Energy Efficient Power Allocation for Distributed Antenna System over Shadowed Nakagami Fading Channel

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Abstract. *In this paper, the energy efficiency (EE) of downlink distributed antenna system (DAS) with multiple receive antennas is investigated over composite fading channel that takes the path loss, shadow fading and Nakagami-m fading into account. Our aim is to maximize EE which is defined as the ratio of the transmission rate to the total consumed power under the constraints of maximum transmit power of each remote antenna. According to the definition of EE and using the upper bound of average EE, the optimized objective function is provided. Based on this, utilizing Karush-Kuhn-Tucker (KKT) conditions and mathematical derivation, a suboptimal energy efficient power allocation (PA) scheme is developed, and closed-form PA coefficients are obtained. The developed scheme has the EE performance close to the existing optimal scheme. Moreover, it has relatively lower complexity than the existing scheme because only the statistic channel information and less iteration are required. Besides, it includes the scheme in composite Rayleigh channel as a special case. Simulation results show the effectiveness of the developed scheme.*

Keywords

Distributed antenna system, energy efficiency, power allocation, path loss, Nakagami channel

1. Introduction

Distributed antenna system (DAS) has emerged as a promising technology for future wireless communications due to its advantage of enhancing the system capacity, improving the signal quality and reducing the power [1–3]. In DAS, the remote antennas (RAs) are separated geographically and connected to a central control module via dedicated wires, fiber optics, or an exclusive radio frequency link [3]. Traditionally, the spectral efficiency (SE) has been used to measure the efficiency of a communication system. However, it fails to evaluate how the energy is efficiently consumed. Green communication, which pursues high energy efficiency (EE), has drawn increasing attentions these days. Due to a growing energy demand and increasing energy price, pursuing high EE is becoming a mainstream for future mobile systems [4–6].

EE is defined as the sum-rate divided by the total power consumption measured in bit/Joule/Hz. Based on this, different energy efficient schemes have been proposed for DAS [7–10]. In [7], an approximate power allocation (PA) scheme through an iterative numerical search is provided for DAS, but the large-scale fading in practice is neglected. In [8], an optimal energy efficient PA scheme is presented with the help of Lagrange optimization method for orthogonal frequency division multiple access systems with distributed antennas. A novel PA algorithm to achieve maximum EE while satisfying SE requirements in downlink multiuser DAS is proposed in [9]. But the schemes in [8] and [9] need plenty of iterative calculations. An adaptive PA for DAS EE maximizing is derived in [10], but the shadow fading is not considered. Moreover, for analysis simplicity, the above schemes only consider single receive antenna case and the small-scale fast fading is modeled as Rayleigh fading, so the derived PA schemes lack generality and the performance is limited. As well known, Nakagami-*m* fading represents a wide range of realistic fast fading conditions and includes Rayleigh channel as a special case. Furthermore, the above-mentioned PA schemes basically need instantaneous small-scale channel state information (CSI) and iterative calculation. Thus, the real-time CSI calculation and feedback are required, and the resulting complexity is higher.

Motivated by the reasons above, the low-complexity energy efficient PA scheme for DAS over shadowed Nakagami channel is studied, where Nakagami-*m* fading, path loss, and log-normal shadowing are all considered for practicability. Based on the upper bound of EE, the energy efficient PA optimization problem for downlink DAS with multiple receive antennas over Nakagami channel is formulated. By using the Karush-Kuhn-Tucker (KKT) conditions and the Lambert *W* function, the closed-form PA expressions are derived, and the resulting suboptimal

PA scheme is attained. This scheme only needs the large-scale channel information and is different from the existing optimal schemes which need both small-scale and large-scale channel information. Moreover, the iterative time is required less due to the closed-form expression. Thus, the calculation complexity is lower. As its special case, the energy efficient PA scheme in shadowed Rayleigh fading channel is also included. Besides, this scheme can obtain the EE performance close to the existing optimal one due to its effectiveness, which is testified by the simulation.

