A Novel Data Association Method for Frequency Based MIMO Systems

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Abstract. *Whenever more than one target exist, the most important problem is to associate associating the received signals to the correct targets. This problem appears for all multiple target applications such as multiple target tracking and it is known as "Data Association". For frequency-based systems, Multiple-Input Multiple-Output (MIMO) configuration together with the frequency diversity of the system enable us to determine the number of moving targets by using the Doppler frequencies. These frequencies include all relevant information about the location, velocity and direction of the targets and hence, they can be used efficiently to estimate the unknown target parameters.*

Keywords

Data association, MIMO radar, frequency-only, Doppler frequency

1. Introduction

Whenever one deals with multiple targets, a new and important problem appears. It is to associate all the received signals with correct scatterers (targets). It is desired to know which signal comes from which target and, sometimes this problem can be more difficult to solve than the problem itself. In the literature, this problem is known as "Data Association" and it is defined as "the decision process of linking measurements (from successive scans) deemed to be of a common origin (i.e., a target or false alarm) such that each measurement is associated with at most one origin" [1]. Since the pioneering work of Sittler [2], many algorithms have been developed ([2]-[9]) over the past three decades to solve the measurement origin uncertainty problem. Two simple solutions proposed were the Strongest Neighbor Filter (SNF) and the Nearest Neighbor Filter (NNF). While these simple Data Association techniques work reasonably well with being targets in sparse scenarios [1], they begin to fail as the false alarm rate increases or with low probability of detection. Instead of using only one measurement among the received ones and discarding the others, an alternative approach is proposed which is known as Probabilistic Data Association (PDA). PDA uses all the latest validated measurements with different weights [5, 10]. The standard PDA and its numerous improved versions have been shown in [5] to be effective in tracking a single target in clutter. On the other hand, Joint Probabilistic Data Association (JPDA) [11] and Multiple Hypothesis Tracking (MHT)[12] can be given as the popular solutions for the multiple target data association problem.

In classical Data Association problem, all estimated parameters included in received signals such as range of the target, time delay, frequency and angle of arrivals of the received signals are used to associate the targets with received signals. On the other hand, for frequency-based systems, only the Doppler frequencies exist and time-of-arrival (TOA) and hence the time delay information either do not exist or not well enough for moving targets. If this is the case, one must rely on these Doppler frequencies and they have to be used for Data Association. By using multiple transmitters and receivers, number of signals which arrive to the receivers can be increased. Hence, the number of Doppler frequencies obtained from the received signals is increased as well. As a result, frequency diversity of the system can be increased using multiple transmitter-receiver pairs which also helps to avoid blind speeds of the targets. Therefore, any radar network such as MIMO or multistatic radar can be utilized for frequency-only systems. In [13], it is shown that multiple target localization and Data Association are possible for frequency-only widely separated MIMO radar. In [13], Data Association is obtained using target positions and velocities which are all previously estimated in 2-dimensional (2D) Cartesian coordinate system. On the other hand, in this current paper, a similar but novel Data association method is proposed and the previous method is also expanded to 3-dimensional (3D) case. Here, Data Association is achieved using Doppler frequencies directly without requiring an intermediate step to estimate target parameters such as position and velocity. The almost same signal model and cost function given in [13] are used together with grid searching in 3D Cartesian coordinate system. The detailed information about frequency-based target localization methods can be found in [14], [15].

2. Signal Model

First of all, the signal model is given for one target case to simplify the procedure. Then, expanding one target case to multiple targets is a simple step. Assume that the system includes N_T transmitters and N_R receivers which are located in physically different locations (known as widely separated MIMO configuration). Transmitters are not able to receive transmitted signals and hence, radars operate as bistatic. Due to this bistatic configuration, the system has a total of $N = N_T \times N_R$ transmitter-receiver pairs which are all stationary and ground based. The target is assumed as moving with a constant speed (V) to simplify the signal model. Actually, the proposed method calculates the cost function in a time instant only and it is independent from whether the target motion is linear or not. Hence, the previous locations or velocities of the target are not important for the proposed method and it can be used not only for targets with linear motion but also for maneuvering targets. N_T transmitters radiate unmodulated Continuous Wave (CW) signals in different frequencies represented as $f_1, f_2, \dots f_{N_T}$ to increase the frequency resolution of the system. N_R receivers intercept these signals with Doppler-shifts and time delays due to the target motions. At the receiver site, N_T radiated frequencies can be measured and the transmitted signals can be detected. The operating frequencies of the transmitters should be chosen carefully to provide the following condition

