Exact Outage Performance Analysis of Multiuser Multi-relay Spectrum Sharing Cognitive Networks

Tao ZHANG, Weiwei YANG, Yueming CAI

College of Communications Engineering, PLA University of Science and Technology, Yudao Street,
Nanjing City, Jiangsu Province, 210007, China.

ztcool@126.com, wwyang1981@163.com, caiym@vip.sina.com

Abstract. *In this paper, we investigate the outage performance of dual-hop multiuser multi-relay cognitive radio networks under spectrum sharing constraints. Using an efficient relay-destination selection scheme, the exact and asymptotic closed-form expressions for the outage probability are derived. From these expressions it is indicated that the achieved diversity order is only determined by the number of secondary user (SU) relays and destinations, and equals to $M+N$ (where M and N are the number of destination nodes and relay nodes, respectively). Further, we find that the coding gain of the SU network will be affected by the interference threshold $\bar{I}$ at the primary user (PU) receiver. Specifically, as the increases of the interference threshold, the coding gain of the considered network approaches to that of the multiuser multi-relay system in the non-cognitive network. Finally, our study is corroborated by representative numerical examples.*

Keywords

Spectrum sharing, cognitive relay, outage probability, diversity order, coding gain

1. Introduction

Recently, due to the ability to alleviate the spectrum shortage problem spectrum sharing cognitive radio has received much interests [1]. In spectrum sharing networks, the secondary users (SUs) are authorized to have a concurrent transmission with primary user (PU) as long as the generated interference is below an interference temperature tolerated by the primary system. In order to extend the coverage of secondary transmission and enhance system performance in spectrum sharing cognitive networks, cooperative relaying techniques can be further exploited. The performance of decode-and-forward (DF) and amplify-and-forward (AF) relaying in spectrum sharing network is widely investigated in the literature [2], [3]. However, all these prior works only consider a single SU user.

Moreover, multiuser diversity (MUD) has attracted significant attention in non-cognitive cooperative networks. In [4], Sun et al. proposed a joint source-relay selection scheme to select the best source-relay pair to access the channel. Furthermore, Ding et al. [5] proposed a source-relay selection scheme with lower system complexity compared to [4] and achieved the same diversity order. Recently, there were also several works to study the multiuser diversity in spectrum sharing cognitive relaying networks. In [6], the impact of multiuser diversity on the performance of SUs in DF spectrum sharing systems over Nakagami-m fading channels was investigated, while the system only consider a single SU relay and the multi-relay cooperative diversity could not be achieved. Combing multiuser diversity and multi-relay cooperative diversity, the authors of [7] analyzed the outage performance of the multiuser multi-relay networks using an efficient relay-destination selection scheme. However, the theoretical analysis in [7, Eq.(10)] assumes that the interference links from SU relays to primary destination are identical. As we known, there are multiple SU relays to primary destination and the interference links should not be identical due to the different locations of the secondary relays. However, the theoretical analysis in [7, Eq.(10)] assuming that the the interference links are identical, based on which the analysis of the system model is simplified. Such that, the derived closed-form expression in [7] is appropriate.

In this manuscript, we investigate the performance of multiuser multi-relay spectrum sharing cognitive networks with one SU source and M users and N relays in presence of primary receiver and considering both interference and peak power constraints on the SU networks. The exact and asymptotic outage performance are analyzed using the efficient relay-destination selection scheme. Specially, the contributions of this paper are summarized as follows:

1) We present a general analysis of the multiuser multi-relay spectrum sharing cognitive network using the efficient relay-destination selection scheme. Different to that in [7], the interference links from SU relays to primary destination are assumed to be not identical in our analysis. Moreover, we derive the generally exact closed-form expression for the outage probability compared to that in [7], which indicates that the result of [7, Eq.(10)] is the appropriate of our achieved exact closed-form expression for the outage probability.

2) Since the exact analysis is too complicated to ren-

der insight on the impact of the interference threshold and the number of the relays and users, the asymptotic analysis is investigated to indicate that the diversity order is $M+N$, which reveals that the diversity order is only affected by the number of SU relays and destinations. Moreover, we find that the interference threshold at PU receiver will affect the coding gain of the considered network. Specifically, as the increases of the interference threshold, the coding gain of the considered network will approach to that of the multiuser multi-relay system in the non-cognitive network.

3) In special cases, we further analyze the outage performance of the multi-user multi-relay networks based on the efficient relay-destination selection scheme without interference threshold. Moreover, we demonstrate the outage performance analysis in [5] for non-cognitive networks can be the special cases of our works without interference constraint. Finally, simulation results are presented to demonstrate the validity of our theoretical analysis.

2. System Model

We consider a spectrum sharing cognitive relay network, where the secondary network consists of one source S$-S$, N relays S$-R_n\,(n=1,\dots,N)$, and M users S$-D_m\,(m=1,\dots,M)$, whereas the primary network consists of a source P $-$ Tx and a receiver P $-$ Rx and all receivers are affected by additive white Gaussian noise (AWGN) which has zero mean and equal variance (N_0). The P $-$ Tx transmitter is assumed to be far away from the SU nodes so that it does not interfere on the selection process of the relay and destination nodes. An example of such system is shown in Fig. 1. Each node is equipped with a single-antenna device and operates in a half-duplex mode. For the secondary network, the channels are mutually independent flat Rayleigh fading and we denote h_{KT} as the coefficients of the channels between the node K and the node T. And $|h_{KT}|^2$ is an exponentially distributed random with variance $\lambda_{KT} \propto d_{KT}^{-\rho}$, where d_{KT} is the distance between node K and node T, and ρ is the path loss factor.