The remainder of this paper is organized as follows. Section 2 describes the system model. In Sec. 3, a suboptimal PA scheme is proposed and the corresponding algorithm procedure is provided. Simulation results are carried out in Sec. 4. The main conclusions are summarized in Sec. 5.

The notations throughout this paper are as follows. Bold lower case letters denote column vectors. The superscript $(\cdot)^{\mathrm{T}}$ denotes transposition $E\{\cdot\}$ denotes statistical expectation, $\{z\}^+$ is expressed as max(z,0), $W(\cdot)$ denotes Lambert W function.

2. System Model

In this paper, we consider a distributed antenna system with multiple remote antennas (RAs) and receive antennas in a single-cell environment as shown in Fig. 1. The RAs are distributed in the cell and linked to the BS (also named as RA_0) via dedicated wireless connection for signal processing, where N_t RAs are considered, and the i-th RA is denoted as RA_i. The mobile station (MS) is equipped with N_r receive antennas. The positions of the i-th RA are denoted by the polar coordinates (D_i, τ_i), $i = 0,1,\ldots, N_t$, where D_i and τ_i are the distance and angle of the i-th RA to cell center. For transmit remote antenna i, the corresponding received signal at mobile station can be expressed as

$$r_i = \sum_{j=1}^{N_r} \sqrt{p_i} h_i^j x_i + w \tag{1}$$

where p_i is the transmit power consumed by the i-th RA with power constraint $0 \le p_i \le P_{max,i}$, $P_{max,i}$ is the maximum transmit power available at RA_i for $i = 0,1,\ldots, N_t$. x_i stands for the transmitted symbol from the i-th RA with unit energy, and w is the complex Gaussian noise variable with zero-mean and variance N_0. h_i^j indicates the composite channel fading coefficient between RA_i and the j-th antenna of MS, and can be modeled as [11]

$$h_i^j = g_i^j \sqrt{L_i S_i} \tag{2}$$

where g_i^j represents the small-scale fading between RA_i and the j-th receive antenna of MS, and it is a constant during a symbol period and varies from one symbol to another. L_i denotes the path gain between RA_i and MS, and the corresponding path loss is expressed as $PL_i = 1/L_i$ [12], [13]. S_i is log-normal shadow fading between RA_i and MS. The large-scale fading changes very slowly so that S_i may

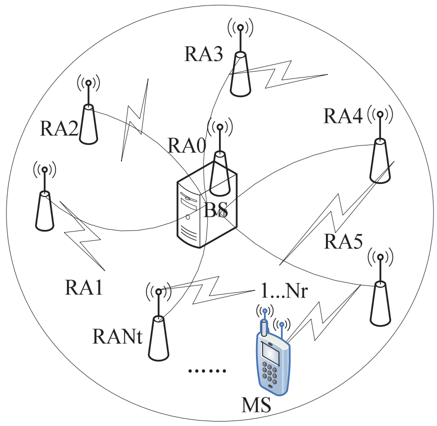

Fig. 1. A circle-cell DAS structure.

be assumed to be a constant over a long time. The path loss PL_i may be expressed as $PL_i = (d_i / d_0)^{\beta_i}$ [12], where β_i is the path loss exponent, d_0 is the reference distance and d_i represents the distance from RA_i to MS. Besides, the envelope of the small-scale fading term $\{g_i^j\}$ is modeled as independent Nakagami random variables of unit power, and correspondingly, the squared envelope $v = |g_i^j|^2$ is gamma distributed. Hence, the probability density function (PDF) of v is written as [14]

$$f_V(v) = \frac{m^m v^{m-1}}{\Gamma(m)} \exp(-mv), \quad v \ge 0 \tag{3}$$

where m is the Nakagami factor and $\Gamma(\cdot)$ is the gamma function.