$$f_{i-1} < f_i \mp f_{max} < f_{i+1}; \quad \text{for all } i \tag{1}$$

where f_{max} is the maximum Doppler frequency of the whole system, and the operation frequencies are chosen as

$$f_1 < f_2 < \cdots < f_{N_T}, \tag{2}$$

N_T transmitters are located at $T_n = (x_{T_n}, y_{T_n}, z_{T_n}),\ n = 1, 2, \dots, N_T$. After the radiated signals are reflected by the target at (x, y, z), they are received by N_R receivers located at different positions as $R_m = (x_{R_m}, y_{R_m}, z_{R_m}),\ m = 1, 2, \dots, N_R$. Assume that the n^{th} receiver receives a signal which is emitted from the m^{th} transmitter after reflected from the same target. The frequency of this signal can be written in $3D$ as [16]

$$\begin{aligned} f_{m,n} = & f_n - \frac{f_n}{\mathrm{c}}\left(\frac{(x - x_{T_n})V_x + (y - y_{T_n})V_y + (z - z_{T_n})V_z}{\sqrt{(x - x_{T_n})^2 + (y - y_{T_n})^2 + (z - z_{T_n})^2}}\right) \\ & - \frac{f_n}{\mathrm{c}}\left(\frac{(x - x_{R_m})V_x + (y - y_{R_m})V_y + (z - z_{R_m})V_z}{\sqrt{(x - x_{R_m})^2 + (y - y_{R_m})^2 + (z - z_{R_m})^2}}\right) \end{aligned} \tag{3}$$

where c is the speed of light, V_x, V_y and V_z are the target velocities in x, y and z directions respectively. All obtained frequencies which are radiated from the same transmitter can be grouped into a matrix-vector equation by using (3). For a system which includes a total of N_T transmitters and N_R receivers, the N_T matrix-vector equations can be obtained as

$$c\mathbf{b_i} = -\mathbf{A_i v} \quad ; \quad i = 1, 2, \dots, N_T \tag{4}$$

where

$$\mathbf{A_i} = \begin{bmatrix} \frac{x-x_{T_i}}{L_{T_i}} + \frac{x-x_{R_1}}{L_{R_1}} & \frac{y-y_{T_i}}{L_{T_i}} + \frac{y-y_{R_1}}{L_{R_1}} & \frac{z-z_{T_i}}{L_{T_i}} + \frac{z-z_{R_1}}{L_{R_1}} \\ \frac{x-x_{T_i}}{L_{T_i}} + \frac{x-x_{R_2}}{L_{R_2}} & \frac{y-y_{T_i}}{L_{T_i}} + \frac{y-y_{R_2}}{L_{R_2}} & \frac{z-z_{T_i}}{L_{T_i}} + \frac{z-z_{R_2}}{L_{R_2}} \\ \dots & \dots & \\ \frac{x-x_{T_i}}{L_{T_i}} + \frac{x-x_{R_{N_R}}}{L_{R_{N_R}}} & \frac{y-y_{T_i}}{L_{T_i}} + \frac{y-y_{R_{N_R}}}{L_{R_{N_R}}} & \frac{z-z_{T_i}}{L_{T_i}} + \frac{z-z_{R_{N_R}}}{L_{R_{N_R}}} \end{bmatrix},$$

$$\mathbf{b_i} = \begin{bmatrix} \frac{f_{i,1}-f_i}{f_i} \\ \frac{f_{i,2}-f_i}{f_i} \\ \dots \\ \frac{f_{i,N_R}-f_i}{f_i} \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}$$

