded as

$$\Gamma(-a,x) \le \frac{2^{1-a}e^{-x}x^{-a}}{x+a} \underbrace{\int_0^\infty \frac{x+a}{x+t} e^{-t}t^a \mathrm{d}t}_{I}. \tag{29}$$

The integral I appearing in (29) above can easily be shown to simplify to

$$I = \Gamma(1+a) + a \underbrace{\int_0^\infty \frac{t}{x+t} e^{-t}t^{a-1}\mathrm{d}t}_{I_1} - \underbrace{\int_0^\infty \frac{t}{x+t} e^{-t}t^a \mathrm{d}t}_{I_2} \tag{30}$$

where we have used the fact that $\Gamma(1+a) = \int_0^\infty e^{-t}t^a \mathrm{d}t$. We formulate an upper-bound for the integral I, by first establishing an upper-bound for the integral I_1. It can readily be observed from (30) that the integrand of I_1 satisfies the inequality

$$\frac{t}{x+t} e^{-t}t^{a-1} \le e^{-t}t^{a-1},\ x > 0, \tag{31}$$

so that from basic calculus it follows that I_1 is upper-bounded as

$$I_1 = \int_0^\infty \frac{t}{x+t} e^{-t}t^{a-1}\mathrm{d}t \le \int_0^\infty e^{-t}t^{a-1}\mathrm{d}t,\ x > 0. \tag{32}$$

Now replacing I_1 in (30) with its upper-bound from (32) and using the Gamma function definition $\Gamma(a) = \int_0^\infty e^{-t}t^{a-1}\mathrm{d}t$ [14, Eq. (8.310.1)] together with (9), I is upper-bounded as

$$I \le 2\Gamma(1+a) - I_2,\ x > 0. \tag{33}$$

Using arguments similar to those for the upper-bound derivation for I_1 above, it can readily be shown that I_2 satisfies the inequality

$$I_2 \le \Gamma(1+a),\ x > 0. \tag{34}$$

Now using (34) in (33) we obtain the relation

$$I \le \Gamma(1+a),\ x > 0 \tag{35}$$

which when used in (29) leads to the inequality

$$\Gamma(-a,x) \le \frac{e^{-x}x^{-a}}{x+a} 2^{1-a}\Gamma(1+a),\ x > 0 \tag{36}$$

which is the desired result.

Appendix B: Proof of (15)

Using [16, Eq. (14)] and setting $k = 1$ for the first moment and $N = 2$ for the 2-hop case, the upper bound for the average SNR of the relayed link can be written as

$$E[S_2^{\text{UB-KG}}] = Z_2 \left[\left(\frac{\bar{\gamma}_1}{m_1} \right) \frac{\Gamma(m_1+1)}{\Gamma(m_1)} \right] \times \left[\left(\frac{\bar{\gamma}_2}{m_2} \right)^{1/2} \frac{\Gamma(m_2+1/2)}{\Gamma(m_2)} \right] \quad (37)$$

where $\bar{\gamma}_1 = \rho\sigma_{sr}^2$ and $\bar{\gamma}_2 = \alpha\rho\sigma_{rd}^2$ represent the average receive SNR for the source-relay and relay-destination links, respectively, and $\Gamma(a)$ is the Gamma function [14, Eq. (8.310)]. Furthermore, Z_2 is a constant dependent on the type of fixed gain used at the relay and is written as [16, Eq. (11)],

$$Z_2 = \frac{1}{2} \prod_{i=1}^{2} K_i^{-\frac{2-i}{2}} = \frac{1}{2\sqrt{K_1}}. \quad (38)$$

For the choice of relay gain given in (3), the constant K_1 in (38) is given by

$$K_1 = \frac{E_r}{A^2 N_o} = 1 + \bar{\gamma}_1. \quad (39)$$

Substituting (38), (39), and $\bar{\gamma}_1, \bar{\gamma}_2$ into (37) and using (9), we obtain (15) after simplification.

References

[1] LANEMAN, J. N., TSE, D. N. C., WORNELL, G. W. Cooperative diversity in wireless networks: Efficient protocols and outage behaviour. *IEEE Transactions on Information Theory*, 2004, vol. 50, no. 12, p. 3062 - 3080.

[2] PABST, R., WALKE, B. H. et al. Relay-based deployment concepts for wireless and mobile broadband radio. *IEEE Communications Magazine*, 2004, vol. 42, no. 9, p. 80 - 89.

[3] NABAR, R. U., BÖLCSKEI, H., KNEUBÜHLER, F. W. Fading relay channels: performance limits and space-time signal design. *IEEE Journal on Selected Areas in Communications*, 2004, vol. 22, no. 6, p. 1099 - 1109.

[4] LIU, K. J. R., SADEK, A. K., SU, W., KWASINSKI, A. *Cooperative Communications and Networking*. Cambridge (UK): Cambridge University Press, 2009.