For the DAS, the achievable data transmission rate for the MS is given by

$$\begin{aligned} R &= \log_2(1+\sum_{i=0}^{N_t} \gamma_i p_i) = \log_2(1+\sum_{i=0}^{N_t}\sum_{j=1}^{N_r} |h_i^j|^2 p_i / N_0) \\ &= \log_2(1+\sum_{i=0}^{N_t}\sum_{j=1}^{N_r} |g_i^j|^2 L_i S_i p_i / N_0) \end{aligned} \tag{4}$$

where $\gamma_i = \sum_{j=1}^{N_r} |g_i^j|^2 L_i S_i / N_0$ is the channel power to noise ratio (CNR).

EE is usually defined as the ratio of data rate to the total power consumption [15]:

$$\eta_{EE} = R / (\sum_{i=0}^{N_t} p_i + p_c) \tag{5}$$

where p_c denotes the circuit power and is a constant throughout this paper .

For analysis convenience, the existing literatures [7–10] basically study optimal PA scheme by maximizing (5) subject to $0 \le p_i \le P_{max,i}$, $\forall i \in \{0,1,\ldots,N_t\}$. As a result, the requirement for instantaneous CSI will be higher because both the large-scale and small-scale channel coefficient need to be known for achieving the optimal PA. Large-scale channel changes very slowly and is easy to obtain, but the small-scale channel changes faster and needs real-time channel estimation and feedback. Thus, the implementation complexity of the optimal scheme is higher.

According to the reasons above, in the next section, we will develop an adaptive energy efficient PA scheme based on the upper bound of EE to eliminate small-scale fading influence and reduce complexity, deduce a low-complexity suboptimal PA algorithm to obtain EE performance close to the optimal one.

3. Suboptimal Power Allocation Scheme

In this section, utilizing the upper bound of EE, the optimized objective function on PA for maximizing average EE is firstly provided. Then, by using the KKT conditions [16] and Lambert W function [17], a suboptimal energy efficient PA scheme is developed for DAS, and the corresponding algorithm procedure is presented.

Utilizing Jensen's inequality, with (4), we have

$$E\{R\} \le \log_2(1+\sum\nolimits_{i=0}^{N_t}\overline{\gamma}_i p_i) \tag{6}$$

where $\overline{\gamma}_i = E\{\gamma_i\}$ is the average CNR, and can be obtained by taking the average of γ_i with respect to small-scale fading as follows:

$$\begin{aligned}\overline{\gamma}_i &= \sum\nolimits_{j=0}^{N_r} E\{|h_i^j|^2\}/N_0 \\ &= \sum\nolimits_{j=0}^{N_r} E\{|g_i^j|^2\}L_i S_i/N_0 = N_r L_i S_i/N_0.\end{aligned} \tag{7}$$

Thus, with (5) and (6), we can achieve the upper bound of average EE, i.e.,

$$\overline{\eta}_{EE}^{(u)} = \log_2(1+\sum\nolimits_{i=0}^{N_t}\overline{\gamma}_i p_i)/[\sum\nolimits_{i=0}^{N_t} p_i + p_c]. \tag{8}$$

From (8), the optimized objective function of the suboptimal PA can be expressed as

$$\begin{aligned}&\max_{\mathbf{P}}\ \overline{\eta}_{EE}^{(u)} = \log_2(1+\sum\nolimits_{i=0}^{N_t}\overline{\gamma}_i p_i)/[\sum\nolimits_{i=0}^{N_t} p_i + p_c] \\ &\text{s.t.}\quad 0 \le p_i \le P_{max,i}, \forall i \in \{0,1,...,N_t\}\end{aligned} \tag{9}$$

where $\mathbf{P}=[p_0, p_1, ..., p_{Nt}]^T$. According to the above analysis, with (5)-(8), the gap between γ_i and $\overline{\gamma}_i$, $\Delta\gamma_i = \gamma_i - \overline{\gamma}_i = (\sum\nolimits_{j=1}^{N_r}|g_i^j|^2 - N_r)L_i S_i/N_0$ will be reduced by increasing N_r and Nakagami parameter m or decreasing L_i. As a result, the obtained PA coefficients from (9) will be very close to those from (5).

