Optimal Energy-Efficient Power Allocation Scheme with Low Complexity for Distributed Antenna System

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Abstract. In this paper, by maximizing the energy efficiency (EE), an optimal power allocation scheme is developed for downlink distributed antenna system (DAS). Different from conventional optimal power allocation schemes that need iterative calculation, the developed scheme can provide closed-form power allocation and no iteration is required. Based on the definition of EE, the optimized objective function is firstly formulated, and then a computationally efficient algorithm is proposed to obtain the optimal number of active remote antennas and the corresponding power allocation. Using the optimal number, the multidimensional solution for the optimized function is transformed into searching one-dimensional solution. As a result, closed-form expression of power allocation coefficients is attained. Numerical results verify the effectiveness of the proposed scheme. The scheme can obtain the same EE as the conventional optimal scheme but with lower complexity, and it has more accuracy than the existing low-complexity scheme.

Keywords
Energy efficiency, distributed antenna system, power allocation, low complexity, Rayleigh channel

1. Introduction

Recently, green communication, where energy efficiency (EE) is pursued [1–3], has received much attention, since energy demand and prices increase dramatically. As an important technology for green communication, distributed antenna system (DAS) can expand cell coverage and improve the EE [4–6]. For this reason, different energy-efficient power allocation (PA) schemes have been developed for DAS [7–11]. In [7], using iterative search and numerical calculation, an approximate PA scheme is proposed for generalized DAS. An adaptive PA scheme for achieving maximum EE while satisfying spectral efficiency requirement in multiuser DAS is developed in [8]. In [9], by maximizing the upper bound of average EE, a suboptimal energy-efficient PA scheme is presented for DAS over composite fading channel. An optimal PA algorithm to maximize EE is proposed in [10], where the optimization problem is solved by using the Karush-Kuhn-Tucker (KKT) conditions and Lambert function. Considering that the above schemes need iterations, a low-complexity PA scheme aiming at maximizing EE is presented in [11], but there exist some errors in the theoretical derivations on PA. Hence, the accuracy of the optimal PA cannot be guaranteed. Besides, the above schemes basically consider single receive antenna for analysis convenience, and thus the corresponding EE performance will be limited.

Motivated by the reasons above, we propose a low-complexity energy-efficient power allocation scheme for DAS in composite Rayleigh channel, where path-loss, shadowing, fading, and multiple receive antennas are all considered. Also we present a new computationally efficient algorithm to achieve the optimal number of active remote antennas and the corresponding closed-form power allocation coefficients. It is shown that the proposed power allocation scheme and algorithm are both valid, the power allocation scheme can obtain the same energy efficiency as the conventional optimal one [10], and the algorithm has more accuracy than the existing low-complexity algorithm [11]. Furthermore, no iteration is required.

The rest of this paper is organized as follows. Section 2 introduces the system model. In Section 3, a low-complexity power allocation scheme is developed, and the comparison between different PA schemes is presented. Simulation results are provided in Sec. 4, and Section 5 concludes the paper.

The notations throughout this paper are as follows. Bold lower case letters denote column vectors. The superscript (·)T denotes transposition, E{·} denotes statistical expectation, W(·) denotes Lambert W function.

2. System Model

We consider a downlink DAS with Nt remote antennas (RAs) and multiple receive antennas operating in
3. Low-complexity Power Allocation Scheme

In this section, we will develop an optimal power allocation scheme with low complexity, and present a computationally efficient algorithm to obtain the optimal number of active remote antennas and the corresponding PA coefficients.

