A Modified Unequally Spaced Array Antenna Synthesis Method for Side Lobe Reduction

Soroush VEISEE, Shahrooz ASADI

Dept. of Electrical Engineering, University of Shahid Beheshti, Tehran, Iran

veisee@aut.ac.ir, sh_asadi@sbu.ac.ir

Submitted October 14, 2016 / Accepted February 7, 2017

Abstract. *The aim of this paper is to demonstrate the application of Unequally Spaced Arrays (USAs) in decreasing side lobe level (SLL) in linear arrays. As well known, solving of a nonlinear equation is needed in USA antenna pattern synthesis. In this paper, an improved algorithm for USA antenna pattern synthesis is presented. This method is based on converting the array factor into a triangular system of equations capable to be solved using a recursive algorithm. This method has more accuracy and speed than reported similar analytical methods based on simulation results, which leads to lower SLL and simulation time. In addition, an improvement of 3dB beamwidth in comparison with uniform spaced array can be observed.*

Keywords

Antenna arrays, unequal spacing, pattern synthesis, side lobe level

1. Introduction

Uniformly spaced array antenna synthesis techniques have been deeply investigated in past decades. Traditional synthesis techniques such as Dolph-Chebyshev [1], Taylor [2], and Fourier transform, can lead to design of a desired pattern. These synthesis procedures change the phase and magnitude of elements current to reduce the side lobe level (SLL). However, the amplitude/phase distribution can complicate the designing of the array antenna feed network. In addition, it cannot optimize the use of all elements in terms of maximum power. Non-uniformly spaced array antenna can solve this issue. This kind of array antenna has also other significant advantages such as smaller size and reduced number of elements. As a result, non-uniformly spaced array antennas are widely used in radar, sonar, and wireless communications.

Variety of numerical and analytical methods [7–20] have been introduced for unequally spaced array (USA) pattern synthesis. The first work on USA pattern synthesis was performed by Unz [3] by using a matrix formulation to obtain the distribution of elements current of an USA to form a desired pattern. Then, Harrington [4] presented a method for reducing SLL using non-uniform element spacing and showed that SLL can be reduced to the level of $2/N^{th}$ of the main lobe level. In [5], Ishimaru used Poisson's formula to reduce SLL in comparison with a linear array with uniform amplitude distribution. Miller and Goodman [6] proposed a simple and non-iterative method for USA pattern synthesis using Prony's method. In [7], [8], Kumar proposed a recursive algorithm for USA pattern synthesis based on Legendre conversion.

On the other side, stochastic methods such as Genetic Algorithm (GA) [11], Particle Swarm Optimization (PSO) and Differential Evolution Algorithm (DEA) [10] have been recently used in pattern synthesis. However, these optimization methods require large CPU time, which further increases with the increment of array element numbers.

In USA pattern synthesis, analytical methods are affordable in time because of the non-iterative property. In this paper, an analytical method for pattern synthesis of unequally spaced arrays is proposed. It is an enhanced version of the method described in [7], in terms of higher precision and speed. Compared with the method in [7], which is used for odd number of elements, the method presented in this paper with some modification, can be used also for even number of elements.

2. Theory

The geometry of a $(2N + 1)$ element non-uniform linear array is shown in Fig. 1, where I_i and d_i state respectively for the element position and current.

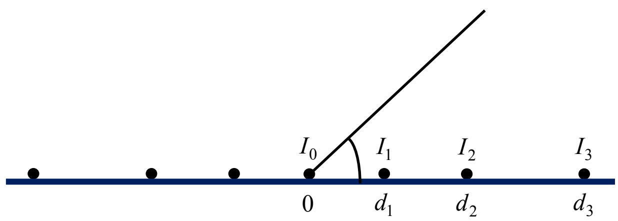

Fig. 1. Geometry of a symmetrical non-uniform linear array.