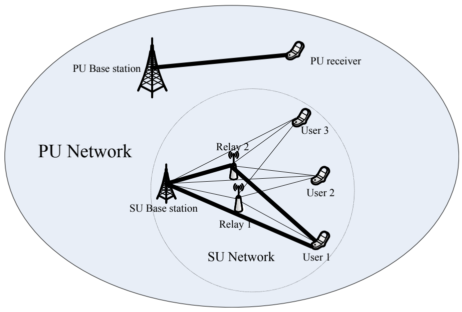

Fig. 1. An example of the considered system. The lines between any two nodes represent the communication links. In this example, the scheduler selects user 1 to access the channel with the help of relay 2 in SU network.

Under the underlay paradigm, S$-S$ and S$-R_n\,(n=1,\dots,N)$ are allowed to use the same frequency as the primary system if the interference generated on P $-$ Rx remains below the interference threshold $\bar{I}$, which is the maximum interference powers tolerable at P $-$ Rx. Thus, the transmit power of S$-S$ and S$-R_n\,(n=1,\dots,N)$ must satisfy $P_S \le \min(\bar{I}/|h_{SP}|^2, P)$ and $P_{R_n} \le \min(\bar{I}/|h_{R_nP}|^2, P)$, respectively, where P is the maximum transmit power constraint of source and relays.

More specifically, the best SU destination D^* is first selected based on the direct links, i.e., $D^* = \arg\max_m [\gamma_{SD_m}]$, where $\gamma_{SD_m} = \min(\bar{I}/|h_{SP}|^2, P)|h_{SD_m}|^2/N_0$ is the instantaneous SNR between the S$-S$ and the mth user S$-D_m$. Secondly, the relay selection process is performed to chose the best relay $R^* = \arg\max_n [\min[\gamma_{SR_n}, \gamma_{R_nD^*}]]$, where $\gamma_{SR_n} = \min(\bar{I}/|h_{SP}|^2, P)|h_{SR_n}|^2/N_0$ is the instantaneous SNR between the S$-S$ and the nth relay S$-R_n$, $\gamma_{R_nD^*} = \min(\bar{I}/|h_{R_nP}|^2, P)|h_{R_nD^*}|^2/N_0$ is the instantaneous SNR between the nth relay S$-R_n$ and the best user S$-D^*$.

For DF protocol, using selection combining (SC) scheme, the achieved system SNR at the destination can be expressed as

$$\gamma_{end}^{DF} = \max\left[\max_m [\gamma_{SD_m}], \max_n [\min[\gamma_{SR_n}, \gamma_{R_nD^*}]]\right]. \quad (1)$$

From (1), the max-min scheme chooses the relay node and the destination node for the multiuser multi-relay spectrum sharing cognitive network. As shown in $\gamma_{SR_n} = \min(\bar{I}/|h_{SP}|^2, P)|h_{SR_n}|^2/N_0$, because the transmission power from the source to every relay is limited according to the same interference threshold at the PU receiver, there exists a common term $|h_{SP}|^2$ in every γ_{SR_n}. This common term implies that the operation in (1) becomes correlated. Note that in [7, Eq(10)], the term $|h_{R_nP}|^2$ in every $\gamma_{R_nD^*} = \min(\bar{I}/|h_{R_nP}|^2, P)|h_{R_nD^*}|^2/N_0$ is assumed to be identical, based on which the analysis of the system model is simplified. However, this assumption is not correct because there are multiple secondary relays in the system, or there are multiple secondary relays to primary destination links. Due to the different locations of the secondary relays, the interference links from SU relays to the primary destination should not be identical.

3. Outage Probability Analysis

Outage event occurs when the achieved system SNR of the selected best channel is below a given threshold γ_{th}. Due to the common terms $|h_{SP}|^2$, the conditional outage probability can be formulated as

$$\Pr\left[\gamma_{end}^{DF} < \gamma_{th} \middle| |h_{SP}|^2\right] = \underbrace{\Pr\left[\max_m \{\gamma_{SD_m}\} < \gamma_{th} \middle| |h_{SP}|^2\right]}_{\Psi_1} \times \underbrace{\Pr\left[\max_{m,n} [\min[\gamma_{SR_n}, \gamma_{R_nD*}]] < \gamma_{th} \middle| |h_{SP}|^2\right]}_{\Psi_2} \quad (2)$$

Firstly, since all the links from S to D_m are statistically independent, Ψ_1 in (2) can be rewritten as

$$\Psi_1 = \prod_{m=1}^{M} F^*_{\gamma_{SD_m}}(\gamma_{th}) \tag{3}$$

where $F^*_{\gamma_{SD_m}}(x)$ is the cumulative distribution function (CDF) of γ_{SD_m} conditioned on $|h_{SP}|^2 = z$, and we firstly derive that

$$\min(\bar{I}/z, P) = \begin{cases} P, \text{when} z \le \bar{I}/P \\ \bar{I}/z, \text{when} z > \bar{I}/P \end{cases} \tag{4}$$

Based on (4), we derive the CDF of $F^*_{\gamma_{SD_m}}(x)$ that

$$F^*_{\gamma_{SD_m}}(x) = \begin{cases} 1 - \exp(-\lambda_{SD_m} x N_0/P), z \le \bar{I}/P \\ 1 - \exp(-\lambda_{SD_m} x N_0/\bar{I} \cdot z), z > \bar{I}/P \end{cases} \tag{5}$$

Then, using the total probability theorem [8, Eq. (12)(13)], Ψ_2 in (2) can be rewritten as

$$\begin{aligned}\Psi_2 &= \Pr\left(\max_n [\min[\gamma_{SR_n}, \gamma_{R_n D^*}]] < \gamma_{th} \,|\, |h_{SP}|^2\right) \\ &= \sum_{m=1}^{M} \Pr(D^* = D_m) \prod_{n=1}^{N} \left[1 - \left(1 - F^*_{\gamma_{SR_n}}(\gamma_{th})\right)\left(1 - F_{\gamma_{R_n D_m}}(\gamma_{th})\right)\right]\end{aligned} \tag{6}$$

where $\Pr(D^* = D_m)$ can be derived with the help of [8, Eq. (14)] as

$$\begin{aligned}&\Pr(D^* = D_m) \\ &= 1 + \sum_{k=1}^{M-1} \sum_{\substack{A_k \subseteq \{1,\ldots,m-1,m+1,\ldots,M\} \\ |A_k| = k}} (-1)^k \frac{\lambda_{SD_m}}{\lambda_{SD_m} + \sum_{j \in A_k} \lambda_{SD_j}}\end{aligned} \tag{7}$$