Subject to the power constraint, we firstly formulate the objective function of the optimal PA as

$$\max_{\mathbf{P}} \eta_{EE} = \frac{\log_2 (1 + \sum_{i=1}^{N_r} \gamma_i P_i)}{\sum_{i=1}^{N_r} P_i + P_c}$$

subject to $0 \leq P_i \leq P_{\text{max},i}$, $\forall i \in \{1, \ldots, N_r\}$

where $\mathbf{P} = [P_1, \ldots, P_{N_r}]^T$. Without loss of generality, we assume that $\gamma_1 \geq \gamma_2 \geq \ldots \geq \gamma_{N_r}$. It has been proved in [10] that the optimal PA solution will have the following general form:

$$y_i = \sqrt{P_i [h_i^1, \ldots, h_i^N]^T} x_i + z$$

where $P_{\text{in}}$, $i = 1, \ldots, N_r$, is the transmit power consumed by the RA, $x_i$ stands for the transmitted symbol from RA, with unit energy, $z$ is the complex Gaussian noise vector with zero mean and variance $\sigma_n^2$, and $h_i^j = g_i^j \Omega$, $j = 1, \ldots, N_r$, is the composite channel fading coefficient between RA and the $j$-th antenna of MS, where $g_i^j$ denotes the small-scale Rayleigh fading between RA and the $j$-th receive antenna, $\Omega$ is the path loss, $d_i$ is the distance from RA to the MS, and $S_i$ denotes the shadowing effect.

In general, EE is defined as the ratio of data transmission rate to the total consumed power. Based on this, the EE of DAS can be written as

$$\eta_{EE} = \frac{\log_2 (1 + \sum_{i=1}^{N_r} \gamma_i P_i)}{\sum_{i=1}^{N_r} P_i + P_c}$$

where $\gamma_i = \sum_{j=1}^{N_r} |g_i^j|^2 \Omega_i^2 / \sigma_n^2$ is defined as the channel to noise ratio (CNR) after maximal ratio combining at the receiver. Per RA power constraint is $0 \leq P_i \leq P_{\text{max},i}$, where $P_{\text{max},i}$ is the maximum transmit power available at RA, $P_c$ denotes the circuit power and is a constant.

According to this, the optimization of PA in (3) will become to search the optimal number of active RAs (i.e., $N_0$) and the corresponding PA. The optimized problem in (3) can be converted into finding the optimal solution of the following segmented function:

$$\psi(k, \mathbf{P}_k) = V_k (P_k) \quad , \quad k = 1, \ldots, N_r$$

where $\mathbf{P}_k = [P_{\text{max},1}, \ldots, P_{\text{max},k-1}, P_{\text{max},k}, 0, \ldots, 0]^T$. The optimal PA solution will have the following general form:

$$V_k (P_k) = \frac{\log_2 (1 + \sum_{i=1}^{k} \gamma_i P_{\text{max},i} + \gamma_k P_k)}{\sum_{i=1}^{k} P_{\text{max},i} + P_k + P_c}$$

Obviously, (5) is a continuous function. Taking the derivative of $V_k (P_k)$ with respect to $P_k$ yields

$$V_k' (P_k) = \frac{\gamma_k \Phi_k - (1 + \Psi_k) \ln (1 + \Psi_k)}{\Psi_k \Phi_k \ln 2} = A_k (P_k) / B_k (P_k)$$

where $\Phi_k = \sum_{i=1}^{k} P_{\text{max},i} + P_k + P_c$, $\Psi_k = \sum_{i=1}^{k} \gamma_i P_{\text{max},i} + \gamma_k P_k$.

Since $B_k (P_k)$ is positive, the sign of $V_k' (P_k)$ is determined by $A_k (P_k)$ only.

Taking the derivative of $A_k (P_k)$ with respect to $P_k$ gives

$$A_k' (P_k) = -\gamma_k \ln \left(1 + \sum_{i=1}^{k} \gamma_i P_{\text{max},i} + \gamma_k P_k\right) < 0.$$  

It means that $A_k (P_k)$ is decreasing in any segment. With (7), we have:

$$A_k (P_{\text{max},k}) = \gamma_k \left( \sum_{i=1}^{k} P_{\text{max},i} + P_k \right) \left(1 + \sum_{i=1}^{k} \gamma_i P_{\text{max},i}\right) \ln \left(1 + \sum_{i=1}^{k} \gamma_i P_{\text{max},i}\right)$$