This paper proposes a new synthesis pattern approach that adjusts the element position in a linear array with a uniform current distribution, in order to quickly and accurately improve the radiation characteristics such as lower peak side lobe level (PSLL). This method can indeed determine the element positions of the array in order to fit the array pattern to a pre-specified array pattern. The desired array pattern is defined as where, $-1 \le u \le 1$ and $u = \cos\theta$, $0 \le \theta \le \pi$ radians. As known, the array pattern of a symmetric array with $(2N + 1)$ elements is defined as:

$$E(u) = \sum_{n=0}^{N} I_n \cos(kd_n u) . \tag{1}$$

By sampling the above relation with a sampling space $\Delta u = 1/(M - 1)$, we obtain:

$$E(u_m) = \sum_{n=0}^{N} I_n \cos(m\beta_n) \tag{2}$$

where $u_m = m\,\Delta u$ and $\beta_n = kd_n\,\Delta u$.

Because of the symmetry of the pattern in the u space this sampling is performed in $0 \le u \le 1$ interval or $m = 0,1,2,3,\ldots, N$.The element position must be chosen in such a way where:

$$E(u_m) = E_\mathrm{d}(u_m) \tag{3}$$

or

$$E_\mathrm{d}(u_m) = \sum_{n=0}^{N} I_n \cos(m\beta_n), \qquad m = 0,1,2,3,\ldots,M-1 \tag{4}$$

where $E_\mathrm{d}(u)$ is the desired pattern.

Relation (1) is a nonlinear equation. To solve it, both sides of (4) are multiplied by a matrix **A** defined as:

$$\mathbf{A}(\alpha) = [a_0(\alpha)\; a_1(\alpha)\; a_2(\alpha)\; ......a_{M-1}(\alpha)] \tag{5}$$

leading to

$$\sum_{m=0}^{M-1} a_m(\alpha) E_\mathrm{d}(u_m) = \sum_{n=o}^{N-1} I_n \sum_{m=0}^{M-1} a_m(\alpha)\cos(m\beta_n d) \tag{6}$$

The values of $a_m(\alpha)$ coefficients are selected as:

$$f(\alpha,\beta) = \sum_{m=0}^{M-1} a_m(\alpha)\cos(m\beta) \tag{7}$$

where [7]

$$f(\alpha,\beta) = \begin{cases} \dfrac{2}{(\cos(\beta)-\cos(\alpha))^{1/2}}, & 0 \le \beta < \alpha \\ 0 & \alpha \le \beta \le \pi \end{cases} \tag{8}$$

Also, by expressing the left side of (6) as:

$$F(\alpha_p) = \sum_{m=0}^{M-1} a_m(\alpha_p) E_\mathrm{d}(u_m) . \tag{9}$$

This later can be simplified to:

$$F(\alpha_p) = \sum_{n=0}^{N} I_n\, f(\alpha_p, \beta_n) . \tag{10}$$

In [7], Legendre coefficients $p_{m-0.5}(\cos\alpha)$ are used instead of $a_m(\alpha)$ in (7), which are the Fourier series coefficients of $f(\alpha,\beta)$. But in this paper the values of a_m coefficients are obtained by minimizing the error term stated below:

$$error = \sum_{i=0}^{M-1} [f(\alpha,\beta_i) - \sum_{m=0}^{M-1} a_m(\alpha)\cos(m\beta_i)]^2,$$
$$\beta_i = i\,\Delta\beta, \Delta\beta = \frac{\pi}{M-1} \tag{11}$$

which leads to

$$f(\alpha,\beta_i) - \sum_{m=0}^{M-1} a_m(\alpha)\cos(m\beta_i) = 0, \quad i = 0,1,\ldots,M-1 . \tag{12}$$

In fact, the left hand side of (7) is considered to be equal to the right hand side in β_i points, which $i = 0,1,\ldots,M-1$.

After some manipulations, equation (12) can be stated as a matrix equation as below:

$$\mathbf{B} \times \mathbf{A}(\alpha) = \mathbf{F} . \tag{13}$$

Therefore, matrix $\mathbf{A}(\alpha)$ can be obtained as:

$$\mathbf{A}(\alpha) = \mathbf{B}^{-1} \times \mathbf{F} \tag{14}$$

which leads to obtain the values of $a_m(\alpha)$. In order to avoid misunderstanding, in continue, instead of $a_m(\alpha)$ which derived from (14), we use $a'_m(\alpha)$.

