Cascade Structure of Digital Predistorter for Power Amplifier Linearization

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Abstract. In this paper, a cascade structure of nonlinear digital predistorter (DPD) synthesized by the direct learning adaptive algorithm is represented. DPD is used for linearization of the power amplifier (PA) characteristics, namely for compensation of PA nonlinear distortion. Blocks of the cascade DPD are described by different models: the functional link artificial neural network (FLANN), the polynomial perceptron network (PPN) and the radially pruned Volterra model (RPVM). At synthesis of the cascade DPD there is possibility to overcome the ill conditionality problem due to reducing the dimension of the DPD nonlinear operator approximation. Results of compensating nonlinear distortion in Wiener–Hammerstein model of PA at the GSM–signal with four carriers are shown. The highest accuracy of PA linearization is produced by the cascade DPD containing PPN and RPVM.

Keywords
Digital predistorter, nonlinear compensation, nonlinear distortion, power amplifier, neural networks

1. Introduction

With development of mobile communication, requirements for transmitting systems containing power amplifiers become increasingly stringent. PA is a nonlinear device, where the transmitted signal is distorted and its spectrum extends beyond the boundaries of the communication channel bandwidth. As a result, in multi-channel communication systems the distortion caused by the influence of adjacent channels is amplified (inter-channel interference is emerged) [1].

To prevent the extension of the PA output signal spectrum and to maintain high PA energy efficiency the linearization of the PA characteristics is fulfilled. One of universal linearization methods is the digital predistortion (precompensation), which is characterized by robustness, simplicity of predistorter hardware implementation, efficiency of nonlinear distortion canceling [3]. The purpose of digital predistorter (DPD, precompensator) is to linearize the PA characteristics by introducing a predistortion compensating nonlinear PA distortion. In wideband communication channels PAs with high efficiency are described by nonlinear dynamic models therefore DPDs have nonlinear dynamic models, too [1].

Both the modification of DPD polynomial models and the DPD synthesis on the basis of neural networks are developed [3]. Neural models can be much simpler than polynomials. This fact is important for DPD hardware implementation.

The present work describes the use of the functional link artificial neural network [2–4] and the polynomial perceptron network [4], [5] for synthesis of the cascade DPD in order to improve the quality of cancellation of nonlinear signal distortion in PA. The comparative analysis of DPD models is considered by the example of compensating nonlinear distortion in the Wiener-Hammerstein model of PA at the GSM-signal with four carriers.

2. Cascade Structure of Adaptive DPD

DPD introduces nonlinear predistortion in order to compensate PA nonlinear distortion or to linearize PA.

A structure of an adaptive DPD link to PA in accordance with the feedforward algorithm of DPD learning is shown in Fig. 1 [6]. Here, DPD is described by the nonlinear operator \( S \) composed of two nonlinear operators \( S_1, S_2 \), which are characteristic of two cascade connected blocks correspondently. The operators \( S_1 \) and \( S_2 \) are introduced in the operational equations

\[
z(n) = S_1[x(n)], \quad z(n) = S_2[z(n)],
\]

where \( n \) is the normalized discrete time, \( x(n) \), \( z(n) \) are the DPD input and output signals respectively, \( z(n) \) is the output signal of the block described by the operator \( S_1 \) and shown in Fig. 1.

Models of nonlinear operators \( S_1 \) and \( S_2 \) are constructed by solving the approximation problem

\[
\|y(z(n)) - x(n)\| \Rightarrow \min_{n \in [0, N_X]}, \quad (1)
\]
The FLANN model is expressed as
\[ y(n) = f \left( \sum_{i=1}^{G} w_i \varphi_i(x(n)) \right) = f \left( W^T \Phi(X(n)) \right) \]  
\hspace{1cm} (2)

where \( f \) is the nonlinear activation function of the network, \( X(n) \) is the vector of input signals, \( X(n) = [x_1(n), x_2(n), \ldots, x_d(n)]^T \), \( T \) denotes transposing of the vector, \( W(n) \) is the vector of network weights, \( W = [w_1, w_2, \ldots, w_G]^T \), \( \Phi(X(n)) \) is the vector of the functions \( \varphi_i, \ \Phi(X(n)) = [\varphi_1(X(n)), \varphi_2(X(n)), \ldots, \varphi_G(X(n))]^T \), \( y(n) \) is the output signal of the model.

