An Introduction to the Source Concept for Antennas

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Manuscript received March 4, 2016

Abstract. *Antenna parameters particularly relevant to electrically small antenna design are reviewed in this paper. Source current definitions are accentuated leading to the introduction of the source concept which advantageously utilize only spatially bounded quantities. The framework of the source concept incorporates powerful techniques such as structural and modal decomposition, operator's inversion and current optimization, thus opening new, challenging possibilities for antenna design, analysis and synthesis.*

Keywords

Poynting theorem, antenna theory, stored energy, quality factor Q, source concept

1. Introduction

Certain antenna parameters, including for example radiated power, are traditionally calculated from radiated fields in space. Recently, a number of papers, see e. g. [1], [2] have been published which attempt to use only antenna geometry and source distribution (electric or magnetic or both currents) when evaluating the performance of antennas using spatially limited integrations over current density. This is advantageous as you do not have to deal with integrals involving radiated field in the whole space. It is the main principle that constitutes part of the framework called the Source Concept and this paper summarizes some of the findings in the field. We concentrate only on electric currents flowing in vacuum.

Brillouin [3] was probably the first scientist who proposed to use source spatial currents to evaluate radiated power, see also [4], [5, Sec. 2.3]. An illustration of this procedure is provided later in Sec. 2.1.

Radiation quality factor Q [6], which indicates the bandwidth potential of an antenna, is another important measure. It is well known [6], [7] that the stored energy of an electromagnetic field is infinite in frequency domain (a time-harmonic state) due to radiated energy. Traditionally, radiated energy was subtracted from the total energy and the methods operated with "sphere enclosing the antenna", but without taking into account the exact shape of the radiator, see [8] and references therein. Geyi [9, Sec. 4] and later Vandenbosch [10] and Gustafsson [11] attempted to obtain stored electromagnetic energies directly from currents and their results lead to so-called measurable or observable energies since they are tightly connected to the frequency changes of an antenna's input impedance. Prior to this, observable energies were proposed by Rhodes [12], but he still relied upon using fields in space, not source currents. Section 2.2 illustrates how the measurable energies can be derived in a simple and intuitive way [15], using only spatially localized currents.

It should be noted that the problem of correct definition of stored energy has still not been completely solved, although, currently, it is being intensively studied, see e. g. [13]. In particular, the time-domain approach seems very promising [14].

In Section 3, a well-known Theory of Characteristic Modes [16] is introduced, though expressed in terms of a power functional involving source current density. Such a formulation allows the study of arbitrary current distribution (for instance one may guess for characteristic currents and test their properties) and may enable the study of the optimal composition of modal currents for minimal Q [17].

The last five years have allowed the capabilities of the Source Concept to be recognized and consequently software tools associated with its implementation have begun to appear. To support this effort, the Antenna Toolbox for Matlab (AToM) [18] is currently being developed. The AToM is written entirely in Matlab, so the user can enjoy its semi-open architecture and friendly operation through graphical interface or direct access to low level functions. Main simulation core is based on Method of Moments [4], both for 3D wire and planar structures. Together with modal decomposition (characteristic modes), the source concept, feeding synthesis and powerful optimization, it will present a unique tool for synthesis of antennas.

2. Source Concept

Source concept can be introduced as a framework utilizing integral equations involving spatially localized sources of radiation.

It will be shown how to express input impedance, radiated power, measurable energies, quality factor and characteristic eigenvalues solely in terms of electric current density. It must be noted that currents can be obtained from a full-wave simulation, by modal decomposition or even specified by analytical approximation. They can be the subject of modal and structural decomposition and optimization which created the possibility of finding optimal current distributions with regard to quality factor Q, gain G, G/Q ratio and polarization properties [19].