and

$$L_{T_n} = \sqrt{(x - x_{T_n})^2 + (y - y_{T_n})^2 + (z - z_{T_n})^2}, \tag{5}$$

$$L_{R_m} = \sqrt{(x - x_{R_m})^2 + (y - y_{R_m})^2 + (z - z_{R_m})^2} \tag{6}$$

are the distances from the target to both the n^{th} transmitter and to the m^{th} receiver respectively for $n = 1, 2, \dots, N_T$ and $m = 1, 2, \dots, N_R$. Here, $\mathbf{A_i}$'s are $N_R \times 3$ size matrices, $\mathbf{b_i}$'s are the size of $N_R \times 1$ vectors. The total of N_T equations as in (4) can be combined into one equation as

$$c\mathbf{b} = -\mathbf{Av} \tag{7}$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{A_1} \\ \mathbf{A_2} \\ \dots \\ \mathbf{A_{N_T}} \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} \mathbf{b_1} \\ \mathbf{b_2} \\ \dots \\ \mathbf{b_{N_T}} \end{bmatrix}.$$

After combining all received frequencies into one equation, the target velocity (as a vector) can be estimated using (7) with a simple matrix inversion as

$$\hat{\mathbf{v}} = -\mathrm{c}\mathbf{A^{-1}b}. \tag{8}$$

By using this estimated target velocity, a new cost function can be defined by inserting it ($\hat{\mathbf{v}}$) into (4) as

$$J_i = \| \mathrm{c}\mathbf{b_i} + \mathbf{A_i}\hat{\mathbf{v}} \|_2 \,; \quad i = 1, 2, \dots, N_T \tag{9}$$

where $||.||_2$ is the Euclidean distance and it is given for a vector $\mathbf{r} = [r_1, r_2, \dots, r_N]^T$ as $||\mathbf{r}||_2 = \sqrt{{r_1}^2 + {r_1}^2 + \cdots + {r_N}^2}$. By inserting (8) into (9), it can be rewritten as

$$\begin{aligned} J_i &= \| \mathrm{c}\mathbf{b_i} + \mathbf{A_i}\hat{\mathbf{v}} \|_2 \\ &= \| \mathrm{c}\mathbf{b_i} - \mathrm{c}\mathbf{A_i A^{-1} b} \|_2 \\ &= \mathrm{c} \, \| \mathbf{b_i} - \mathbf{A_i A^{-1} b} \|_2 \end{aligned} \tag{10}$$

where c is constant and can be dropped. Hence

$$J_i = \| \mathbf{b_i} - \mathbf{A_i A^{-1} b} \|_2 \quad ; \quad i = 1, 2, \dots, N_T \tag{11}$$

and in the general case the cost function becomes

$$J = \frac{1}{N_T}\sqrt{\sum_{i=1}^{N_T} {J_i}^2} \tag{12}$$

where $J_i' s$ are defined as in (11).

Note that $\mathbf{A}$ is a square matrix only when $N_R = 3$ and $N_T = 1$. In other cases, $\mathbf{A}$ is not square and the inverse of $\mathbf{A}$ can be obtained using the pseudo-inverse matrix as, $\mathbf{A}^+ = (\mathbf{A^H A})^{-1}\mathbf{A^H}$. Here, $\mathbf{A^H}$ represents the Hermitian (complex conjugate and transpose) of matrix $\mathbf{A}$.

In the equations above, $\mathbf{b_i}$'s include the estimated frequencies whereas $(\mathbf{A_i A^{-1} b})$'s are the estimations of $(\mathbf{b_i})$'s $(\mathbf{\hat{b}_i} = \mathbf{A_i A^{-1} b})$ and they include frequencies as well. When these frequencies are estimated accurately, frequency estimation error becomes zero $(\mathbf{\hat{b}_i} = \mathbf{b_i})$. But, in general it is not true and some amount of frequency estimation errors occur due to the many kind of reasons, such as multipath, clutter, carrier frequency offset, etc. The frequency estimation error can be given as

$$e \triangleq \mathbf{b_i} - \mathbf{A_i A^{-1} b}\,; \quad i = 1, 2, \ldots, N_T. \tag{13}$$

The cost function defined in (11) tries to minimize the frequency estimation error. Actually, this function tries to find the closest Doppler frequency to the estimated Doppler frequency using the grid searching for all possible target positions. Hence, the target localization can be achieved easily by using this cost function together with grid searching. In simulations, frequency estimation errors are added to investigate the performance of the proposed method with respect to the frequency estimation errors.