Notice that $F^*_{\gamma_{SR_n}}(x)$ is the CDF of γ_{SR_n} conditioned on $|h_{SP}|^2 = z$. Based on (5), the conditioned CDF $F^*_{\gamma_{SR_n}}(x)$ and $F^*_{\gamma_{R_n D_m}}(x)$ can be written as

$$F^*_{\gamma_{SR_n}}(x) = \begin{cases} 1 - \exp(-\lambda_{SR_n} x N_0/P), z \le \bar{I}/P \\ 1 - \exp(-\lambda_{SR_n} x N_0/\bar{I} \cdot z), z > \bar{I}/P \end{cases} \tag{8}$$

$$F^*_{\gamma_{R_n D_m}}(x) = \begin{cases} 1 - \exp(-\lambda_{R_n D_m} x N_0/P), z \le \bar{I}/P \\ 1 - \exp(-\lambda_{R_n D_m} x N_0/\bar{I} \cdot z), z > \bar{I}/P \end{cases} \tag{9}$$

In addition, $F_{\gamma_{R_n D_m}}(x)$ in (6) is the CDF of $\gamma_{R_n D_m}$. Performing the integration in (9), $F_{\gamma_{R_n D_m}}(x)$ can be written as

$$\begin{aligned}F_{\gamma_{R_n D_m}}(x) &= \int_0^\infty F^*_{\gamma_{R_n D_m}}(x) f_{|h_{SP}|^2}(z)\,dz \\ &= \int_0^{\bar{I}/P} (1 - \exp(-\lambda_{R_n D_m} x N_0/P)) \lambda_{SP} \exp(-\lambda_{SP} z)\,dz \\ &+ \int_{\bar{I}/P}^{\infty} \left(1 - \exp\left(-\lambda_{R_n D_m} x N_0/\bar{I} \cdot z\right)\right) \lambda_{SP} \exp(-\lambda_{SP} z)\,dz \\ &= 1 - \exp(-\lambda_{R_n D_m} x N_0/P) \left(1 - \frac{\exp\left(-\lambda_{R_n P} \bar{I}/P\right)}{\lambda_{R_n P}/\lambda_{R_n D_m} \bar{I}/N_0/x + 1}\right)\end{aligned} \tag{10}$$

By substituting (3) and (6) into (2) and the general expression for the outage probability can be found by the following integral

$$\begin{aligned}P_{out}^{DL} &= \int_0^\infty \Pr\left(\gamma_{end}^{DF} < \gamma_{th} \,|\, |h_{SP}|^2 = z\right) f_{|h_{SP}|^2}(z)\,dz \\ &= \underbrace{\int_0^{\bar{I}/P} \prod_{m=1}^{M} [1 - \exp(-\lambda_{SD_m} \gamma_{th} N_0/P)] \sum_{m=1}^{M} \Pr(D^* = D_m) \times \prod_{n=1}^{N} [1 - \Phi_n \exp(-\lambda_{SR_n} \gamma_{th} N_0/P)] \lambda_{SP} \exp(-\lambda_{SP} z)\,dz}_{\xi_1} \\ &+ \underbrace{\int_{\bar{I}/P}^{\infty} \prod_{m=1}^{M} \left[1 - \exp\left(-\lambda_{SD_m} \gamma_{th} N_0/\bar{I} \cdot z\right)\right] \sum_{m=1}^{M} \Pr(D^* = D_m) \times \prod_{n=1}^{N} \left[1 - \Phi_n \exp\left(-\lambda_{SR_n} \gamma_{th} N_0/\bar{I} \cdot z\right)\right] \lambda_{SP} \exp(-\lambda_{SP} z)\,dz}_{\xi_2}\end{aligned} \tag{11}$$

where $$\begin{aligned}\Phi_n &= 1 - F_{\gamma_{R_n D_m}}(\gamma_{th}) \\ &= \exp(-\lambda_{R_n D_m} \gamma_{th} N_0/P) \left(1 - \frac{\exp\left(-\lambda_{R_n P} \bar{I}/P\right)}{\lambda_{R_n P}/\lambda_{R_n D_m} \bar{I}/N_0/\gamma_{th} + 1}\right).\end{aligned}$$

Thus, performing the appropriate substitutions in (11) we can derive that

$$\begin{aligned}\xi_1 &= \prod_{m=1}^{M} [1 - \exp(-\lambda_{SD_m} \gamma_{th} N_0/P)] \\ &\times \sum_{m=1}^{M} \Pr(D^* = D_m) \prod_{n=1}^{N} [1 - \Phi_n \exp(-\lambda_{SR_n} \gamma_{th} N_0/P)] \\ &\times \left(1 - \exp\left(-\lambda_{SP} \bar{I}/P\right)\right)\end{aligned} \tag{12}$$

$$\xi_2 = \sum_{m=1}^{M} \Pr(D* = D_m) \left(\exp\left(-\lambda_{SP} \bar{I}/P\right) + \Delta_1 + \Delta_2 + \Delta_3\right) \tag{13}$$

where
$$\Delta_1 = \sum_{m=1}^{M} \frac{(-1)^m}{m!} \underbrace{\sum_{k_1=1}^{M} \cdots \sum_{k_m=1}^{M}}_{k_1 \ne k_2 \ne \cdots \ne k_m} \frac{\lambda_{SP} \exp\left(-\sum_{b=1}^{m} \lambda_{SD_{k_b}} \gamma_{th} N_0/P - \lambda_{SP} \bar{I}/P\right)}{\sum_{b=1}^{m} \lambda_{SD_{k_b}} \gamma_{th} N_0/\bar{I} + \lambda_{SP}},$$