[5] HASNA, M. O., ALOUINI, M.-S. A performance study of dual-hop transmissions with fixed gain relays. *IEEE Transactions on Wireless Communications*, 2004, vol. 3, no. 6, p. 1963 - 1968.

[6] FAN, Z., GUO, D., ZHANG, B. Outage probability and power allocation for two-way DF relay networks with relay selection. *Radioengineering*, 2012, vol. 21, no. 3, p. 795 - 801.

[7] POLAK, L., KRATOCHVIL, T. DVB-T and DVB-T2 performance in fixed terrestrial TV channels. In *Proceedings of International Conference on Telecommunications and Signal Processing (TSP)*. Prague (Czech Republic), 2012, p. 725 - 729.

[8] TANEDA, M. A., TAKADA, J., ARAKI, K. The problem of the fading model selection. *IEICE Transactions on Communications*, 2001, vol. E84-B, no. 3, p. 355 - 358.

[9] KIM, D., INGRAM, M. A., SMITH, W. W. Jr. Measurements of small-scale fading and path loss for long range RF tags. *IEEE Transactions on Antennas and Propagation*, 2003, vol. 51, no. 8, p. 1740 - 1749.

[10] FROLIK, J. On appropriate models for hyper-Rayleigh fading. *IEEE Transactions on Wireless Communications*, 2008, vol. 7, no. 12, p. 5202 - 5207.

[11] SEN, I., MATOLAK, D. W. Vehicle-vehicle channel models for the 5-GHz band. *IEEE Transactions on Intelligent Transportation Systems*, 2008, vol. 9, no. 2, p. 235 - 245.

[12] TABATABA, F. S., SADEGHI, P., PAKRAVAN, M. R. Outage probability and power allocation of amplify and forward relaying with channel estimation errors. *IEEE Transactions on Wireless Communications*, 2011, vol. 10, no. 1, p. 124 - 134.

[13] ZHANG, Y., MA, Y., TAFAZOLLI, R. Power allocation for bidirectional AF relaying over Rayleigh fading channels. *IEEE Communications Letters*, 2010, vol. 14, no. 2, p. 145 - 147.

[14] GRADSHTEYN, I. S., RYZHIK, I. M. *Table of Integrals, Series and Products*. 7th ed. Burlington (MA, USA): Academic Press, 2007.

[15] HASNA, M. O., ALOUINI, M.-S. Outage probability of multihop transmision over Nakagami fading channels. *IEEE Communications Letters*, 2003, vol. 7, no. 5, p. 216 - 218.

[16] KARAGIANNIDIS, G. K., ZOGAS, D. A., SAGIAS, N. C., TSIFTSIS, T. A., MATHIOPOULOS, P. T. Multihop communications with fixed-gain relays over generalized fading channels. In *Proceedings of IEEE Global Telecommunications Conference (GLOBECOM'04)*. Dallas (TX, USA), 2004, p. 36 - 40.

[17] KARAGIANNIDIS, G. K., TSIFTSIS, T. A., MALLIK, R. K. Bounds for multihop relayed communications in Nakagami-*m* fading. *IEEE Transactions on Communications*, 2006, vol. 54, no. 1, p. 18 - 22.

[18] ZHAI, C., LIU, J., ZHENG, L., CHEN, H. New power allocation schemes for AF cooperative communication over Nakagami-*m* fading channels. In *Proceedings of International Conference on Wireless Communications and Signal Processing (WCSP'09)*. Nanjing (China), 2009, p. 1 - 5.

[19] DENG, X., HAIMOVICH, A. M. Power allocation for cooperative relaying in wireless networks. *IEEE Communications Letters*, 2005, vol. 9, no. 11, p. 994 - 996.

[20] MOLISCH, A. F. *Wireless Communications*. 2nd ed. Chichester (UK): Wiley-IEEE Press, 2011.

[21] SIMON, M. K., ALOUINI, M.-S. *Digital Communications over Fading Channels*. 2nd ed. New York (USA): Wiley, 2005.

[22] GOLDSMITH, A. *Wireless Communications*. Cambridge (UK): Cambridge University Press, 2005.

[23] CAO, L., BEAULIEU, N. C. Simple efficient methods for generating independent and bivariate Nakagami-*m* fading envelope samples. *IEEE Transactions on Vehicular Technology*, 2007, vol. 56, no. 4, p. 1573-1579.

[24] XIA, M., XING, C., WU, Y.-C., AISSA, S. Exact performance analysis of dual-hop semi-blind AF relaying over arbitrary Nakagami-*m* fading channels. *IEEE Transactions on Wireless Communications*, 2011, vol. 10, no. 10, p. 3449 - 3459.

[25] LAFORGIA, A., NATALINI, P. On some inequalities for the Gamma function. *Advances in Dynamical Systems and Applications*, 2013, vol. 8, no. 2, p. 261 - 267.

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