2.1 Illustration of the Source Concept

The following example illustrates the philosophy of the source concept. For the sake of simplicity consider only the linear z-directed current which generates the θ component in the far-field. It is well known [20] that radiated power can be calculated from far-field power density (the so called "Poynting vector method") as

$$P_\mathrm{r} = \frac{1}{2Z_0} \oint\!\!\oint_{4\pi} \left| E_\theta \left(r, \theta, \phi \right) \right|^2 \mathrm{d}S \tag{1}$$

where Z_0 is the impedance of free space. To eliminate the surface integral involving the radiated field, $\boldsymbol{E} = -\mathrm{j}\omega\boldsymbol{A}$ is inserted into (1)

$$P_\mathrm{r} = \frac{\omega^2}{2Z_0} \oint\!\!\oint_{4\pi} \left| A_\theta \left(r, \theta, \phi \right) \right|^2 \mathrm{d}S \tag{2}$$

where the vector potential is

$$A_\theta \left(r, \theta, \phi \right) = \frac{\mu}{4\pi} \frac{e^{-\mathrm{j}kr}}{r} \sin\theta \int_{-L/2}^{L/2} I\left(z'\right) e^{\mathrm{j}kz'\cos\theta} \mathrm{d}z' . \tag{3}$$

We now have to evaluate

$$P_\mathrm{r} = \frac{\mu^2}{16\pi^2 r^2} \oint\!\!\oint_{4\pi} \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} I\left(z\right) I^*\left(z'\right) \sin^2\theta e^{\mathrm{j}k\cos\theta\left(z-z'\right)} \mathrm{d}z\mathrm{d}z'\mathrm{d}S . \tag{4}$$

There are two integrals over the spatial domain of sources and one surface integral, which can be worked out in closed form

$$\int_0^\pi \sin^3\theta e^{\mathrm{j}kR\cos\theta} \mathrm{d}\theta = 4 \left(\frac{\sin kR}{\left(kR\right)^3} - \frac{\cos kR}{\left(kR\right)^2} \right) \tag{5}$$

where $R = |z - z'|$ has been substituted as a distance between interacting currents.

Hence, the radiated power can now be alternatively written as follows

$$P_\mathrm{r} = 30k^2 \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} I\left(z\right) I^*\left(z'\right) \left(\frac{\sin kR}{\left(kR\right)^3} - \frac{\cos kR}{\left(kR\right)^2} \right) \mathrm{d}z\, \mathrm{d}z' . \tag{6}$$

As a check, constant current $I(z) = I(z') = I$ is inserted and the kernel is expanded into the Taylor series. The first term integrates to zero and the second gives $P_\mathrm{r} = 10(kIL)^2$, a well known result [20] for an elementary dipole when $L \to 0$, though obtained in a completely different way.

2.2 Complex Power and Input Impedance

Brillouin [3] and Papas [5] have shown that the "Poynting vector method" and the above presented method (which will be revealed in its extended form involving reactive power as the "EMF method" [20]) are equivalent. Physically, the EMF method is based on shrinking the enclosing surface to the boundary of the antenna while allowing to capture the reactive components, too.

The derivation starts with the Complex Poynting Theorem (PT) in frequency domain [6]

$$P_\mathrm{in} = -\frac{1}{2} \int_V \boldsymbol{E} \cdot \boldsymbol{J}^* \mathrm{d}V = P_\mathrm{r} + \mathrm{j}2\omega\left(W_\mathrm{m} - W_\mathrm{e}\right) = P_\mathrm{r} + \mathrm{j}P_\mathrm{x} \tag{7}$$

where $P_\mathrm{in} = P_\mathrm{r} + \mathrm{j}P_\mathrm{x}$ is complex power measured at the antenna ports, V is the volume of the antenna and W_m and W_e are energies of the magnetic and electric fields respectively.