In (14), the matrices **B** and **F** are defined as below:

$$\mathbf{B} = \begin{bmatrix} 1 & 1 & 1 .... 1 \\ 1 & \cos(\beta_1) & \cos(2\beta_1)........\cos((M-1)\beta_1) \\ 1 & \cos(\beta_2) & \cos(2\beta_2).......\cos((M-1)\beta_2) \\ . & & \\ . & & \\ . & & \\ 1 & \cos(\beta_{M-1}) & \cos(2\beta_{M-1})...\cos((M-1)\beta_{M-1}) \end{bmatrix}$$

and

$$\mathbf{F} = \begin{bmatrix} f(\alpha,\beta_0) \\ f(\alpha,\beta_1) \\ f(\alpha,\beta_2) \\ . \\ . \\ . \\ f(\alpha,\beta_{M-1}) \end{bmatrix} .$$

The value of α is defined as:

$$\alpha_p = \beta_p + V_p \tag{15}$$

in which the value of V_p is defined such that:

$$\alpha_p < \beta_{p+1}\,. \tag{16}$$

From (9) and (14), we obtain:

$$F(\alpha_p) = \sum_{n=0}^{p} I_n\, f(\alpha_p, \beta_n) \tag{17}$$

which can be reformatted as:

$$\begin{aligned} &F(\alpha_0) = I_0\, f(\alpha_0, \beta_0) \\ &F(\alpha_1) = I_0\, f(\alpha_1, \beta_0) + I_1\, f(\alpha_1, \beta_1) \\ &\ldots\ldots \\ &F(\alpha_p) = \sum_{n=0}^{p-1} I_n\, f(\alpha_p, \beta_n) + I_p\, f(\alpha_p, \beta_p) \end{aligned} \tag{18}$$

The locations of the array elements for a defined current distribution can be then obtained from the above equation using a recursive algorithm. In the proposed algorithm, the distance between elements was selected as:

$$0.5\lambda \le d_p - d_{p-1} \le 0.5\lambda + \Delta(p), \quad p = 1,2,\ldots,N \tag{19}$$

where $\Delta(p)$ is the variable space broadening factor for the p^{th} element. The lower limit of (17) is able to prevent mutual coupling effects and the upper limit is determined in order to prevent from grating lobes. It is noted that in comparison with [7] which used a constant value for $\Delta(p)$, in this paper we obtained an optimized value for each value of $\Delta(p)$. Let us define some parameters such as:

d_0, the place of the first element, assumed to be zero, which leads to $\beta_0 = 0$,

I_0, the normalized value of the first element current as:

$$I_0 = \frac{F(\alpha_0)}{f(\alpha_0, \beta_0)}\,, \tag{20}$$

α such as:

$$\alpha_0 = k(d_0 + \frac{\lambda}{2})\Delta u \tag{21}$$

and

$$\alpha_p = k(d_{p-1} + \frac{\lambda}{2} + \Delta(p))\Delta u\,. \tag{22}$$

Before starting the algorithm, the initial values of all $\Delta(p)$ are considered to be equal with Δ, where Δ is a constant space broadening factor to be specified by the user. Then the values of $\Delta(p)$ should be optimized in order to minimize PSLL. This algorithm includes N steps. In the n^{th} step, the optimized value of $\Delta(n)$ is obtained and the other values of $\Delta(p)$, $p \ne n$ are considered as below:

For $p > n$, $\Delta(p) = \Delta$.

For $p < n$, $\Delta(p)$ is equal to the optimized value obtained in previous steps.

In order to get the optimized value of $\Delta(n)$, the value of $\Delta(n)$ varies from 0 to 0.5λ with 0.01λ step size. For each value of $\Delta(n)$, by utilizing the following procedure, a set of element positions ($d_0, d_1, \ldots, d_N$) can be obtained.

Assuming $\beta_0 = 0$, the values of β_p for $p = 1,2,3,\ldots,N$, are calculated as [7]:

$$\begin{aligned} &\beta_p = \cos^{-1}\left(g(\alpha_p)\right), \\ &g(\alpha_p) = \frac{2I_p^{\,2}}{\left[F(\alpha_p) - \sum_{n=0}^{p-1} I_n f(\alpha_p, \beta_n)\right]^2} + \cos(\alpha_p). \end{aligned} \tag{23}$$

Thus,

$$d_p = \beta_p / (k\Delta u)\,.$$

For each value of d_p from (23), the conditions below should be applied.