For solving the above constrained optimization in (9), we first consider the solution of the following optimization problem:

$$\max_{z\ge 0}\ y(z) = \ln(az+b)/(z+c),\ a>0,\ b\ge 1,\ c>0. \tag{10}$$

By taking the derivative of the objective function $y(z)$ with respect to z and equating the derivative to zero yields

$$a(z+c) = (az+b)\ln(az+b). \tag{11}$$

Let $t = az + b$, using the Lambert W function, the closed-form solution of t can be obtained as

$$t = (ac-b)/W\left((ac-b)/\mathrm{e}\right). \tag{12}$$

Considering the non-negative of z, with (12) and $t = az + b$, the optimal solution z^* may be given by

$$z^* = \{\hat{z}\}^+ \tag{13}$$

with

$$\hat{z} = a^{-1}[(ac-b)\big/W\left((ac-b)\mathrm{e}^{-1}\right) - b]. \tag{14}$$

With (10) and (14), the following conclusions can be proved. Namely, if $0 \le z < z^*$, $y(z)$ in (10) strictly increases; if $z > z^*$, $y(z)$ strictly decreases, and for a special case of $b = 1$, $\hat{z}$ in (13) is always positive. For this, Appendix provides the corresponding proof.

Considering that the distances between RA_i and MS are different, $\overline{\gamma}_i$ may be different, and thus we can sort them in descending order as

$$\overline{\gamma}_0 > \overline{\gamma}_1 > \cdots > \overline{\gamma}_{N_t}. \tag{15}$$

With (9), the Lagrangian function can be constructed as

$$\begin{aligned}J(\{p_i,\lambda_i,\nu_i\}) &= \ln\left(1+\sum_{i=0}^{N_t} p_i\overline{\gamma}_i\right)/(\sum\nolimits_{i=0}^{N_t} p_i + p_c) \\ &+ \sum_{i=0}^{N_t}\lambda_i p_i + \sum_{i=0}^{N_t}\upsilon_i(P_{max,i} - p_i)\end{aligned} \tag{16}$$

where λ_i and υ_i are the introduced Lagrange multipliers. According to KKT conditions, the optimal values $\{p_i^*, \lambda_i^*, \upsilon_i^*\}(i=0,1,\cdots N_t)$ should satisfy the following equations:

$$\partial J/\partial p_i = f_i(p_0^*,...,p_{N_t}^*) + \lambda_i^* - \nu_i^* = 0, \tag{17}$$

$$\lambda_i^* p_i^* = \nu_i^*(P_{max,i} - p_i^*) = 0 \tag{18}$$

where $0 \le p_i^* \le P_{max,i}, \lambda_i^* \ge 0, \nu_i^* \ge 0$.

$$\begin{aligned}&f_i(p_0^*,...,p_{N_t}^*) = \\ &\frac{-\ln(1+\sum\nolimits_{i=0}^{N_t}\overline{\gamma}_i p_i^*)}{(\sum\nolimits_{i=0}^{N_t} p_i^* + p_c)^2} + \frac{\overline{\gamma}_i}{(\sum\nolimits_{i=0}^{N_t} p_i^* + p_c)(1+\sum\nolimits_{i=0}^{N_t}\overline{\gamma}_i p_i^*)}.\end{aligned} \tag{19}$$

For notation simplicity, we rewrite $f_i(p_0^*,\dots, p_{N_t}^*)$ as f_i. With (15) and (19), we have $f_0 > f_1 > \dots > f_{N_t}$. Through analysis and mathematical derivation, the following conclusions can be reached.

Conclusions:

1) For a positive p_i^*, f_i is always non-negative.
2) For any j ($j = 0,1,\dots, N_t$), if $f_j < 0$ then $p_k^* = 0$, ($k > j$); if $f_j > 0$ then $p_k^* = P_{max,k}$ ($k < j$); if $f_j = 0$, then $p_k^* = P_{max,k}$ ($k < j$) and $p_l^* = 0$ ($l > j$).

According to the above conclusions and using the nature of Lambert W function, a suboptimal PA scheme is presented, and the corresponding algorithm procedure is summarized as follows:

Suboptimal Power Allocation Algorithm

1) Given the system parameters, compute $\bar{\gamma}_i$ with (7).