Fig. 1. A circle-cell DAS structure.
\[ A_{\gamma_{k+1}}(0) = \gamma_{k+1} \left[ \sum_{i=1}^{k} P_{\text{max},j} + P_{i} \right] - \left[ 1 + \sum_{i=1}^{k} \gamma_{i} P_{\text{max},j} \right] \ln \left[ 1 + \sum_{i=1}^{k} \gamma_{i} P_{\text{max},j} \right]. \tag{10} \]

Because \( \gamma_{k} > \gamma_{k+1} \), \( A_{\gamma_{k+1}}(P_{\text{max},j}) > A_{\gamma_{k}}(0) \). Based on this, considering that \( A_{\gamma_{k}}(P_{i}) \) is decreasing function in a segment, the \( A_{\gamma_{k}}(P_{i}) \) in different segments has the following relation:

\[ \forall P_{i}, P_{\gamma_{k+1}}, A_{\gamma_{k}}(P_{i}) > A_{\gamma_{k}}(P_{\gamma_{k+1}}). \tag{11} \]

It means that \( A_{\gamma_{k}}(P_{i}) \) is always greater than \( A_{\gamma_{k}}(P_{\gamma_{k+1}}) \). According to the analysis above, the optimal PA, \( P_{\gamma_{k+1}}^{*} \), will be achieved.

Specifically:

If \( A_{\gamma_{k}}(P_{\text{max},j}) > 0 \) and \( A_{\gamma_{k+1}}(0) < 0 \) hold, then we have \( V_{\gamma_{k}}(P_{\text{max},j}) > 0 \) and \( V_{\gamma_{k+1}}(0) < 0 \). From (11), it is easily found that \( V_{\gamma_{k+1}}(P_{\text{max},j}) \) is decreasing function for \( 0 < P_{\gamma_{k+1}} < P_{\gamma_{k}}, \) and \( V_{\gamma_{k}}(P_{\text{max},j}) \) is increasing function for \( 0 < P_{\gamma_{k}} < P_{\gamma_{k+1}} \). Moreover, (5) is a continuous function. Hence, \( P_{\gamma_{k}}^{*} \) in (4) equals \( P_{\gamma_{k+1}}^{*} \) under this case.

If \( \exists 0 < P_{\gamma_{k}}^{*} < P_{\gamma_{k+1}}, A_{\gamma_{k}}(P_{\gamma_{k}}^{*}) = 0 \), then with (11), for \( k < N_{0}, A_{\gamma_{k}}(P_{i}) > A_{\gamma_{k}}(P_{\gamma_{k+1}}^{*}) = 0, \) so \( V_{\gamma_{k}}(P_{i}) > 0 \) for \( k < N_{0} \). Similarly, \( V_{\gamma_{k}}(P_{i}) < 0 \) for \( k > N_{0} \). According to (8), and considering \( V_{\gamma_{k}}(P_{\gamma_{k+1}}^{*}) = A_{\gamma_{k}}(P_{\gamma_{k+1}}^{*}) = 0 \), we have:

\[ V_{\gamma_{k}}(P_{i}) > 0 \text{ for } 0 < P_{\gamma_{k}} < P_{\gamma_{k+1}}, \text{ and } V_{\gamma_{k}}(P_{i}) < 0 \text{ for } P_{\gamma_{k+1}} < P_{\gamma_{k}} < P_{\gamma_{k+1}}, \text{ Hence } A_{\gamma_{k}}(P_{i}) = 0 \text{ has sole solution, i.e., } P_{\gamma_{k}}^{*}. \]

From (7), \( A_{\gamma_{k}}(P_{i}) \) can be written as:

\[ A_{\gamma_{k}}(P_{i}) = \gamma_{k} \left( \sum_{j=1}^{N_{k}-1} P_{\text{max},j} + P_{i} \right) - \left( 1 + \sum_{j=1}^{N_{k}-1} \gamma_{j} P_{\text{max},j} \right) \ln \left( 1 + \sum_{j=1}^{N_{k}-1} \gamma_{j} P_{\text{max},j} \right). \tag{12} \]