$$\Delta_2 = \sum_{n=1}^{N} \frac{(-1)^n}{n!} \underbrace{\sum_{t_1=1}^{N} \cdots \sum_{t_n=1}^{N}}_{t_1 \ne t_2 \ne \cdots \ne t_n} \prod_{a=1}^{n} \Phi_{t_a} \cdot \frac{\lambda_{SP} \exp\left(-\sum_{a=1}^{n} \frac{\lambda_{SR_{t_a}} \gamma_{th} N_0}{P} - \frac{\lambda_{SP} \bar{I}}{P}\right)}{\sum_{a=1}^{n} \lambda_{SR_{t_a}} \gamma_{th} N_0/\bar{I} + \lambda_{SP}},$$

$$\begin{aligned}\Delta_3 &= \sum_{m=1}^{M} \sum_{n=1}^{N} \frac{(-1)^n}{n!} \frac{(-1)^m}{m!} \underbrace{\sum_{k_1=1}^{M} \cdots \sum_{k_m=1}^{M}}_{k_1 \ne k_2 \ne \cdots \ne k_m} \underbrace{\sum_{t_1=1}^{N} \cdots \sum_{t_n=1}^{N}}_{t_1 \ne t_2 \ne \cdots \ne t_n} \prod_{a=1}^{n} \Phi_{t_a} \\ &\times \frac{\lambda_{SP} \exp\left(-\sum_{b=1}^{m} \lambda_{SD_{k_b}} \gamma_{th} N_0/P - \sum_{a=1}^{n} \lambda_{SR_{t_a}} \gamma_{th} N_0/P - \lambda_{SP} \bar{I}/P\right)}{\sum_{b=1}^{m} \lambda_{SD_{k_b}} \gamma_{th} N_0/\bar{I} + \sum_{a=1}^{n} \lambda_{SR_{t_a}} \gamma_{th} N_0/\bar{I} + \lambda_{SP}}.\end{aligned}$$

The outage probability analysis of the multiuser multi-relay spectrum sharing cognitive networks can be used to assess the feasibility of the application of multi-relay transmission and the multiuser diversity techniques in the cognitive cellular networks [9]. In addition, it will guide the designment of the interference threshold of PU receiver and the number of the SU relays and users in practical system designing.

Special Case: *Multiuser Multi-relay Networks without Interference Threshold* $\left(\bar{I} \to \infty\right)$

In the case when the multiuser multi-relay networks without interference threshold, and the PU can tolerate an unlimited interference from the SU ($\bar{I} \to \infty$), taking ($\bar{I} \to \infty$)

in $P_{out}^{DL} = \xi_1 + \xi_2$.

When $(\bar{I} \to \infty)$, $\exp\left(-\lambda_{SP}\bar{I}/P\right) \to 0$, ξ_1 in (12) can be written as

$$\begin{aligned}\xi_1 = &\prod_{m=1}^{M}\left[1-\exp\left(-\lambda_{SD_m}\gamma_{th}N_0/P\right)\right]\sum_{m=1}^{M}\Pr\left(D* = D_m\right)\\ &\times\prod_{n=1}^{N}\left[1-\Phi_n\exp\left(-\lambda_{SR_n}\gamma_{th}N_0/P\right)\right]\end{aligned} \tag{14}$$

where $\Phi_n = \exp\left(-\lambda_{R_nD_m}\gamma_{th}N_0/P\right)$.

When $(\bar{I} \to \infty)$, in order to obtain ξ_2 in (13), we firstly derive that

$\exp\left(-\lambda_{SP}\bar{I}/P\right) \to 0$ in (14)

$\exp\left(-\sum_{b=1}^{m}\lambda_{SD_{k_b}}\gamma_{th}N_0/P - \lambda_{SP}\bar{I}/P\right) \to 0$ in Δ_1

$\exp\left(-\sum_{a=1}^{n}\lambda_{SR_{t_a}}\gamma_{th}N_0/P - \lambda_{SP}\bar{I}/P\right) \to 0$ in Δ_2

$\exp\left(-\sum_{b=1}^{m}\lambda_{SD_{k_b}}\gamma_{th}N_0/P - \sum_{a=1}^{n}\lambda_{SR_{t_a}}\gamma_{th}N_0/P - \lambda_{SP}\bar{I}/P\right) \to 0$ in Δ_3

Therefore, $\xi_2 \to 0$ when $\bar{I} \to \infty$. Then, we can obtain the outage probability of the multiuser multi-relay network without interference threshold as

$$\begin{aligned}P_{out}^{DL} = &\prod_{m=1}^{M}\left[1-\exp\left(-\lambda_{SD_m}\gamma_{th}N_0/P\right)\right]\sum_{m=1}^{M}\Pr\left(D* = D_m\right)\\ &\times\prod_{n=1}^{N}\left[1-\exp\left(-\lambda_{R_nD_m}\gamma_{th}N_0/P\right)\exp\left(-\lambda_{SR_n}\gamma_{th}N_0/P\right)\right]\end{aligned} \tag{15}$$

The closed-form expression of the outage probability for this special case is obtained, which is equivalent to the result obtained in non-cognitive cooperative system [5, Eq. (18)].