The $\boldsymbol{E} \cdot \boldsymbol{J}^*$ part of the above equation for input power may also be illustratively written in terms of dynamic potentials [6] if we substitute $\boldsymbol{E} = -\mathrm{j}\omega\boldsymbol{A} - \nabla\varphi$ into (7):

$$P_\mathrm{in} = \frac{\mathrm{j}\omega}{2} \int_V \left(\boldsymbol{A} \cdot \boldsymbol{J}^* - \varphi\rho^* \right) \mathrm{d}V , \tag{8}$$

in which φ is scalar electric potential and ρ is charge density. The PT thus gives us access to radiated power and *difference* of magnetic and electric energies, the *only* quantities which are measurable through the input impedance, compactly expressed as

$$Z_\mathrm{in} = \frac{\mathrm{j}\omega}{\left|I_0\right|^2} \int_V \left(\boldsymbol{A} \cdot \boldsymbol{J}^* - \varphi\rho^* \right) \mathrm{d}V \tag{9}$$

where I_0 is the input current at the antenna terminals.

By inserting the continuity equation $\rho = -\nabla\cdot\boldsymbol{J}/\mathrm{j}\omega$, the self-impedance may now be written as double integral involving current density, frequency and antenna geometry:

$$Z_\mathrm{in} = \frac{\mathrm{j}30}{k\left|I_0\right|^2} \iint_{VV'} \left[\Psi - \Upsilon \right] \frac{e^{-\mathrm{j}kR}}{R} \mathrm{d}V\, \mathrm{d}V' \tag{10}$$

where $\Psi = k^2\boldsymbol{J}(\boldsymbol{r})\cdot\boldsymbol{J}^*(\boldsymbol{r}')$ originated from vector potential, $\Upsilon = \nabla\cdot\boldsymbol{J}(\boldsymbol{r})\nabla'\cdot\boldsymbol{J}^*(\boldsymbol{r}')$ originated from scalar potential and the integration is performed over the antenna occupying volume V with $R = |\boldsymbol{r} - \boldsymbol{r}'|$ being the distance between currents[1].

Equation (10) is a departure point for the impedance quality factor [21]. It occurred many times, and in various forms and is known as the "EMF method" or EFIE for evaluating the impedance of an antenna using the prescribed current [20] or in the MoM formulation [22]. The spatial derivative of current can also be transferred to Green's function $\exp(-\mathrm{j}kR) / R$, though the resulting kernel

[1] Modification of (10) for evaluating the mutual impedance is straightforward [20].

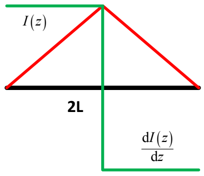

Fig. 1. Distribution of current and charge along a short dipole.

has strong singularities [5], see also (6) where such a differentiated kernel appeared naturally.

Inspecting equation (10) reveals that the contribution to the input impedance (or input complex power) is both magnetic (from currents, Ψ term) and electric (from charges, ϒ term).

The following illustrative example considers the evaluation of the radiation resistance of a short dipole with a triangular current, see Fig. 1.

From (10) we have

$$R_{\mathrm{r}} = \frac{30}{k|I_0|^2} \iint_{VV'} [\Psi - \Upsilon] \frac{\sin kR}{R} \mathrm{d}z\,\mathrm{d}z' = R_{\mathrm{r}}^{\Psi} - R_{\mathrm{r}}^{\Upsilon} \quad (11)$$

where R_{r}^{Ψ} and $R_{\mathrm{r}}^{\Upsilon}$ are contributions from a current and charge respectively. By using two terms of the Taylor series for the kernel $\sin(kR)/R$ we obtain $R_{\mathrm{r}} = R_{\mathrm{r}}^{\Psi} - R_{\mathrm{r}}^{\Upsilon} = 30(kL)^2 - 10(kL)^2 = 20(kL)$, another well-known result, but this is obtained only from sources. Noted that the closed-form calculation of reactance is much more delicate because the dominant (static) part of the kernel $\cos(kR)/R \cong 1/R$ contains singularity. However, results for first-order linear sinusoidal currents are known and widely available [20].