- If d_p is a complex number, $d_p = d_{p-1} + 0.5\lambda$.
- If d_p is larger than α_p, $d_p = d_{p-1} + 0.5\lambda$.
- If the element spacing between d_p and d_{p-1} is less than 0.5λ, $d_p = d_{p-1} + 0.5\lambda$.

Then, utilizing the obtained set of element positions from above procedure and using (1), the array pattern and PSLL of the related set of positions can be acquired. Therefore, in the n^{th} step, for each value of $\Delta(n)$, there is a related value of PSLL. The optimized value of $\Delta(n)$ is the one which has the minimum related value of PSLL. This selected value of $\Delta(n)$ should be used for the $(n + 1)^{\text{th}}$ step.

In order to apply the mentioned method to an array of even number of elements ($2N$), α_0 and d_0 should be selected as:

$$\alpha_0 = k(\Delta_0 + \frac{\lambda}{2})\Delta u,\ d_0 = 0.25\lambda\,. \tag{24}$$

In the above equation, Δ_0 is called the initial space broadening factor and its value is changed from 0 to λ. In this way, Δ_0 and Δ should be initially defined by users.

3. Numerical Results

In this section, the presented method is demonstrated through examples. In all these examples, it is assumed that the current distribution amplitude is uniform and unequally spacing is used in order to decrease SLL. The desired pattern to design an unequally spaced array with q elements is defined as:

$$E_{\mathrm{d}}(u) = \begin{cases} 1 & 0 \le u \le u_m \\ E_{\min} & u_m \le u \le 1 \end{cases} \tag{25}$$

The value of $E_{\min}$ is typically selected as 1×10^{-3}. It is shown that this value should be decreased with the element number increment. u_m is equal to the place of the first pattern null of an equally spaced array with the same number of elements q.

As mentioned before, the proposed method in this paper is more accurate vs. similar existing methods [7]. It is noted that the right hand side of (7) is an estimation of $f(\alpha,\beta)$ function defined in (8). Using the $a'_m(\alpha)$ coefficients and also the Legendre coefficients $p_{m-0.5}(\cos\alpha)$ [7], instead of $a_m(\alpha)$ in the right hand side of (7), two approximate function can be obtained to estimate the function $f(\alpha,\beta)$ defined in (8). In Fig. 2, these two approximate functions and $f(\alpha,\beta)$ in (8) are plotted for M = 107 and α = 1. As can be seen, the $a'_m(\alpha)$ coefficients can better estimate $f(\alpha,\beta)$ than the Legendre coefficients. Another advantage of this method is its speediness since the $a'_m(\alpha)$ coefficients can be determined faster than $p_{m-0.5}(\cos\alpha)$. The reason is that in [7], it is needed to calculate the integral operation for M times to get the values of the Legendre coefficients:

$$p_{m-0.5}(\cos\alpha) = \frac{1}{\pi}\int_0^{\alpha} \frac{2}{(\cos(\beta)-\cos(\alpha))^{1/2}}\mathrm{d}\beta \qquad (26)$$

but in our proposed algorithm, it is needed just to obtain the reverse of matrix **B** which is an $M \times M$ matrix. This speediness makes the proposed approach more suitable for arrays with large number of elements. For example, the comparison of measured simulation time in both cases, shows that $a'_m(\alpha)$ coefficients can be determined 37 times faster than $p_{m-0.5}(\cos\alpha)$ for M = 107 and α = 1.

To validate the proposed synthesis method, a linear unequally spaced array of 39 elements was evaluated. In its pattern, shown in Fig. 3, the SLL has been decreased by 8.15 dB in comparison with a uniform array with the same number of elements. In addition, this value was improved by 1.75 dB while compared to the one obtained by the method described in [7]. Also as can be seen, the beamwidth is narrower than the case with uniform space. The 3-dB beamwidth of the uniform and non-uniform arrays are 2.52° and 2.25° respectively. In this algorithm, the values of Δ and M were set to 0.33λ and 107, respectively. The obtained elements positions are reported in Tab. 1. As can be inferred from (23), in order to obtain the value of element positions ($d_0, d_1,\ldots, d_N$), we need to calculate the value of $F(\alpha_p)$ function with using (9) and then calculating the values of $a_m(\alpha)$. As mentioned previously, since the $a'_m(\alpha)$ coefficients can be determined faster than $p_{m-0.5}(\cos\alpha)$, therefore, using $a'_m(\alpha)$ instead of $p_{m-0.5}(\cos\alpha)$ makes our proposed method faster than the Legendre method. For the linear unequally spaced array of 39 elements discussed above, with $\Delta = 0.33\lambda$ and M = 107, the measured simulation time shows that for this case, the simulation speed of our proposed method is 79 time faster in comparison with the Legendre method. Table 2 provided information about the simulation time of the proposed method and Legendre method, respectively, for different number of elements. In this case, the values of Δ and M were considered to be 0.4λ and 120. According to the data shown in Tab. 2, it can be deduced, with increasing the number of elements the simulation time of the proposed method increases with a slower rate in comparison with Legendre method. As a result, the proposed method is an appropriate analytical method for synthesis of large unequally spaced array.