3. Data Association

When the number of targets increases, a new problem appears. As easily predicted, the radiated signals arrive at different receivers after scattered from many scatterers (targets). Hence, a signal arrives to the same receiver unit after scattered from many different targets. The problem is to find which signal is scattered from which targets, which is known as "Data Association". It is an important problem not only for multiple target localization and multiple target tracking but also for all other multiple target applications. Besides associating the signals with the correct targets, determining the number of targets is another important problem and is also a prerequisite for Data Association. MIMO configuration provides an extra advantage for this problem. The total number of targets can be found using the Doppler frequencies which arise from the same transmitter. Due to the widely separated MIMO configuration, each receiver is faced with different Doppler frequencies for different targets depending on their velocities and positions. In some cases, some frequencies may not be resolved at some receivers, but at various receivers all Doppler frequencies are resolved. As a result, the total number of targets can be identified at the fusion center easily. Moreover, due to the system geometry, blind speeds can be occurred for some transmitter-receiver pairs and as a result Doppler frequency can be estimated as zero. On the other hand, system includes many extra receivers and transmitters and they can estimate Doppler frequencies different from zero. Therefore, blind speeds can be eliminated easily as well.

If the system includes more than one target, all equations given in the previous section can be written for all targets separately. The estimated Doppler frequencies can be represented as $f_{l,m,n}$ which is the Doppler frequency of the signal radiated by the n^{th} transmitter, and received by the m^{th} receiver just after scattered by the l^{th} target. For MIMO radar with 2 transmitters and 2 receivers (2×2 MIMO radar) and for two targets case, a total of 8 Doppler frequencies are generated. All possible Doppler frequencies can be written as in Tab. 1 with respect to the notation defined above.

	Rec1, Tr1	**Rec1, Tr2**	**Rec2, Tr1**	**Rec2, Tr2**
Target1	$f_{1,1,1}$	$f_{1,1,2}$	$f_{1,2,1}$	$f_{1,2,2}$
Target2	$f_{2,1,1}$	$f_{2,1,2}$	$f_{2,2,1}$	$f_{2,2,2}$

Tab. 1. Possible Doppler frequencies for 2×2 MIMO radar and for two targets.

Due to the reasons explained above, the fusion center knows that 2 targets exist. Moreover, the system includes 2×2 MIMO radar, hence, 4 of 8 Doppler frequencies (for each target) must be associated with the correct target. We can choose randomly as $f_{1,1,1}$ from target 1 and $f_{2,1,1}$ from target 2. In this case, all possible associated frequency groups which are called pre-associations can be formed as in Tab. 2.

In Tab. 2., the first group represents the correct association whereas the others are all possible but wrong associations. In this case, the problem is reduced to choose the correct association group from all possible groups. If we have 2×2 MIMO radar, $\mathbf{A_1}$, $\mathbf{A_2}$, $\mathbf{b_1}$, $\mathbf{b_2}$ and $\mathbf{v}$ can be obtained for each of the targets by using (4). Then, cost function can be written as

$$J = \| \mathbf{b_1} - \mathbf{A_1 A_2}^{-1}\mathbf{b_2} \|_2\,. \tag{14}$$

For each pre-association group given in Tab. 2, two cost functions can be written as $J_{1_{\text{target1}}}$ and $J_{1_{\text{target2}}}$ by using (14). Finally, one cost function for each group can be obtained by averaging them as

$$J_i = \frac{1}{2}\left(J_{i_{\text{target1}}} + J_{i_{\text{target2}}}\right)\,; \quad i = 1, 2, \ldots, 8. \tag{15}$$