2.3 The Radiation Quality Factor

The untuned quality factor in the frequency domain is generally defined as [6]

$$Q = \frac{\omega W_{\mathrm{sto}}}{P_{\mathrm{lost}}} = \frac{\omega(W_{\mathrm{m}} + W_{\mathrm{e}})}{P_{\mathrm{r}}} \quad (12)$$

where W_{sto} is the stored energy and where we assumed that power is lost only by radiation. While radiated power is uniquely defined, the energy of the radiating system in a time-harmonic state is infinite. Therefore, modified finite energies have been proposed by many authors, see e. g. [14]. Unfortunately, deficiencies such as coordinate dependence [21] and negative values [23] exist.

In (12), we may interpret ωW_{sto} as a kind of reactive power, hence (12) is in fact a power ratio. But in practice, an antenna designer is usually not primarily interested in the value of stored energy, but in the bandwidth. For this purpose, reasonable definition[2] of quality factor Q, based on the frequency sensitivity of input impedance, have been proposed [12], [25]

$$Q_{\mathrm{Z}} = \frac{\omega}{2R_{\mathrm{in}}} \left| \frac{\partial Z_{\mathrm{in}}}{\partial \omega} \right| = \frac{k}{2R_{\mathrm{in}}} \left| \frac{\partial Z_{\mathrm{in}}}{\partial k} \right| = \left| Q_{\mathrm{R}} + \mathrm{j}Q_{\mathrm{X}} \right| \quad (13)$$

where Q_{R} and Q_{X} represent change of input resistance and reactance, which is dominant, respectively.

Insert $Z_{\mathrm{in}} = 2P_{\mathrm{in}}/|I_0|^2$ to obtain

$$Q_{\mathrm{Z}} = \frac{k}{2R_{\mathrm{in}}} \left| \frac{\partial}{\partial k} \frac{2P_{\mathrm{in}}}{|I_0|^2} \right| = \frac{P_{\mathrm{A}}}{P_{\mathrm{r}}} \quad (14)$$

where $P_{\mathrm{A}} = \frac{\partial}{\partial k}\sqrt{P_{\mathrm{r}}^2 + \mathrm{j}P_{\mathrm{x}}^2}$ is the frequency derivative of the apparent input power. The port current I_0 is assumed to be constant.

Indeed the quality factor Q_{z} can be evaluated by differentiation of the input impedance obtained from the electromagnetic field simulator. However, we can use (10) to great advantage and perform the derivation in (13) analytically[3]. It is then possible to insert a modal or even arbitrary current density and examine how much a given part of the antenna affects the overall Q.

The result is composed of three complex terms (which we loosely call measurable or observable energies [13]) of different nature

$$\frac{\partial Z_{\mathrm{in}}}{\partial k} |I_0|^2 = \hat{W}_{\mathrm{me}} + \hat{W}_{\mathrm{r}} + \hat{W}_{\mathrm{k}} \quad (15)$$

where

$$\hat{W}_{\mathrm{me}} = \frac{\mathrm{j}30}{k^2} \iint_{VV'} [\Psi + \Upsilon] \frac{e^{-\mathrm{j}kR}}{R} \mathrm{d}V \mathrm{d}V', \quad (16)$$

$$\hat{W}_{\mathrm{r}} = \frac{30}{k} \iint_{VV'} [\Psi - \Upsilon] e^{-\mathrm{j}kR} \mathrm{d}V \mathrm{d}V', \quad (17)$$

$$\hat{W}_{\mathrm{k}} = \frac{\mathrm{j}30}{k} \iint_{VV'} \frac{\partial}{\partial k} [\Psi - \Upsilon] \frac{e^{-\mathrm{j}kR}}{R} \mathrm{d}V \mathrm{d}V'. \quad (18)$$

The structure of the first term $\hat{W}_{\mathrm{me}}$ is similar to the Z_{in}, but includes the sum of parts Ψ and ϒ. This strongly resembles the Foster theorem [21] where the reactance derivation produces a sum of energies. It should be stressed that the Foster theorem is valid only for a lossless reactance network, which is not the case of an radiating system. This term is nonzero, even for non-radiating system and it comprises a major contribution.