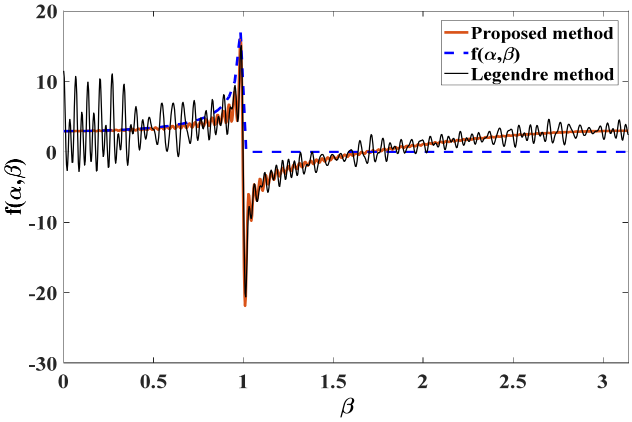

Fig. 2. Estimation of $f(\alpha,\beta)$ function using the proposed method and Legendre method [7].

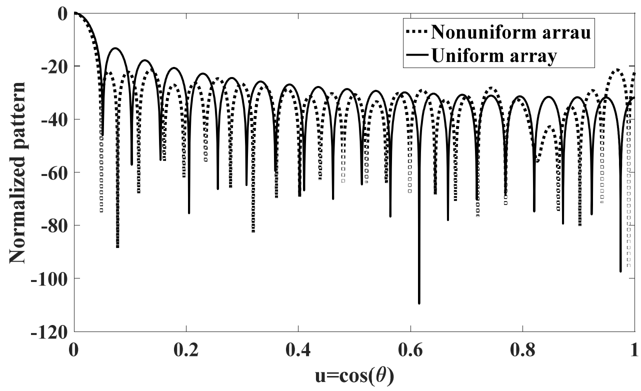

Fig. 3. Synthesized pattern of a 39 elements nonuniform array.

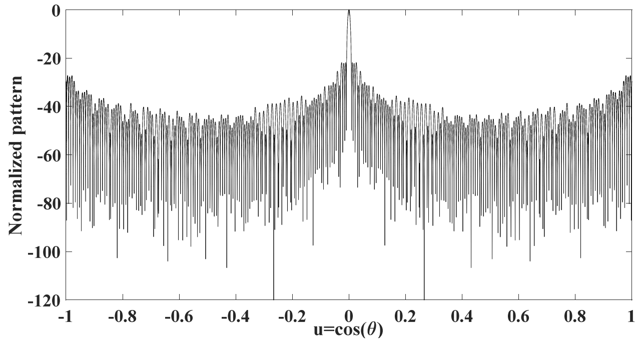

Fig. 4. Synthesized pattern of a 200 elements nonuniform array.

Element number	Element position	Element number	Element position	Element number	Element position
p	d_p/λ	p	d_p/λ	p	d_p/λ
1	0.5	8	4.00	15	8.63
2	1.0	9	4.52	16	9.46
3	1.5	10	5.12	17	10.41
4	2.0	11	5.71	18	11.32
5	2.5	12	6.40	19	12.12
6	3.0	13	7.12		
7	3.5	14	7.87		

Tab. 1. Element position for 39 elements array.

2*N*+1 element array	**Simulation time (seconds)**	
N	**Proposed Method**	**Legendre Method**
9	19.5	1052
14	50.2	3295.7
19	86.5	6644.5
24	139.0	11837

Tab. 2. Simulation time of the proposed method and Legendre method for different number of array elements.