2) Set the $\bar{\gamma}_i$ as $\bar{\gamma}_0 > \bar{\gamma}_1 > \cdots > \bar{\gamma}_{N_t}$.

3) Initialize $i = 0$, for the power of RA_0 (i.e. p_0^*), by substituting $a = \bar{\gamma}_0$, $b = 1$, $c = p_c$ into (14) yields $z_0^* = \{\hat{z}_0\}^+$, where

$$\hat{z}_0 = \frac{1}{\bar{\gamma}_0}\left[(\bar{\gamma}_0 p_c - 1) \Big/ W\left(\frac{\bar{\gamma}_0 p_c - 1}{e}\right) - 1 \right] . \tag{20}$$

If $z_0^* = 0$, then $(p_0^*, \lambda_0^*, \nu_0^*) = (0, \lambda_0^*, 0)$ and $f_0 = -\lambda_0^* + \nu_0^* < 0$. According to conclusion (2), p_i^* should be zero for $i > 0$. The PA solution becomes $(p_0^*, ..., p_{N_t}^*) = (0, ..., 0)$. This is unpractical, so $p_i^* = 0$ will never occur.

If $0 < z_0^* < P_{max,0}$, then the solution of PA is $(p_0^*, ..., p_{N_t}^*) = (z_0^*, 0, ..., 0)$, and the algorithm stops.

If $z_0^* > P_{max,0}$, then $p_0^* = P_{max,0}$; if $i < N_t$, go to step 4; otherwise stop the algorithm.

4) **while** ($p_i^* = P_{max,i}$ and $i < N_t$)

$i = i + 1$;

Substituting $b = 1 + \sum_{k=0}^{i-1} \bar{\gamma}_k P_{max,k}$, $c = p_c + \sum_{k=0}^{i-1} P_{max,k}$, and $a = \bar{\gamma}_i$ into (14) to compute $z_i^* = \{\hat{z}_i\}^+$ yields

$$\begin{aligned}\hat{z}_i = \bar{\gamma}_i^{-1} \Big[& \Big(\bar{\gamma}_i (p_c + \textstyle\sum_{k=0}^{i-1} P_{max,k}) - (1 + \sum_{k=0}^{i-1} \bar{\gamma}_k P_{max,k})\Big) \\ & \Big/ W\Big(e^{-1}\big(\bar{\gamma}_i (p_c + \textstyle\sum_{k=0}^{i-1} P_{max,k}) - (1 + \sum_{k=0}^{i-1} \bar{\gamma}_k P_{max,k})\big)\Big) \\ & - (1 + \textstyle\sum_{k=0}^{i-1} \bar{\gamma}_k P_{max,k}) \Big]\end{aligned} \tag{21}$$

Hence, the solution of PA coefficients is given by

$$p_i^* = \min(z_i^*, P_{max,i}) \tag{22}$$

end

5) Set $p_j^* = 0$ for $j > i$.

Based on the algorithm above, the suboptimal power allocation coefficients $\{p_i^*\}$ can be obtained. These $\{p_i^*\}$ will have the values close to the optimal one. Substituting these coefficients into (5) will yield the suboptimal energy efficiency. Besides, with (22), we provide a closed-form solution for each p_i^* for $0 \le p_i^* < P_{max,i}$. Under this case, p_i^* can be calculated by (22) directly. Only when $p_i^* = P_{max,i}$ and $i < N_t$, we need to calculate the remaining p_j^* ($j > i$) by iteration, and the iteration will stop immediately when $p_i^* \ne P_{max,i}$ is fulfilled. Moreover, most of the PA coefficients $\{p_0^*, p_1^*, \dots, p_{N_t}^*\}$ can be obtained directly by using (22), and only few coefficients need iteration.

The optimal scheme is presented in [8], and the corresponding PA is obtained by maximizing (5) subject to $0 \le p_i \le P_{max,i}$, $\forall i \in \{0, 1, \dots, N_t\}$, but it needs plenty of numerical iterative computation. Moreover, single receive antenna and Rayleigh channel are considered.