Alternatively, cost functions for two targets can be combined in one maxrix-vector equation by writing $\mathbf{A_{1_n}}$, $\mathbf{A_{2_n}}$, $\mathbf{b_{1_n}}$, $\mathbf{b_{2_n}}$, and $\mathbf{v_n}$ where $n = 1, 2$ and n represents the target number. The other vectors and matrices are the same as in (14) for two targets separately. Then, they are combined as

$$\mathbf{A_n} = \begin{bmatrix} \mathbf{A_{1_n}} & \mathbf{0_2} \\ \mathbf{0_2} & \mathbf{A_{2_n}} \end{bmatrix}, \mathbf{b_n} = \begin{bmatrix} \mathbf{b_{1_n}} \\ \mathbf{b_{2_n}} \end{bmatrix}, \mathbf{v_n} = \begin{bmatrix} \mathbf{v_1} \\ \mathbf{v_2} \end{bmatrix}, n = 1, 2$$

Group	Freq. Assoc. for Target 1	Freq. Assoc. for Target 2
1	$f_{1,1,1}, f_{1,1,2}, f_{1,2,1}, f_{1,2,2}$	$f_{2,1,1}, f_{2,1,2}, f_{2,2,1}, f_{2,2,2}$
2	$f_{1,1,1}, f_{1,1,2}, f_{1,2,1}, f_{2,2,2}$	$f_{2,1,1}, f_{2,1,2}, f_{2,2,1}, f_{1,2,2}$
3	$f_{1,1,1}, f_{1,1,2}, f_{2,2,1}, f_{1,2,2}$	$f_{2,1,1}, f_{2,1,2}, f_{1,2,1}, f_{2,2,2}$
4	$f_{1,1,1}, f_{1,1,2}, f_{2,2,1}, f_{2,2,2}$	$f_{2,1,1}, f_{2,1,2}, f_{1,2,1}, f_{1,2,2}$
5	$f_{1,1,1}, f_{2,1,2}, f_{1,2,1}, f_{1,2,2}$	$f_{2,1,1}, f_{1,1,2}, f_{2,2,1}, f_{2,2,2}$
6	$f_{1,1,1}, f_{2,1,2}, f_{1,2,1}, f_{2,2,2}$	$f_{2,1,1}, f_{1,1,2}, f_{2,2,1}, f_{1,2,2}$
7	$f_{1,1,1}, f_{2,1,2}, f_{2,2,1}, f_{1,2,2}$	$f_{2,1,1}, f_{1,1,2}, f_{1,2,1}, f_{2,2,2}$
8	$f_{1,1,1}, f_{2,1,2}, f_{2,2,1}, f_{2,2,2}$	$f_{2,1,1}, f_{1,1,2}, f_{1,2,1}, f_{1,2,2}$

Tab. 2. All possible frequency associations (pre-associations) for Target 1 and Target 2.

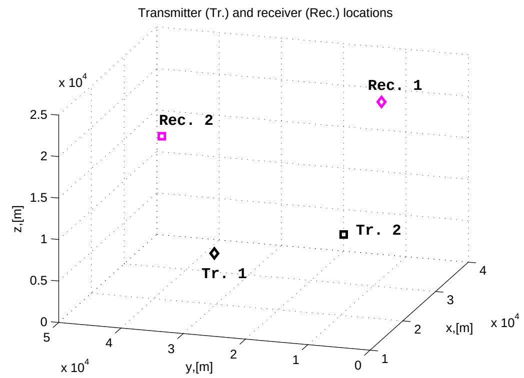

Fig. 1. Transmitter and receiver locations.