4. Asymptotic Performance Analysis

Since the exact analysis is too complicated to render insight on the performance of the multiuser multi-relay spectrum sharing cognitive network, we turn our attention to the asymptotic outage probability at high SNR regime. Without loss of generality, let $\bar{\gamma} \triangleq P/N_0$ be the system SNR and assume that $P = \eta\bar{I}$. For given $|h_{R_nP}|^2$ and $|h_{SP}|^2$, the outage probability can be rewritten as

$$\begin{aligned}P_{out}^{DL}\left(\gamma_{th}|\alpha_n,\beta\right) = &\prod_{m=1}^{M}\left(1-\exp\left(-\frac{\lambda_{SD_m}N_0}{\beta}\gamma_{th}\right)\right)\\ &\times\Pr\left(D*=D_m\right)\prod_{n=1}^{N}\left\{1-\exp\left(-\frac{\lambda_{SR_n}N_0\gamma_{th}}{\beta}-\frac{\lambda_{R_nD_m}N_0\gamma_{th}}{\alpha_n}\right)\right\}\end{aligned} \tag{16}$$

where $\frac{1}{\alpha_n} = \frac{1}{\min\left[1/\left(\eta|h_{R_nP}|^2\right),1\right]}$, $\frac{1}{\beta} = \frac{1}{\min\left[1/\left(\eta|h_{SP}|^2\right),1\right]}$.

For high SNR regime, $\bar{\gamma} \to \infty$, with the help of the Taylor series $\lim_{x\to 0} e^x = 1+x$, (16) can be approximated as

$$\begin{aligned}P_{out}^{DL,\infty}\left(\gamma_{th}|\alpha_n,\beta\right) \approx &\frac{1}{\beta^M}\prod_{m=1}^{M}\left(\frac{\lambda_{SD_m}\gamma_{th}}{\bar{\gamma}}\right)\\ &\times\Pr\left(D* = D_m\right)\prod_{n=1}^{N}\left\{\frac{\lambda_{SR_n}\gamma_{th}}{\bar{\gamma}\beta}+\frac{\lambda_{R_nD_m}\gamma_{th}}{\bar{\gamma}\alpha_n}\right\}\end{aligned} \tag{17}$$

Averaging over the random variables α_n and β, (17) can be approximated as

$$\begin{aligned}P_{out}^{DL,\infty}\left(\gamma_{th}\right) = &\prod_{m=1}^{M}\left(\frac{\lambda_{SD_m}\gamma_{th}}{\bar{\gamma}}\right)\Pr\left(D* = D_m\right)\\ &\times\prod_{n=1}^{N}\left(\frac{\lambda_{R_nD_m}\gamma_{th}}{\bar{\gamma}}\right)E\left[\frac{1}{\beta^M}\prod_{n=1}^{N}\left\{\frac{1}{\mu_n\beta}+\frac{1}{\alpha_n}\right\}\right]\end{aligned} \tag{18}$$

where $\mu_n = \lambda_{R_nD_m}/\lambda_{SR_n}$, in order to derive $E\left[\frac{1}{\beta^M}\prod_{n=1}^{N}\left\{\frac{1}{\mu_n\beta}+\frac{1}{\alpha_n}\right\}\right]$, using the binomial expansion, we firstly write $\frac{1}{\beta^M}\prod_{n=1}^{N}\left(\frac{1}{\alpha_n}+\frac{1}{\mu_n\beta}\right)$ as

$$\begin{aligned}&\frac{1}{\beta^M}\prod_{n=1}^{N}\left(\frac{1}{\alpha_n}+\frac{1}{\mu_n\beta}\right)\\ &= \sum_{a=0}^{N}\sum_{\substack{A_a\subseteq\{1,\dots,N\}\\ |A_a|=a,|\overline{A_a}|=N-a}}\prod_{i\in A_a}\frac{1}{\alpha_i}\prod_{j\in\overline{A_a}}\frac{1}{\mu_j}\frac{1}{\beta^{M+N-a}}\end{aligned} \tag{19}$$

In order to derive $E\left[\frac{1}{\beta^M}\prod_{n=1}^{N}\left(\frac{1}{\mu_n\alpha_n}+\frac{1}{\beta}\right)\right]$, we perform the integration of α_i and β in (19) and obtain that

$$\begin{aligned}&E\left[\frac{1}{\beta^M}\prod_{n=1}^{N}\left(\frac{1}{\alpha_n}+\frac{1}{\mu_n\beta}\right)\right]\\ &= \sum_{a=0}^{N}\sum_{\substack{A_a\subseteq\{1,\dots,N\}\\ |A_a|=a,|\overline{A_a}|=N-a}}\underbrace{\int_0^\infty\cdots\int_0^\infty}_{a+1}\prod_{i\in A_a}\frac{1}{\alpha_i}\frac{\prod_{j\in\overline{A_a}}\frac{1}{\mu_j}}{\beta^{M+N-a}}\lambda_{SP}\\ &\times\exp\left(-\lambda_{SP}x\right)\prod_{i\in A_a}\lambda_{R_iP}\exp\left(-\lambda_{R_iP}y_i\right)dx\prod_{i\in A_a}dy_i\\ &= \sum_{a=0}^{N}\sum_{\substack{A_a\subseteq\{1,\dots,N\}\\ |A_a|=a,|\overline{A_a}|=N-a}}\underbrace{\prod_{i\in A_a}\int_0^\infty\frac{1}{\alpha_i}\lambda_{R_iP}\exp\left(-\lambda_{R_iP}y_i\right)dy_i}_{\Xi_1}\\ &\underbrace{\times\int_0^\infty\frac{\prod_{j\in\overline{A_a}}\frac{1}{\mu_j}}{\beta^{M+N-a}}\lambda_{SP}\exp\left(-\lambda_{SP}x\right)dx}_{\Xi_2}\end{aligned} \tag{20}$$

Along with some algebraic manipulations, Ξ_1 in (20) can be derived as

$$\begin{aligned}\Xi_1 &= \int_0^{\frac{1}{\eta}}\frac{1}{P}f_{|h_{R_iP}|^2}\left(y_i\right)dy_i + \int_{\frac{1}{\eta}}^{\infty}\eta y_i f_{|h_{R_iP}|^2}\left(y_i\right)dy_i\\ &= \left(1-\exp(-\lambda_{R_iP}/\eta)\right)+\eta\lambda_{R_iP}^{-1}\Gamma\left(2,\lambda_{R_iP}/\eta\right)\end{aligned} \tag{21}$$