Let \( \phi_{1} = \gamma_{k} \left( \sum_{j=1}^{N_{k}-1} P_{\text{max},j} + P_{i} \right), \) \( \phi_{2} = 1 + \sum_{j=1}^{N_{k}-1} \gamma_{j} P_{\text{max},j}, \) then (12) is changed to:

\[ A_{\gamma_{k}}(P_{i}) = \phi_{1} + \gamma_{k} P_{i} - (\phi_{2} + \gamma_{k} P_{i}) \ln(\phi_{1} + \gamma_{k} P_{i}) \tag{13} \]

Equating \( A_{\gamma_{k}}(P_{i}) \) to zero gives:

\[ \phi_{1} - \phi_{2} = (\phi_{2} + \gamma_{k} P_{i}) \ln(\phi_{1} + \gamma_{k} P_{i}) - 1. \tag{14} \]

With (14), we have:

\[ \frac{\phi_{1} - \phi_{2}}{e} = \frac{\phi_{2} + \gamma_{k} P_{i} \ln(\phi_{1} + \gamma_{k} P_{i})}{e}. \tag{15} \]

Using (15) and the Lambert function [12], we can obtain the solution of \( P_{i} \), i.e., \( P_{i}^{*} \), as follows:

\[ P_{i}^{*} = \frac{1}{\gamma_{k}} \exp \left[ W\left( \frac{\phi_{1} - \phi_{2}}{e} + 1 \right) - \frac{\phi_{2}}{e} \right]. \tag{16} \]

where \( W(\cdot) \) denotes the Lambert function. Equation (16) provides a closed-form expression of \( P_{i}^{*} \) in (4). Based on the analysis above, a low-complexity optimal PA algorithm is proposed, and correspondingly, the algorithm procedure is summarized as follows:

**Algorithm 1** Efficient PA calculation method

1: Sort the \( \gamma_{i} \) with descending order.

2: Calculating the \( V_{k}(0) \) for \( k = 1, \ldots, N_{0} \), and finding \( K \) that makes \( V_{K}(0) \) be the largest among \( \{V_{k}(0)\} \).

3: If \( V_{K}(0) > 0 \) & \( V_{K-1}(P_{\text{max},K-1}) > 0 \), then Return \( N_{0} = K \), go to 4.

   Else

   If \( V_{K}(0) < 0 \) & \( V_{K-1}(P_{\text{max},K-1}) < 0 \) then

   Return \( N_{0} = K - 1 \), go to 4.

   Else

   Return \( N_{0} = K - 1, P_{N_{0}} = P_{\text{max},N_{0}} \), go to 5.

4: Using (16) to obtain the \( P_{i}^{*} \).

5: Return \( P^{*} \) in (4), end the algorithm.

Using the above algorithm, the optimal number of active RAs is determined correctly, and the multi-dimensional root finding for the optimized objective function becomes one-dimensional root finding. As a result, closed-form power allocation is computed and no iteration is required. For this algorithm, the number of Lambert \( W \) function is calculated by one or zero. Note that in [11], a low-complexity PA scheme is also proposed for maximizing the energy efficiency in DAS, but the scheme cannot always guarantee the optimal values since there are errors in [11, (9)] and [11, (11)], i.e., the derived derivative \( V_{i}(P_{i}) \) and optimal PA \( P_{i}^{*} \) are not accurate so that the obtained power value may exceed the given maximum transmit power. For example, using the CNRs as \( \{\gamma_{i}\}_{i=1,\ldots,7} = \{6.3608e+05, 4.8028e+05, 1.6016e+05, 1.3177e+05, 6.6877e+04, 6.4877e+03, 1.2921e+03\} \), we plot the corresponding numerical results of the energy efficiency in Fig. 2.