The functions \( \varphi_i, i = 1, 2, \ldots, G \) of the model (2) transform input signals into basic functions by, for example, trigonometric polynomials, Legendre or Chebyshev polynomials and carry out a multidimensional transformation of the obtained basis functions. The basic functions are formed for reducing the condition number at solving high nonlinear approximation problem.

The FLANN structure is shown in Fig. 2.

\[ x_1(n) \rightarrow \varphi_1(X(n)) \rightarrow f(\bullet) \rightarrow y(n) \]
\[ x_2(n) \rightarrow \varphi_2(X(n)) \rightarrow w_2 \]
\[ \vdots \]
\[ x_G(n) \rightarrow \varphi_G(X(n)) \rightarrow w_G \]

3. Functional Link Artificial Neural Network

FLANN is a single-layer network [2-4]. Therefore the algorithm of this network learning has a more rapid convergence to the solution of the approximation problem and it is simpler in comparison with the algorithms of traditional neural networks learning.
products generated by nonlinear PA [8].

The mentioned multiplication results in producing the vector \( \Phi(X(n)) \), which is then multiplied by the vector \( W^T \) in (3). As a result we form the CFLANN model of odd degree.

4. Polynomial Perceptron Network

PPN is a single-layer network [4], [5]. This network is characterized by the simplicity of its learning algorithm and high speed of convergence to the solution of the approximation problem.

The PPN model is described by the expression [4], [5]

\[
y(n) = f\left( W^T F(X(n)) \right)
\]

(4)

where \( f \) is the nonlinear activation function of the network, \( X(n) \) is the vector of input signals, \( X(n) = [x_1(n), x_2(n), \ldots, x_Q(n)]^T \). \( T \) denotes transposing of the vector, \( W(n) \) is the vector of network weights, \( W = [w_1, w_2, \ldots, w_d]^T \), \( F(X(n)) \) is the vector containing elements with multidimensional transformation,

\[
F(X(n)) = [1, x_1(n), \ldots, x_Q(n), x_1^2(n), \ldots, x_Q^2(n), \ldots, x_1(n)x_2(n), \ldots, x_Q(n)x_1(n), \ldots, x_Q^P(n), \ldots, x_1^P(n), \ldots, x_Q^P(n)]^T
\]

\( P \) is the degree of the function-vector element, \( y(n) \) is the output signal of the model (4).

The PNN structure is shown in Fig. 4.
From a comparison of PPN and FLANN it follows that FLANN (Fig. 2) is obtained from PPN (Fig. 4) by the additional transformation of input signals in basic functions by using Legendre polynomials, Chebyshev polynomials, etc.

For DPD synthesis let us assume that the activation function is linear in the model (4). We can rewrite (4) as

\[ y(n) = \mathbf{w}^T \mathbf{f}(\mathbf{x}(n)). \]  

(5)

The input signal vector \( \mathbf{x}(n) \) is formed on the basis of delay line. Multidimensional transformation \( \mathbf{f} \) in the model (5) is performed under conditions of constructing intermodulation spectral components of the DPD output [8], [9].

As a result we obtain the following PPN model

\[ y(n) = y_1(n) + y_2(n) \]  