And Ξ_2 in (20) can be obtained by following similar lines as in (21), along with some simple algebraic manipulations, it can be written as

$$\begin{aligned}\Xi_2 &= \int_0^{\frac{1}{\eta}}\prod_{j\in\overline{A_a}}\frac{1}{\mu_j}f_{|h_{SP}|^2}\left(x\right)dx\\ &+\int_{\frac{1}{\eta}}^{\infty}\prod_{j\in\overline{A_a}}\frac{1}{\mu_j}\left(\eta x\right)^{M+N-a}f_{|h_{SP}|^2}\left(x\right)dx\\ &= \left(1-\exp(-\lambda_{SP}/\eta)\right)\prod_{j\in\overline{A_a}}\frac{1}{\mu_j}\\ &+\eta^{M+N-a}\lambda_{SP}^{a-M-N}\Gamma\left(M+N-a+1,\lambda_{SP}/\eta\right)\prod_{j\in\overline{A_a}}\frac{1}{\mu_j}\end{aligned} \tag{22}$$

By substituting (20) into (18), the asymptotic outage probability can be finally obtained as

$$
\begin{aligned}
&\mathrm{P}_{out}^{DL,\infty}(\gamma_{th}) \approx \sum_{a=0}^{N} \sum_{\substack{A_a \subseteq \{1,\ldots,N\} \\ |A_a|=a, |\overline{A_a}|=N-a}} \Bigg((1-\exp(-\lambda_{SP}/\eta)) \prod_{j\in\overline{A_a}} \frac{1}{\mu_j} \\
&+\eta^{M+N-a}\lambda_{SP}^{a-M-N}\Gamma(M+N-a+1,\lambda_{SP}/\eta) \prod_{j\in\overline{A_a}} \frac{1}{\mu_j} \Bigg) \\
&\times \prod_{i\in A_a} \left((1-\exp(-\lambda_{R_iP}/\eta)) + \eta\lambda_{R_iP}^{-1}\Gamma(2,\lambda_{R_iP}/\eta) \right) \\
&\times \prod_{m=1}^{M} \left(\frac{\lambda_{SD_m}\gamma_{th}}{\bar{\gamma}} \right) \sum_{m=1}^{M} \Pr(D* = D_m) \prod_{n=1}^{N} \left(\frac{\lambda_{R_nD_m}\gamma_{th}}{\bar{\gamma}} \right) \\
&= \varepsilon_A \left(\frac{\gamma_{th}}{\bar{\gamma}} \right)^{M+N}
\end{aligned} \tag{23}
$$

where

$$
\begin{aligned}
\varepsilon_A = &\sum_{a=0}^{N} \sum_{\substack{A_a \subseteq \{1,\ldots,N\} \\ |A_a|=a, |\overline{A_a}|=N-a}} \Bigg((1-\exp(-\lambda_{SP}/\eta)) \prod_{j\in\overline{A_a}} \frac{1}{\mu_j} \\
&+\eta^{M+N-a}\lambda_{SP}^{a-M-N}\Gamma(M+N-a+1,\lambda_{SP}/\eta) \prod_{j\in\overline{A_a}} \frac{1}{\mu_j} \Bigg) \\
&\times \prod_{i\in A_a} \left((1-\exp(-\lambda_{R_iP}/\eta)) + \eta\lambda_{R_iP}^{-1}\Gamma(2,\lambda_{R_iP}/\eta) \right) \\
&\times \prod_{m=1}^{M} (\lambda_{SD_m}) \sum_{m=1}^{M} \Pr(D* = D_m) \prod_{n=1}^{N} (\lambda_{R_nD_m})
\end{aligned} .
$$

5. Remarks

As can be observed from (23), the diversity order of the SU network is only determined by the number of SU relays and destinations, as the diversity gain $\mathcal{G}_d \triangleq \lim_{\bar{\gamma}\to\infty} \frac{-\log P_{out}^{DL,\infty}}{\log\bar{\gamma}} = M+N$. It is noteworthy that the primary network affects the coding gain $\mathcal{G}_c = \frac{\varepsilon_A^{-\frac{1}{M+N}}}{\gamma_{th}}$ of the SU network. Moreover, we note that there exits a coding gain gap [10, Eq. (31)] between the multiuser multi-relay system in the cognitive network and the same multiuser multi-relay system in the non-cognitive system due to the interference threshold $\bar{I}$ at the PU receiver, which can be written as

$$
G = 10\log_{10}\left(\frac{\varepsilon_A}{\varepsilon_B} \right)^{\frac{1}{M+N}} \tag{24}
$$

where ε_B is the gain of the multiuser multi-relay system in the non-cognitive network, which can be derived with the help of (15) as

$$
\begin{aligned}
&P_{out}^{DL,\infty} \approx \prod_{m=1}^{M} [\lambda_{SD_m}\gamma_{th}/\bar{\gamma}] \sum_{m=1}^{M} \Pr(D* = D_m) \\
&\times \prod_{n=1}^{N} [\lambda_{R_nD_m}\gamma_{th}/\bar{\gamma} + \lambda_{SR_n}\gamma_{th}/\bar{\gamma}] \\
&= \prod_{m=1}^{M} [\lambda_{SD_m}] \sum_{m=1}^{M} \Pr(D* = D_m) \prod_{n=1}^{N} [\lambda_{R_nD_m} + \lambda_{SR_n}] \left(\frac{\gamma_{th}}{\bar{\gamma}} \right)^{M+N} \\
&= \varepsilon_B \left(\frac{\gamma_{th}}{\bar{\gamma}} \right)^{M+N}
\end{aligned} \tag{25}
$$

Thus, we can derive ε_B that

$$
\varepsilon_B = \prod_{m=1}^{M} [\lambda_{SD_m}] \sum_{m=1}^{M} \Pr(D* = D_m) \prod_{n=1}^{N} [\lambda_{R_nD_m} + \lambda_{SR_n}] \tag{26}
$$

This result indicates that for the same outage probability, the performance of multiuser multi-relay system in non-cognitive network outperforms the same system in the cognitive network by an gap of $10\log_{10}\left(\frac{\varepsilon_A}{\varepsilon_B}\right)^{\frac{1}{M+N}}$ dB, because of the multiuser multi-relay system in the cognitive network is affected by the interference threshold at the PU receiver.