The second term $\hat{W}_{\mathrm{r}}$ arises from the frequency change of Green's function and is associated with radiation, see discussion in [15]. It is zero for a non-radiating system. The third term $\hat{W}_{\mathrm{k}}$ accounts for the change of current density with frequency [13] and is, again, zero for a non-radiating system.

[2] Another question arises of whether the quality factor is exactly proportional to the bandwidth. It can be shown [24] that it is, but only for a series/parallel single-resonant circuit or for higher values of Q.

[3] This derivation will be performed in k instead of ω in order to keep the easy notation of (10).

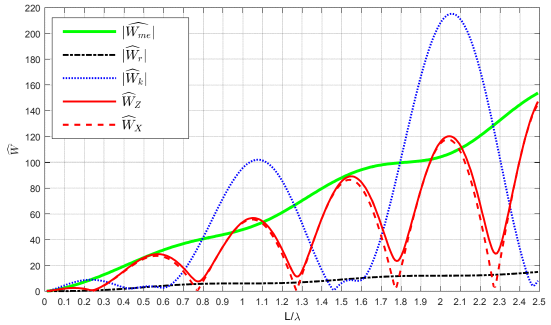

Fig. 2. The module of measurable energies for a dipole of length L. In general, the radiation term $|\hat{W}_\text{r}|$ is small. The $|\hat{W}_\text{k}|$ term is small in resonance, where the current distribution is stable and peaks in antiresonances. In resonance, the dominant contribution comes from $|\hat{W}_\text{me}|$.

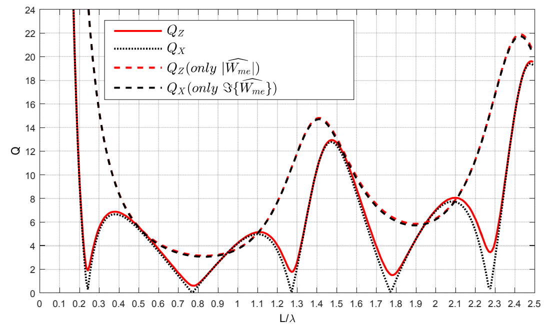

Fig. 3. Untuned quality factors for a thin-wire dipole.

We define the measurable energies, available from the antenna terminals and consider input impedance, or only input reactance, as

$$\hat{W}_\text{Z} = \left|\hat{W}_\text{me} + \hat{W}_\text{r} + \hat{W}_\text{k}\right|, \tag{19}$$

$$\hat{W}_\text{X} = \Im\left\{\hat{W}_\text{me} + \hat{W}_\text{r} + \hat{W}_\text{k}\right\}. \tag{20}$$

The behavior of $|\hat{W}_\text{me}|$, $|\hat{W}_\text{r}|$, $|\hat{W}_\text{k}|$, $\hat{W}_\text{z}$, and $\hat{W}_\text{x}$, is illustrated in the case of a thin-wire dipole with a prescribed sinusoidal current, see Fig. 2. For easy plotting we show only modules of terms (16)–(18).

Various quality factors were calculated from the above energies. They all agree very well in resonance, as seen in Fig. 3.