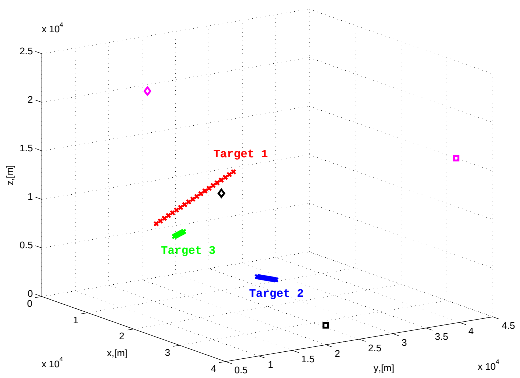

Fig. 2. Linearly moving 3 targets.

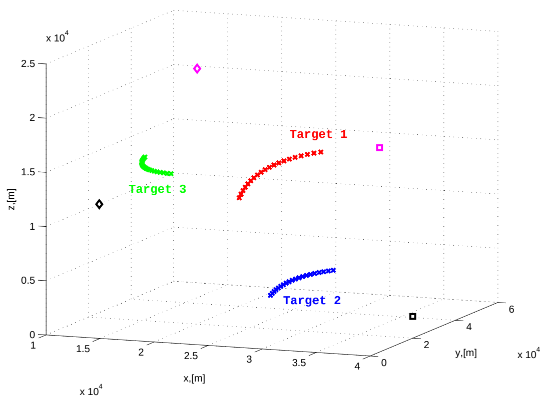

Fig. 3. Maneuvering 3 targets.

	x	y	z	V_x	V_y	V_z
Rec1	23	5	25	0	0	0
Tr1	10	25	10	0	0	0
Rec2	32	45	15	0	0	0
Tr2	40	20	2	0	0	0
Target1	6	18	7	120	120	100
Target2	17, 5	28	2	157.14	−157.14	50
Target3	8.5	19	6	57	−15	20

Tab. 3. Simulation parameters of radar units and targets (initial positions and velocities) for linear motion case.

	x	y	z	V_x	V_y	V_z
Rec1	23	5	25	0	0	0
Tr1	10	25	10	0	0	0
Rec2	32	45	15	0	0	0
Tr2	40	20	2	0	0	0
Target1	20	40	10	10	200	100
Target2	22.5	42	1	20	150	50
Target3	15	21	15	−50	50	−50

Tab. 4. Simulation parameters of radar units and targets (initial positions and velocities) for maneuvering motion case.

where $\mathbf{0_2}$ is 2×3 zero matrix. Then, costs for each pre-association group (J_i) can be calculated directly using (14). After calculating the costs (J_i) for each pre-association group, the final Data Association is achieved by choosing the frequency group with minimum cost (J_i).

4. Simulation Results

The Data Association performance of the proposed method is analyzed with simulations using Matlab©. The widely separated MIMO radar with 2 receivers and 2 transmitters is used in simulations for 2 (Targets 1, 2) and 3 (Targets 1, 2, 3) targets cases. The locations of the transmitters and receivers can be seen in Fig. 1. Similarly, motions of 3 targets are simulated not only for linear motion case but also for maneuvering motion case. These two motion models can be seen in Fig. 2 and Fig. 3 respectively.

The detailed simulation parameters can be seen in Tab. 3 and Tab. 4 for both motion models.

In Tab. 3 and Tab. 4, the positions are given in km and the initial velocities are in mps. By using these parameters, initial speeds ($|\mathbf{V}|$) of target 1, target 2 and target 3 can be calculated as $709, 820$ and 224 kmph respectively for linear motion case and 805, 574 and 312 kmph respectively for maneuvering motion case. Two transmitters operate in X-band and radiate unmodulated, CW tone signals with $f_1 = 10$ GHz, and $f_2 = 10.3$ GHz. In grid search, grid points are chosen being 100 meters apart from each and maximum grid position error for search points are assumed as 50 meters (half of the grid separation). For frequency error analysis, zero mean, normal distributed random variable with different variances (σ^2) is added to exact Doppler frequencies. The Monte-Carlo method is applied and 1000 measurements are averaged. For this simulation setup, the Data Association performances of the proposed methods are the same. The percentage of successful Data Association (represented by the % symbol) with respect to the frequency error variance can be seen in Tab. 5 and Tab. 6 for linearly moving and maneuvering targets respectively. The Data Association performance of the proposed method can also be seen for 2 and 3 targets. This result can be visualized as given in Fig. 4 and Fig. 5.