6. Simulation Results and Analysis

In this section, we confirm our outage probability analysis through comparisons with simulation results. In all cases, we assume that the distance between the SU source and the first SU destination $S-D_1$ is $d_{SD_1} = 1$ without loss of generality. And those the links $S-D_2$, $S-R_1$, $S-R_2$, R_1-D_1, R_1-D_2, R_2-D_1 and R_2-D_2 are $d_{SD_2} = 1.1d_{SD_1}$, $d_{SR_1} = 0.5d_{SD_1}$, $d_{SR_2} = 0.6d_{SD_1}$, $d_{R_1D_1} = 0.5d_{SD_1}$, $d_{R_1D_2} = 0.55d_{SD_1}$, $d_{R_2D_1} = 0.5d_{SD_1}$, $d_{R_2D_2} = 0.55d_{SD_1}$, respectively. In addition, the distance between the SU source and the PU receiver $S-P$ is $d_{SP} = 0.8d_{SD_1}$, the distance between two SU relays and the PU receiver R_1-P and R_2-P are $d_{R_1P} = 0.5d_{SD_1}$ and $d_{R_2P} = 0.6d_{SD_1}$, respectively, so that the overall transmit power is governed by the interference at the PU receiver as well as by the maximum transmission power at the respective nodes. The variance of the Rayleigh channel fading between any two nodes is determined by the distance between them, and the path loss exponent $\rho = 3$. The threshold γ_{th} is set to 3 dB.

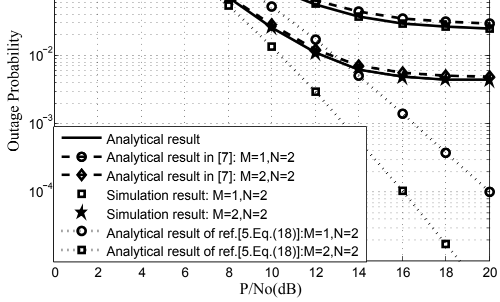

Fig. 2. Outage probability of the system for different numbers of M and N.

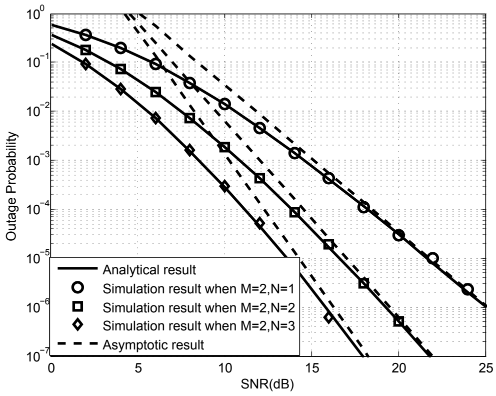

Fig. 3. Outage probability versus SNR $\bar{\gamma}$ with direct links assuming DF relays.

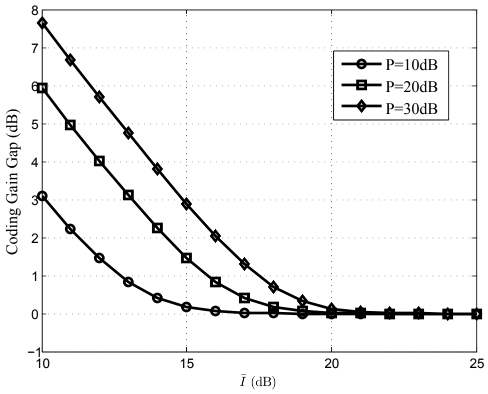

Fig. 4. Coding gain gap between the cognitive network and non-cognitive network versus $\bar{I}$ under different P.

Figure 2 shows that the outage probability when $(M=1, N=2)$ and $(M=2, N=2)$ and $\bar{I}$ =15 dB. We can see that the simulation results match exactly with our proposed analysis results and the outage performance is better than that in [7], which demonstrates that the result of [7, Eq.(10)] is the appropriate of our achieved exact closed-form expression for the outage probability. A floor in the outage performance curve is observed, which is due to the interference threshold constraint. Moreover, as interference threshold gets ∞, the analytical results of [5, Eq.(18)] are derived. It shows that the analysis of the multiuser multi-relay networks based on the efficient source-relay selection scheme in [5] can be the special cases of our works without interference constraint.

Figure 3 illustrates the outage probability and the asymptotic results based on the analysis in section IV. The following parameter values are used: $P = \eta\bar{I}$, $\eta = 1$ and $\bar{\gamma} \overset{\Delta}{=} P/N_0 \to \infty$. In addition, the asymptotic curves are shown to be very tight with the exact curves at high SNR regions, which confirm the correctness of our analysis. As can be observed from the figure that the diversity order of the considered system is only determined by the number of SU relays and destinations, and equals to $M+N$, which reveals that the diversity order of the considered system is not affected by the interference threshold at the PU receiver.