(6)

where

\[ y_1(n) = \sum_{i=0}^{M} x(n-i) \sum_{k=0}^{(P-1)/2} w_{2k+1}^1 x(n-i)^{2k} + \]

\[ \sum_{i=1}^{M} x(n-i) \sum_{k=0}^{(P-1)/2} w_{2k+1}^2 x(n-i)^{2k} + \]

\[ + \sum_{i=0}^{M} x^*(n-i) x^2(n) + \sum_{k=0}^{(P-3)/2} w_{2k+3}^3 x(n-i)^{2k} + \]

\[ + \sum_{i=1}^{M} x(n) x^*(n-i) \sum_{k=0}^{(P-3)/2} w_{2k+3}^4 x(n-i)^{2k} + \]

\[ + \sum_{i=1}^{M} x(n) \sum_{k=1}^{(P-5)/2} w_{2k+5}^5 x(n-i)^{2k} \]

\[ + x^2(n) \sum_{i=0}^{M} x^*(n-i) \sum_{k=0}^{(P-3)/2} w_{2k+3}^6 x(n-i)^{2k} + \]

\[ + |x(n)|^2 \sum_{i=0}^{M} x(n-i) \sum_{k=1}^{(P-3)/2} w_{2k+3}^7 x(n-i)^{2k} + \]

\[ + \sum_{i=0}^{M} x^*(n-i)^2 x^3(n-i) + \sum_{k=0}^{(P-5)/2} w_{2k+5}^8 x(n-i)^{2k} \]

\[ \]  

(7)

\( * \) is the sign of complex conjugation, \( M \) is the memory length, \( P \) is the odd degree of polynomial,

\[ y_2(n) = \sum_{i=1}^{M} x^*(n-i)^2 x^3(n) \sum_{k=0}^{(P-5)/2} w_{2k+5}^9 |x(n)|^{2k} + \]

\[ + \sum_{i=1}^{M} x^*(n) x^2(n-i) \sum_{k=1}^{(P-3)/2} w_{2k+3}^{10} |x(n)|^{2k} + \]

\[ + \sum_{i=1}^{M} x^*(n-i)^2 x^3(n) \sum_{k=1}^{(P-5)/2} w_{2k+5}^{11} |x(n-i)|^{2k} + \]

\[ + \sum_{i=1}^{M} x^*(n)^2 x^3(n-i) \sum_{k=1}^{(P-5)/2} w_{2k+5}^{12} |x(n)|^{2k}. \]  

(8)

The expression (7) describes the radially pruned Volterra model (RPVM). RPVM is the regressive form of the truncated Volterra series. Volterra kernels in RPVM are built on a hypercube grid and radial directions are selected on the basis of the 3rd order kernel [9], [10].

5. Compensation of Nonlinear Distortion in PA Wiener-Hammerstein Model

In practice a nonlinear PA exhibits a memory effect that is why a nonlinear predistorter with memory should be used for compensation of PA nonlinearity.

In the presented example, we assume the PA model is Wiener-Hammerstein model composed of a linear time-invariant (LTI) system, followed by a memoryless nonlinearity, which is in turn followed by another LTI system [9], [11]. The LTI blocks before and after the memoryless nonlinearity have the system functions given by

\[ H_1(z) = \frac{1 + 0.5z^{-2}}{1 - 0.2z^{-1}}, \quad H_2(z) = \frac{1 - 0.1z^{-2}}{1 - 0.4z^{-1}}, \]

correspondently. The memoryless nonlinear part of the described PA model is given by

\[ w(n) = b_1 v(n) + b_2 v(n) |v(n)|^2 + b_3 v(n) |v(n)|^4, \]

where \( b_1 = 1.0108 + 0.0858 j, \quad b_2 = 0.0879 - 0.1583 j, \quad b_3 = -1.0992 - 0.8891 j, \quad v(n) \) is the output signal of the LTI system with the function \( H_1(z) \).

The input signal for the PA model is the complex envelope of a GSM-signal with four carriers in the frequency baseband with the bandwidth of 20 MHz. The sampling frequency of the GSM-signal complex envelope is 184.32 MHz.

The dependences of the normalized magnitude and phase change of the PA output signal on the PA input signal normalized magnitude are depicted in Fig. 5 (a), (b). In Fig.5 (c) the power spectral densities (PSD) of the described PA model input and output signals without DPD application are shown by dotted (1) and solid (2) lines correspondingly.