As can be seen from tables and figures above, the Data Association is possible for different simulation scenarios including 2 and 3 targets. Especially, when the Doppler frequencies are estimated with low error, they can be associated with correct targets. Similar performances are observed when targets are not only moving linearly but also maneuvering. As already explained, the proposed method is independent from target's direction. It calculates a cost function for a time instant by using Doppler frequencies only for an instant of time. Then, the Data Association is investigated for that time only. This process is repeated for each observation time separately. Hence, the Data Association performances are almost the same for both motion models as expected. On the other hand, the number of targets has effect on performance as well. When the number of targets increase, the possible number of pre-associations increases and choosing the correct association with low cost becomes a more complex problem. Hence, the Data Association performance of the proposed method decrease slightly. As a result, it can be seen from the simulation results that when Doppler frequencies are estimated with low error, frequency based Data Association can be achieved using Doppler frequencies only for multiple targets which are moving linearly or maneuvering.

σ^2 [Hz]	0	1	10	100	1k	10k	100k
% (2 targets)	100	100	100	100	99.2	55.4	21.1
% (3 targets)	100	100	100	100	95.4	51.2	18.7

Tab. 5. Data Association performance with respect to the received frequency error variance (σ^2) for 2 and 3 targets when targets are moving linearly.

σ^2 [Hz]	0	1	10	100	1k	10k	100k
% (2 targets)	100	100	100	100	98.7	54.8	20.6
% (3 targets)	100	100	100	100	94.3	50.5	17.4

Tab. 6. Data Association performance with respect to the received frequency error variance (σ^2) for 2 and 3 targets when targets are maneuvering.

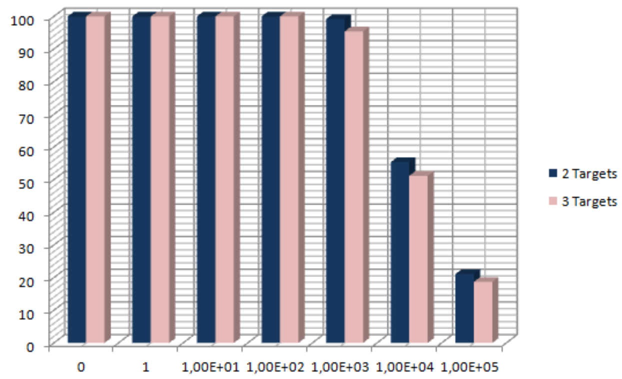

Fig. 4. Results for Linearly moving targets (Tab. 5).

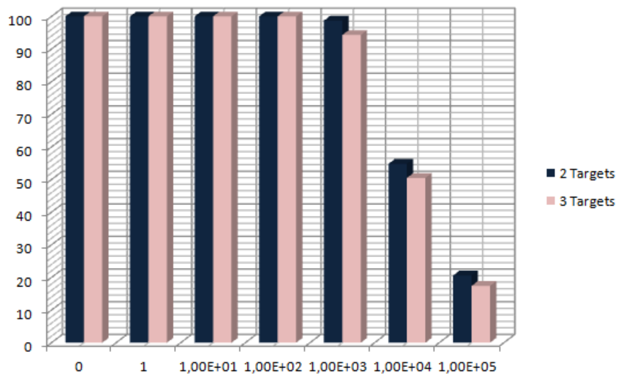

Fig. 5. Results for maneuvering targets (Tab. 6).

5. Conclusion

In this paper, a novel Data Association method for frequency based MIMO radar is proposed in three dimensional (3D) Cartesian coordinate system. It is shown that, without calculating the target positions and velocities, the Data Association can be achieved by using Doppler frequencies only. Hence, the proposed method can be used efficiently when time-of-arrival information of the received signal is missing or not good enough. Two cost functions are defined and both of them calculate the costs for all possible frequencies. One of them calculates the cost by averaging the costs calculated for two targets separately whereas the other one calculates the cost directly using one cost function which combines distinct cost functions for two targets in one vector-matrix equation. Both methods give exactly the same results but the complexity of the second method is higher than the complexity of the first one due to the higher matrix dimension included in it. It is shown that, the proposed methods are robust to the frequency estimation errors. The proposed two methods can be used for Data Association efficiently especially when only frequency information exists at the receiver site. For future work, frequency-based multiple target tracking can be thought by using proposed Data Association method.