Figure 4 plots the coding gain gap between the multiuser multi-relay system in the cognitive network and the multiuser multi-relay system in the non-cognitive network. From the figure, we can see that the coding gain of the multiuser multi-relay system in the cognitive network is worse than that of the system in the non-cognitive system due to the impact of the interference threshold $\bar{I}$ at the PU receiver. However, the coding gain gap approaches to 0 dB for $\bar{I}$ beyond 20 dB. The main reason is that the performance of the system in the cognitive network approaches to that of the system in the non-cognitive network. This observation can also be analytically supported by (24). For given P, when $\bar{I} \to \infty$, $\eta = P/\bar{I} \to 0$. we have $\exp(-\lambda_{SP}/\eta) \to 0$, $\exp(-\lambda_{R_iP}/\eta) \to 0$, $\eta^{M+N-a}\lambda_{SP}^{a-M-N}\Gamma(M+N-a+1, \lambda_{SP}/\eta) \to 0$ and $\eta\lambda_{R_iP}^{-1}\Gamma(2, \lambda_{R_iP}/\eta) \to 0$ in ε_A, hence, we will derive that $\varepsilon_A = \varepsilon_B$.

7. Conclusions

In this paper, we derive the closed-form expressions for outage probability of the multiuser multi-relay spectrum sharing cognitive networks with both interference and maximum allowable transmit power constraint. We derive the exact closed-form expression for outage probability using the efficient relay-destination selection scheme compared to that in [7]. Furthermore, asymptotic analysis indicates that the achieved diversity order is $M+N$, which notes that the diversity order is same to that in non-cognitive cooperative system and the interference threshold at the PU receiver only affects the coding gain of the SU network. Our analysis can be used to guide the designment of the multiuser multi-relay spectrum sharing cognitive system in the cognitive cellular networks. Further study is to derive the optimum power allocation strategy and energy efficiency for the multiuser multi-relay spectrum sharing cognitive network.

Acknowledgements

This work is supported by the Project of Natural Science Foundations of China (No. 61301162 and 61301163) and the Jiangsu Provincial Natural Science Foundation of China (No. BK20130067).

References

[1] MITOLA, J. *Cognitive Radio: An Integrated Agent Architecture for Software Defined Radio*. Ph. D. dissertation, Royal Institute of Technology (KTH), Stockholm, Sweden, 2000.

[2] XU, W., ZHANG, J., ZHANG, P., TELLAMBURA, C. Outage probability of decode-and-forward cognitive relay in presence of primary user's interference, *IEEE Communication Letters,* vol. 16, no. 8, p. 1252–1255, 2012. DOI: 10.1109/LCOMM.2012.061912.120770

[3] XIA, M., AISSA, S. Cooperative AF relaying in spectrum-sharing systems: performance analysis under average interfer-

ence power constraints and Nakagami-m fading. *IEEE Transactions on Communications,* vol. 60, no. 6, p. 1523–1533, 2012. DOI: 10.1109/TCOMM.2012.042712.110410

[4] SUN, L., ZHANG, T., LU, L., NIU, H. On the combination of cooperative diversity and multiuser diversity in multi-source multi-relay wireless networks. *IEEE Signal Processing Letters*, vol. 17, no. 6, p. 535–538, 2010. DOI: 10.1109/LSP.2010.2046350

[5] DING, H., GE, J., DA COSTA, D. B., JIANG, Z. A new efficient low complexity scheme for multi-source multi-relay cooperative networks. *IEEE Transactions on Vehicular Technology*, vol. 60, no. 2, p. 716–722, 2011. DOI: 10.1109/TVT.2010.2100416

[6] HUANG, Y., AL-QAHTANI, F., WU, Q., ZHONG, C., WANG, J., ALNUWEIRI, H. Outage analysis of spectrum sharing relay systems with multi-secondary destinations under primary user's interference. *IEEE Transactions on Vehicular Technology*, vol. 63, no. 7, p. 3456–3463, 2014. DOI: 10.1109/TVT.2014.2297973

[7] GUIMARAES, F. R. V., DA COSTA, D. B., TSIFTSIS, T. A., CAVALCANTE, C. C., ET AL. Multi-user and multi-relay cognitive radio networks under spectrum sharing constraints. *IEEE Transactions on Vehicular Technology*, vol. 63, no. 1, p. 433–439, 2014. DOI: 10.1109/TVT.2013.2275201

[8] DE MELO, M. A. B., DA COSTA, D. B. An efficient relay-destination selection scheme for multiuser multirelay downlink cooperative networks. *IEEE Transactions on Vehicular Technology*, vol. 61, no. 5, p. 2354–2360, 2012. DOI: 10.1109/TVT.2012.2192488

[9] XU, T., GE, J., DING, H. Opportunistic scheduling for uplink cognitive cellular networks with outage protection of the primary user, *IEEE Communication Letters*, vol. 17, no. 1, p. 71–74, 2013. DOI: 10.1109/LCOMM.2012.111612.121872

[10] DUONG, T. Q., DA COSTA, D. B., ELKASHLAN, M., BAO, V. N. Q. Cognitive amplify-and-forward relay networks over Nakagami-m fading, *IEEE Transactions on Vehicular Technology*, vol. 61, no. 5, p. 2368–2374, 2012. DOI: 10.1109/TVT.2012.2192509

About the Authors...

Tao ZHANG received the B.S. degree in Communication Engineering from PLA University of Science and Technology, Nanjing, China in 2011. He is currently pursuing for the Ph.D. degree in Communications and Information Systems at the College of Communications Engineering, PLA University of Science and Technology, Nanjing, China. His current research interest includes cooperative communications, wireless sensor networks and physical layer security.

Weiwei YANG received the B.S. and M.S. degrees in College of Communications Engineering, PLA University of Science and Technology, Nanjing, China in 2003 and 2006, respectively. His research interests are mainly on OFDM systems, signal processing in communications, cooperative communications and wireless sensor networks and physical layer security.

Yueming CAI received the B.S. degree in Physics from Xiamen University, Xiamen, China in 1982, the M.S. degree in Micro-electronics Engineering and the Ph.D. degree in Communications and Information Systems both from Southeast University, Nanjing, China in 1988 and 1996 respectively. His current research interest includes MIMO systems, OFDM systems, signal processing in communications, cooperative communications, wireless sensor networks and physical layer security.