Element number	**Element position**	**Element number**	**Element position**	**Element number**	**Element position**
p	d_p / λ	**p**	d_p / λ	**p**	d_p / λ
1	0.2500	35	17.6657	69	39.2361
2	0.7500	36	18.1657	70	39.9993
3	1.3000	37	18.6657	71	40.7753
4	1.9516	38	19.1657	72	41.5557
5	2.5571	39	19.7145	73	42.3460
6	3.1529	40	20.2219	74	43.1430
7	3.6657	41	20.7825	75	43.9469
8	4.1657	42	21.3521	76	44.7617
9	4.6657	43	21.9279	77	45.5825
10	5.1657	44	22.5179	78	46.4118
11	5.6657	45	23.1082	79	47.2480
12	6.1657	46	23.7168	80	48.0910
13	6.6657	47	24.2958	81	48.8706
14	7.1657	48	24.9018	82	49.7688
15	7.6657	49	25.5218	83	50.6699
16	8.1657	50	26.1451	84	51.5754
17	8.6657	51	26.7853	85	52.4859
18	9.1657	52	27.4052	86	53.4023
19	9.6657	53	28.0433	87	54.3240
20	10.1657	54	28.6978	88	55.2507
21	10.6657	55	29.3386	89	56.1823
22	11.1657	56	29.9970	90	57.1186
23	11.6657	57	30.6702	91	58.0594
24	12.1657	58	31.3342	92	59.0043
25	12.6657	59	32.0142	93	59.9532
26	13.1657	60	32.7085	94	60.9062
27	13.6657	61	33.3974	95	61.8625
28	14.1657	62	34.1000	96	62.8219
29	14.6657	63	34.8174	97	63.7842
30	15.1657	64	35.5323	98	64.7491
31	15.6657	65	36.2583	99	65.7165
32	16.1657	66	36.9881	100	66.6862
33	16.6657	67	37.7323		
34	17.1657	68	38.4785		

Tab. 3. Element position for 200 elements array.

To further demonstrate the proposed approach, a larger array with 200 elements (even number) was studied. The values of Δ_0, Δ, and M are 0.06λ, 0.46λ, and 237, respectively. As shown in Fig. 4, the PSLL is equal to –21.9 dB, i.e., an improvement of 8.64 dB in comparison with a uniform array with the same number of elements. In addition, the 3-dB beamwidth is 0.4140° which has decreased in comparison with 0.5040° beamwidth of the uniform array. The synthesized elements positions are presented in Tab. 3. Also in this case, as can be expected, the simulation time has been decreased to 1/89 of its value in comparison with the method using $p_{m-0.5}(\cos\alpha)$ instead of $a_m(\alpha)$ coefficients.

4. Conclusion

In this paper, a fast and accurate analytic method for synthesizing of unequally spaced arrays with uniform amplitude distribution was proposed. It can be implemented in arrays with both even and odd number of elements. This approach allows obtaining a radiation pattern with a narrow beamwidth and lowest PSLL value. The simulation results show a significant reduction of simulation time. In fact, the obtained results demonstrated its strong ability to design low side lobe level phased array antennas with unequally spaced elements.

References

[1] ELLIOT, R. S. *Antenna Theory and Design*. Englewood Cliffs (NJ): Prentice-Hall, 1981.

[2] BALANIS, C. A. *Antenna Theory*. New York: Wiley, 1997.

[3] UNZ, H. Linear arrays with arbitrarily distributed elements. *IRE Transactions on Antennas and Propagation*, 1960, vol. AP-8, no. 2, p. 222–223. DOI: 10.1109/TAP.1960.1144829

[4] HARRINGTON, R. F. Sidelobe reduction by nonuniform element spacing. *IRE Transactions on Antennas and Propagation,* 1961, vol. 9, no. 2, p. 187–192. DOI: 10.1109/TAP.1961.1144961

[5] ISHIMARU, A. Theory of unequally-spaced arrays. *IRE Transactions on Antennas and Propagation,* 1962, vol. AP-11, no. 6, p. 691–702. DOI: 10.1109/TAP.1962.1137952

[6] KUMAR, B. P., BRANNER, G. R. Generalized analytical technique for the synthesis of unequally spaced arrays with linear, planar, cylindrical or spherical geometry. *IEEE Transactions on Antennas and Propagation*, 2005, vol. 53, no. 2, p. 621–634. DOI: 10.1109/TAP.2004.841324