In what follows, we compare the optimal scheme [8] and the developed suboptimal scheme. Firstly, the developed scheme depends on large-scale fading coefficients only, while the optimal scheme requires both large-scale and small-scale channel information to calculate PA coefficients. Thus, the optimal scheme has a higher requirement for instantaneous CSI. Secondly, for the optimal scheme, during the iterative process of computing each p_i^*, the dual loops are performed, and both the inner and outer loop need $O(\log(1/\varepsilon))$ iterations to guarantee the error tolerance of ε. Whereas in our proposed scheme, each p_i^* can be directly computed by the closed-form expression (22). Based on this, the suboptimal scheme will have lower calculation complexity than the optimal one, which can also be seen from Tab. 2, where the average numbers of iterations of two schemes are compared.

In addition, when $m = 1$, the proposed suboptimal scheme and the corresponding power allocation coefficients are reduced to those over composite Rayleigh fading channels.

4. Simulation Results and Analysis

In this section, the validity of the proposed scheme will be evaluated via computer simulations. In simulation, we assume $P_{max,i} = P_{max}$ ($i = 0, \dots, N_t$) for analysis convenience. The BS (RA_0) is in the center of the cell and the RAs are evenly and symmetrically placed in the cell. The main parameters used in simulations are listed in Tab. 1. The simulation results are illustrated in Figs. 2–5.

In Fig. 2, we plot the energy efficiency of DAS with different receive antennas, where the developed suboptimal scheme and the existing optimal scheme [8] are employed for comparison. The path loss exponent $\beta = \beta_i = 4$ and the Nakagami factor $m = 1.5$. Considering that only single receive antenna is employed in the optimal scheme, we extend the method in [8] to DAS with multiple receive antennas after some modifications for fair comparison. As shown in Fig. 2, the system EE can be improved as the number of receive antenna N_r increases. This is because the increase of the receive antennas will bring about more space diversity gain. In other words, the application of multiple receive antennas can improve the EE performance obviously. Besides, the suboptimal scheme can obtain the EE performance very close to the optimal scheme as N_r increases. This is because the CNR γ_i will be very close to the average CNR $\bar{\gamma}_i$ when N_r increases. As a result, almost the same PA coefficients can be achieved for these two schemes, which accord with the analysis in Sec. 3. The above results indicate that the proposed PA scheme with multiple receive antennas is valid for improving EE. In Tab. 2, we compare the average numbers of iteration of the proposed scheme and the optimal scheme. It is found that the suboptimal scheme requires much less iteration than the optimal one, which accords with the complexity analysis in Sec. 3.

Parameters	value
Number of remote antennas N_t+1	5, 7
Path loss exponent β_i	3, 4
Shadow fading standard deviation σ_i	8 dB
Noise power N_0	-104 dBm
Circuit Power p_c	40 dBm
Cell radius R	1000 m
MS distribution	Uniform
(D_0, τ_0)	(0,0)
(D_i, τ_i), $i = 1,\ldots N_t$	$(2R/3, 2\pi i/N_t)$
Reference distance d_0	80 m
Number of receive antennas N_r	1, 2, 3

Tab. 1. Simulation parameters.

	Proposed suboptimal scheme	Optimal scheme [8]
Average number of iterations	1.6	64.1

Tab. 2. Average number of iterations for different schemes.

The energy efficiency of DAS with different Nakagami factors is shown in Fig. 3, where $N_r = 1$ and $\beta = \beta_i = 4$. As can be seen, the system EE can increase as the Nakagami factor m increases. The reason for this is that the increase of m means the decrease of channel fading, which will bring about the increase of the EE performance. Furthermore, when m is larger, the EE of suboptimal scheme is very close to that of optimal scheme. It is known that the square-root of a sum of squares of $2m$ zero-mean, identically distributed Gaussian random variables ($r.v.s$) has a Nakagami distribution with parameter m for integer and half-integer m [18]. Thus, $|g_i^j|^2$ can be equivalent to $(1/2m)\sum_{k=1}^{2m}(u_k)^2$, where $\{u_k\}$ are independent identically distributed Gaussian $r.v.s$ with zero mean and unit variance. According to this relation, when m increases, the γ_i will approach the $\bar{\gamma}_i$. As a result, the PA coefficients of these two schemes will become very close. The above results show that the proposed suboptimal scheme can be applied to Nakagami channel with large m and obtain the superior EE performance.