References

[1] BAR-SHALOM, Y., BLAIR, W. D. *Multitarget-Multisensor Tracking: Applications and Advances*. Boston (USA): Artech House, 2000, vol. 3. ISBN: 9781580530910

[2] SITTLER, R. W. An optimal data association problem in surveillance theory. *IEEE Transactions on Military Electronics*, 1964, vol. 8, p. 125–139. DOI: 10.1109/TME.1964.4323129

[3] BLACKMAN, S. S., POPOLI, R. F. *Design and Analysis of Modern Tracking Systems*. Norwood (MA, USA): Artech House, 1999.

[4] BAR-SHALOM, Y., FORTMAN, T. E. *Tracking and Data Association*. Orlando (FL, USA): Academic Press, 1988.

[5] BAR-SHALOM, Y. LI, X. R. *Multitarget-Multisensor Tracking: Principles and Techniques*. Storrs (CT, USA): YBS Publishing, 1995.

[6] KIRUBARAJAN, T., BAR-SHALOM, Y. Probabilistic data association techniques for target tracking in clutter. *Proceedings of IEEE*, 2004, vol. 92, p. 536–557. DOI: 10.1109/JPROC.2003.823149

[7] MUSICKI, D., SUVOROVA, S. Tracking in clutter using IMM-IPDA-based algorithms. *IEEE Transactions on Aerospace and Electronic Systems*, 2008, vol. 44, no. 1, p. 111–126. DOI: 10.1109/TAES.2008.4516993

[8] MUSICKI, D., LA SCALA, B. Multi-target tracking in clutter without measurement assignment. *IEEE Transactions on Aerospace and Electronic Systems*, 2008, vol. 44, no. 3, p. 877–896. DOI: 10.1109/TAES.2008.4655350

[9] XU, L., JIN, S., YIN, G. A track association algorithm based on leader-follower on-line clustering in dense target environments. *Radioengineering*, 2014, vol. 23, no. 1, p. 259–265.

[10] BAR-SHALOM, Y. TSE, E. Tracking in a cluttered environment with probabilistic data association. *Automatica*, 1975, vol. 11, no. 5, p. 451–460. DOI: 10.1016/0005-1098(75)90021-7

[11] FORTMANN, T., BAR-SHALOM, Y., SCHEFFE, M. Sonar tracking of multiple targets using joint probabilistic data association. *IEEE Journal of Oceanic Engineering*, 1983, vol. 8, no. 3, p. 173–184. DOI: 10.1109/JOE.1983.1145560

[12] REID, D. B. An algorithm for tracking multiple targets. *IEEE Transactions on Automatic Control*, 1979, vol. 24, no. 6, p. 423–432. DOI: 10.1109/TAC.1979.1102177

[13] KALKAN, Y., BAYKAL, B. Multiple target localization & data association for frequency-only widely separated MIMO radar. *Digital Signal Processing*, 2014, vol. 25, p. 51–61. DOI: 10.1016/j.dsp.2013.09.015

[14] KALKAN, Y., BAYKAL, B. Frequency-based target localization methods for widely separated MIMO radar. *Radio Science*, 2014, vol. 49, no. 1, p. 53–67. DOI: 10.1002/2013RS005245

[15] KALKAN, Y. Cramer-Rao bound for target localization for widely separated MIMO radar. *Radioengineering*, 2013, vol. 22, no. 4, p. 1156–1161.

[16] RICHARDS, M. A. *Fundamentals of Radar Signal Processing*. New-York (USA): McGraw-Hill, 2005. ISBN: 9780071798327

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