[7] KUMAR, B. P., BRANNER, G. R. Design of unequally spaced arrays for performance improvement. *IEEE Transactions on Antennas and Propagation,* 1999, vol. 47, no. 3, p. 511–523. DOI: 10.1109/8.768787

[8] JIN, N., RAHMAT-SAMII, Y. Advances in particle swarm optimization for antenna designs: Real-number, binary, single-objective and multi-objective implementations. *IEEE Transactions on Antennas and Propagation,* 2007, vol. 55, no. 3, p. 556–567. DOI: 10.1109/TAP.2007.891552

[9] KURUP, D. G., HIMDI, M., RYDBERG, A. Synthesis of uniform amplitude unequally spaced antenna arrays using the differential evolution algorithm. *IEEE Transactions on Antennas and Propagation,* 2003, vol. 51, no. 9, p. 2210–2217. DOI: 10.1109/TAP.2003.816361

[10] CHEN, K., HE, Z., HAN, C. A modified real GA for the sparse linear array synthesis with multiple constraints. *IEEE Transactions on Antennas and Propagation,* 2006, vol. 54, no. 7, p. 2169–2173. DOI: 10.1109/TAP.2006.877211

[11] DE LUCCIA, C. S., WERNER, D. H. Nature-based design of aperiodic linear arrays with broadband elements using a combination of rapid neural-network estimation techniques and genetic algorithms. *IEEE Antennas and Propagation Magazine,* 2007, vol. 49, no. 5, p. 13–23. DOI: 10.1109/MAP.2007.4395292

[12] DONELLI, M., CAORSI, S., DE NATALE, F., et al. Linear antenna synthesis with a hybrid genetic algorithm. *Progress in Electromagnetic Research PIER*, 2004, vol. 49, p. 1–22. DOI: 10.2528/PIER03121301

[13] MURINO, V., TRUCCO, A., REGAZZONI, C. S. Synthesis of unequally spaced arrays by simulated annealing. *IEEE Transactions on Signal Processing*, 2006, vol. 44, no. 1, p. 119–122. DOI: 10.1109/78.482017

[14] CAORSI, S., LOMMI, A., MASSA, A., et al. Peak side lobe level reduction with a hybrid approach based on GAs and difference sets. *IEEE Transactions on Antennas and Propagation,* 2004, vol. 52, no. 4, p. 1116–1121. DOI: 10.1109/TAP.2004.825689

[15] OLIVERI, G., MASSA, A. Bayesian compressive sampling for pattern synthesis with maximally sparse non-uniform linear arrays. *IEEE Transactions on Antennas and Propagation,* 2011, vol. 59, no. 2, p. 467–481. DOI: 10.1109/TAP.2010.2096400

[16] WANG, W. B., FENG, Q., LIU, D. Application of chaotic particle swarm optimization algorithm to pattern synthesis of antenna arrays. *Progress in Electromagnetic Research PIER*, 2011, vol. 115, p. 173–189. DOI: 10.2528/PIER11012305

[17] RUPCIC, S., MANDRIC, V., ZAGAR, D. Reduction of side lobes by non-uniform elements spacing of a spherical antenna array. *Radioengineering,* 2011, vol. 20, no. 1, p. 299–306.

[18] LIU, Y., NIE, Z. P., LIU, Q. H. A new method for the synthesis of non-uniform linear arrays with shaped power patterns. *Progress in Electromagnetic Research PIER*, 2010, vol. 107, p. 349–363. DOI: 10.2528/PIER10060912

[19] YANG, K., ZHAO, Z., LIU, Q. H. Fast pencil beam pattern synthesis of large unequally spaced antenna arrays. *IEEE Transactions on Antennas and Propagation,* 2013, vol. 61, no. 2, p. 627–634. DOI: 10.1109/TAP.2012.2220319

[20] GOUDOS, S. K., MOYSIADOU, V., SAMARAS, T., et al. Application of a comprehensive learning particle swarm optimizer to unequally spaced linear array synthesis with sidelobe level suppression and null control. *IEEE Antennas and Wireless Propagation Letters*, 2010, vol. 9, p. 125–129. DOI: 10.1109/LAWP.2010.2044552