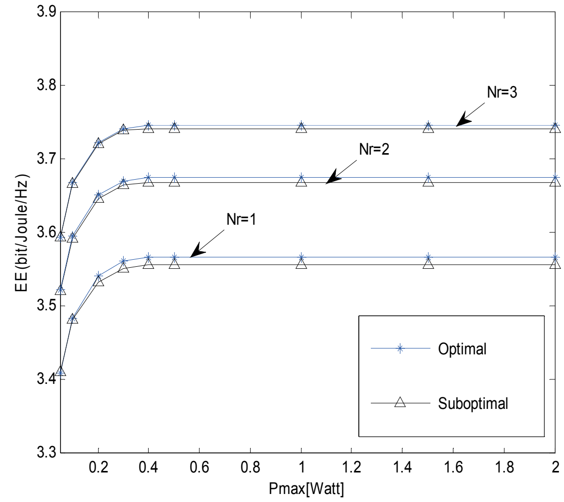

Fig. 2. EE of DAS with different receive antennas.

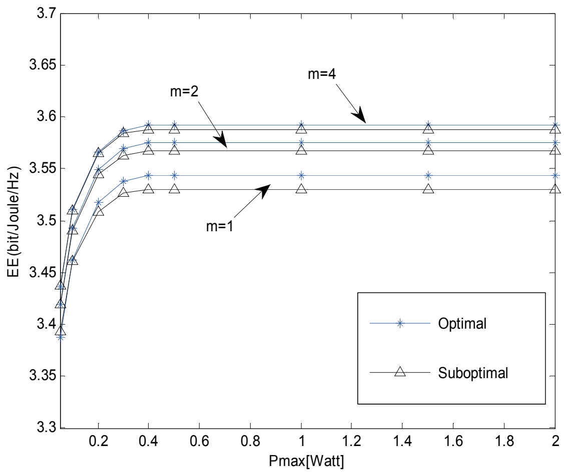

Fig. 3. EE of DAS with different Nakagami factors.

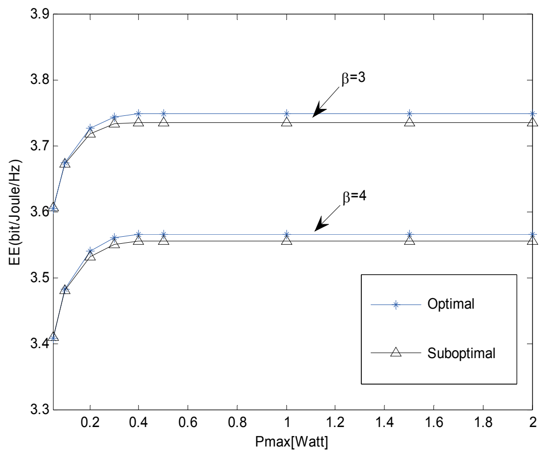

Fig. 4. EE of DAS with different path loss exponents.