The adaptive DPD based on CFLANN of the 9th degree, RPVN (7) with \( P = 7 \), PPN (6) with \( P = 7 \) and the cascade connection of these models are constructed to linearize the above-mentioned PA model. It should be noted, that all the top indexes of the sums in expressions
(7) and (8) are $(P - 1)/2$. The memory length of the investigated models is 4 ($M = 4$ in (7), (8)).

The PA linearization error is estimated by normalized mean-square error (NMSE). This error is calculated from the expression

$$NMSE = 10 \log_{10} \left( \frac{\sum_{n=0}^{N-1} \left| y(z(n)) - x(n) \right|^2}{\sum_{n=0}^{N-1} \left| x(n) \right|^2} \right) \text{ dB},$$

where $x(n)$ is the complex-valued input signal of the cascade connection between DPD and PA shown in Fig. 1, $N$ is the number of samples in the input signal $x(n)$, $N = 106339$, $y(z(n))$ is the PA complex-valued output signal.

<table>
<thead>
<tr>
<th>Model</th>
<th>NMSE</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFLANN of the $9^{th}$ degree and CFLANN of the $7^{th}$ degree</td>
<td>-65.04</td>
<td>226</td>
</tr>
<tr>
<td>RPVM (7) at $P = 7$</td>
<td>-72.87</td>
<td>145</td>
</tr>
<tr>
<td>PPN (6) at $P = 7$</td>
<td>-75.50</td>
<td>221</td>
</tr>
<tr>
<td>Cascade connection</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CFLANN of the $9^{th}$ degree and CFLANN of the $7^{th}$ degree</td>
<td>-69.42</td>
<td>375</td>
</tr>
<tr>
<td>RPVM (7) at $P = 7$ and RPVM (7) at $P = 7$</td>
<td>-78.14</td>
<td>290</td>
</tr>
<tr>
<td>PPN (6) at $P = 7$ and RPVM (7) at $P = 7$</td>
<td>-79.14</td>
<td>366</td>
</tr>
</tbody>
</table>

Tab. 5. NMSE and $Q$ in DPD models. NMSE estimated on the 45th iteration of the DPD adaptive algorithm and the number $Q$ of coefficients in DPD models are presented in Tab. 5.
As can be seen from Tab. 5, the PPN model provides higher accuracy of PA linearization than RPVM and CFLANN. The use of the cascade DPD structure leads to an increased PA linearization accuracy. The highest accuracy is achieved by the cascade DPD structure with PPN and RPVM.

Taking into account the cascade DPD with PPN and RPVM models the dependences of the normalized magnitude and phase change of the PA output signal on the normalized magnitude of the DPD input signal are shown in Fig. 6 (a), (b). The PSD of the PA output signal at the mentioned cascade DPD is depicted in Fig. 6 (c).

From comparison of Fig. 6 with Fig. 5 it follows that the cascade DPD consisting of PPN and RPVM models gives a high quality of PA Wiener-Hammerstein model linearization.

6. Conclusions

On the basis of the decomposition of the DPD nonlinear operator approximation problem into two sub-problems solved consecutively, it is possible to remove the ill conditionality from the DPD nonlinear operator approximation due to reduction of the approximation problem dimension.

This decomposition is realized in a cascade DPD synthesis. DPD block models are built as the functional link artificial neural network, the polynomial perceptron network and the radially pruned Volterra model.

DPD is synthesized for compensation of nonlinear distortion in PA Wiener-Hammerstein model at the complex envelope of a GSM-signal with four carriers. The predistortion results in the facts that the cascade DPD composed of PPN and RPVM linearizes PA with the highest accuracy, and the cascade DPD based on CFLANN yields the lowest accuracy of PA linearization.

References


About the Author ...

Elena B. SOLOVYEVA was born in 1964. She received her M.Sc., Ph.D. and D.Sc. degrees from Saint-Petersburg State Electrotechnical University “LETI” in 1987, 1990 and 2002, respectively. She is the Head of the Theoretical Electrical Engineering Department in the Saint-Petersburg State Electrotechnical University “LETI”, Russia. Her research interests include modeling and synthesis of nonlinear analog and digital systems on the basis of functional series, multidimensional polynomials and neural networks, as well as digital signal processing.