Explicitly, the simplest version of the measurable quality factor considers only an imaginary part of the $\hat{W}_\text{me}$ term and reads

$$Q_\text{me} = \frac{\iint\limits_{VV'} [\Psi + \Upsilon] \frac{\cos kR}{R} \mathrm{d}V \mathrm{d}V'}{2\iint\limits_{VV'} [\Psi - \Upsilon] \frac{\sin kR}{R} \mathrm{d}V \mathrm{d}V'}. \tag{21}$$

3. Modal Decomposition

The characteristic modes [16] are a handy framework for the source concept. Eigen-currents $\boldsymbol{J}_n$ and their corresponding eigenvalues λ_n are usually defined through the weighted eigenvalue equation involving real and imaginary parts of the impedance operator (10) [16]. Alternatively, they can also be expressed as a basis which minimizes the following source concept power functional:

$$\mathcal{F}(\boldsymbol{J}_n) = \frac{P_\text{x}}{P_\text{r}} = \frac{\iint\limits_{VV'} [\Psi - \Upsilon] \frac{\cos kR}{R} \mathrm{d}V \mathrm{d}V'}{\iint\limits_{VV'} [\Psi - \Upsilon] \frac{\sin kR}{R} \mathrm{d}V \mathrm{d}V'} = \kappa_n \tag{22}$$

where κ_n is the Rayleigh quotient, which is equal to characteristic number λ_n when a true characteristic current $\boldsymbol{J}_n$ enters into (22). The functional is minimized by characteristic currents that simultaneously maximizes radiated power and minimizes reactive power, representing external resonances of the radiator. For very special cases, a stationary solution of (22) can be obtained even in closed form. It is clear that we may assign antenna parameters (energies, powers, losses, directivities, near and far fields) directly to the modes and gain extra physical insight into the antenna behavior. In [26] we derived the so-called coupling matrix β_{mn} which connects the modes with the real world and is represented by modal excitation coefficients V_m [27]

$$\beta_{mn} = \frac{V_m V_n (1 + \lambda_m \lambda_n)}{(1 + \lambda_m^2)(1 + \lambda_n^2)}. \tag{23}$$

This matrix is a good subject for efficient optimization since it is possible to move the feeding along a given structure, control the amount of modal parameters and quickly evaluate total behavior, which became just simple matrix operation. The radiation quality factor Q, radiation efficiency and directivity can, then, be compactly expressed using modal quantities as [26], [28]

$$Q = 2\omega \frac{\sum\limits_{m,n} \beta_{mn} \hat{W}^\text{e}_{mn} + \sum\limits_{m,n} \beta_{mn} \hat{W}^\text{m}_{mn}}{\sum\limits_{m,n} \beta_{mn} P^\text{r}_{mn}}, \tag{24}$$

$$\eta = \frac{\sum\limits_{m} \beta_{mm}}{\sum\limits_{m} \beta_{mm} + \sum\limits_{m,n} \beta_{mn} P^\text{L}_{mn}}, \tag{25}$$

$$D = 15k^2 \frac{\sum\limits_{m,n} \beta_{mn} U_{mn}}{\sum\limits_{m} \beta_{mm}} \tag{26}$$

where $\hat{W}^\text{e}_{mn}$ and $\hat{W}^\text{m}_{mn}$ are modified modal energies, P^r_{mn} is modal radiated power, P^L_{mn} is modal lost power and

$$U_{mn} = \iint\limits_{VV'} \boldsymbol{J}_m(\boldsymbol{r}) \cdot \boldsymbol{J}_n^*(\boldsymbol{r}') e^{\mathrm{i}k(R \cdot \boldsymbol{r}_0)} \mathrm{d}V \mathrm{d}V' \tag{27}$$

in (26) is related to the modal radiation intensity.

4. Conclusions

An overview of the source concept definition has been presented. The source concept, as proposed, considers all parameters expressed solely as a function of the source quantities, namely the electric and magnetic currents. This approach offers many appealing properties, notably fast and uncomplicated evaluations via bilinear integral forms, the possibility to perform structural and modal decomposition, linear, quadratic and heuristic optimization and other advanced techniques of a current's modification and analysis. Recent applications of various antenna parameters expressed by source current have been mentioned in conjunction with potential applications.

Acknowledgments

This work has been supported by project no. TA 04010457 of the Technology Agency of the Czech Republic and by project SGS16/226/OHK3/3T/13.

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