Figure 4 shows the energy efficiency of DAS with different schemes and path loss exponents, where $N_r = 1$ and $m = 1.5$. From Fig. 4, it is observed that the system EE can increase as the path loss exponent β decreases, which accords with the existing knowledge. Namely, the decrease of β means the decrease of path loss, which reduces the impact on EE. Besides, the suboptimal scheme has the EE

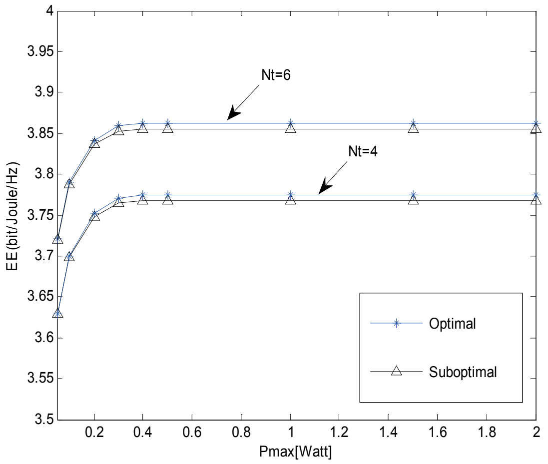

Fig. 5. EE of DAS with different remote antennas.

performance close to the optimal scheme for larger β. This is because as β increases, the path loss increases accordingly, and the resulting path gain L_i will decrease, which narrows the gap between $\bar{\gamma}_i$ and γ_i. Thus, the corresponding EE is very close to that from the optimal scheme.

In Fig. 5, we plot the energy efficiency of DAS with different remote antennas for both the optimal and suboptimal schemes, where $\beta = 3$, $N_r = 2$, $m = 1.5$, $N_t = 4$, 6. It is found that the system EE can be improved as the remote antenna number N_t increases. This is because the system can obtain higher space diversity gain as N_t increases. Besides, the suboptimal scheme has the energy efficiency performance close to the optimal scheme. The above results further verify the effectiveness of the developed suboptimal scheme.

5. Conclusions

We have studied the energy efficiency for the downlink DAS with multiple receive antennas in composite Nakagami-m fading channel, and developed a suboptimal energy-efficient power allocation scheme for DAS. This scheme can provide closed-form expression of PA coefficients, and only needs large-scale channel information and less iteration. Thus, the calculation complexity is lower. Moreover, it includes the scheme under composite Rayleigh fading channel as a special case. Simulation results illustrate that the developed scheme has the EE performance close to the optimal schemes, and the EE can be effectively improved by increasing the receive antenna number N_r, remote antenna number N_t, Nakagami factor m, and/or decreasing the path loss exponents β.

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Appendix

In this appendix, we give the proof of the conclusions below (14). Firstly, taking the derivative of the objective function $y(z)$ in (10) with respect to z yields

$$y'(z) = \frac{-(az+b)\ln(az+b) + a(z+c)}{(az+b)\left(z+c\right)^2} . \quad \text{(A1)}$$

Then, we define the denominator of (A1) as

$$g(z) = -(az+b)\ln(az+b) + a(z+c) . \quad \text{(A2)}$$

With (A2), the derivative of $g(z)$ with respect to z is

$$g'(z) = -a\ln(az+b) < 0 . \quad \text{(A3)}$$

Thus, $g(z)$ is a strictly decreasing function. If $z^* > 0$, we will have $g(z) > 0$ ($0 \le z < z^*$) and $g(z) < 0$ ($z > z^*$). Thus, $y(z)$ will reach the maximum value at $z = z^*$. If $z^* = 0$, then $g(z)$ always has a negative value for $z \ge 0$. So $y(z)$ is a strictly decreasing function, and correspondingly, $y(z)$ obtains the maximum value at $z = 0$. Thus, if $0 \le z < z^*$, $y(z)$ in (10) strictly increases, and if $z > z^*$, $y(z)$ strictly decreases.

In addition, the Lambert W function is an increasing function for $x > (-1/\mathrm{e})$, and thus we have $W(-1/\mathrm{e}) = -1$. Based on this, (14), $\hat{z} = a^{-1}[(ac-b)/W\left((ac-b)\mathrm{e}^{-1}\right) - b]$, can be transformed into the following equation

$$\hat{z} = a^{-1}[\exp\left\{W\left((ac-b)/e\right) + 1\right\} - b] . \quad \text{(A4)}$$

When $b = 1$, $W((ac-1)/\mathrm{e}) > -1$ holds for positive a and c. Hence, it can be easily obtained that $\hat{z}$ must be positive from (A4).

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