As shown in Fig. 2, the proposed scheme can be in good agreement with the conventional optimal scheme [10], but the existing low-complexity scheme [11] has obvious difference because the derived (9) and (11) in [11] have errors, which result in inaccurate power allocation.
posed scheme via computer simulation. In simulation, it is shown that the proposed scheme has lower calculation complexity than the other two and is more accurate than the existing scheme [11].

The above results indicate that the proposed scheme is valid. Besides, the complexity analysis measured by the number of Lambert function calculation and comparison among three schemes are listed in Tab. 1. From Tab. 1, it is shown that the proposed scheme has lower calculation complexity than the other two and is more accurate than the existing scheme [11].

### 4. Simulation Results

In this section, we will assess the validity of the proposed scheme via computer simulation. In simulation, it is assumed that \( P_{\text{max},i} = P_{\text{max}} (i = 0, \ldots, N_r) \) for analysis convenience. The BS (RA1) is in the center of the cell and the RAs are evenly and symmetrically placed in the cell. The main parameters used in simulations are listed in Tab. 2. The simulation results are shown in Fig. 3 and Fig. 4.

Figure 3 illustrates the EE performance of DAS with different receive antennas. Considering that the optimal scheme in [10] is based on single receive antenna, we extend the scheme in [10] to multiple receive antennas case after some modifications. From Fig. 3, it is observed that the EE of the proposed scheme is the same as that of the conventional optimal scheme [10]. Moreover, the EE increases with the growth of the number of receive antennas. Namely, the EE of DAS with three receive antennas \((N_r=3)\) is higher than that with two receive antennas \((N_r=2)\) because of more spatial diversity gain, and due to the same reason, the EE of DAS with two receive antennas \((N_r=1)\) (with single receive antenna) is higher than that with single receive antenna \((N_r=1)\). The results above indicate that the developed scheme is effective and reasonable, and the application of multiple receive antennas does improve the EE performance effectively.

In Fig. 4 we plot the EE performance of DAS with different path loss exponents, where two receive antennas are considered. It is found that the EE of the proposed scheme is still consistent with that of conventional optimal

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**Fig. 2.** EE comparison of DAS with different PA schemes.

**Fig. 3.** EE of DAS with different receive antennas.

**Fig. 4.** EE of DAS with different path loss exponents.

**Fig. 5.** EE of DAS with different shadow fading standard deviations.

**Fig. 6.** EE of DAS with different circuit powers.

**Fig. 7.** EE of DAS with different noise powers.

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**Tab. 1.** Comparison of different schemes.

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<td>( N_r )</td>
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<td>inaccurate</td>
<td>accurate</td>
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<tr>
<td>( P_{\text{max}} )</td>
<td>accurate</td>
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**Tab. 2.** Simulation parameters.

<table>
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<td>Number of remote antennas ( N_t )</td>
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</tr>
<tr>
<td>Path loss exponent ( \alpha )</td>
<td>3, 3.5, 4</td>
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<tr>
<td>Shadow fading standard deviation ( \sigma_0 )</td>
<td>8 dB</td>
</tr>
<tr>
<td>Noise power ( N_0 )</td>
<td>(-104 \text{ dBm})</td>
</tr>
<tr>
<td>Circuit power ( P_c )</td>
<td>10 W</td>
</tr>
<tr>
<td>Cell radius ( D_r )</td>
<td>1000 m</td>
</tr>
<tr>
<td>MS distribution</td>
<td>Uniform</td>
</tr>
<tr>
<td>((D_r, \tau), i = 1, 2, \ldots, N_r)</td>
<td>(</td>
</tr>
<tr>
<td>Number of receive antennas ( N_r )</td>
<td>1, 2, 3</td>
</tr>
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scheme [10]. Besides, the EE decreases with the growth of the path loss exponent $\alpha$ as expected. Namely, the EE of DAS with $\alpha = 4$ is lower than that with $\alpha = 3.5$, and the EE of DAS with $\alpha = 3.5$ is lower than that with $\alpha = 3$. This is because the system with higher path loss exponent will experience greater path loss, which leads to a decline in EE performance.

5. Conclusions

We have developed an optimal energy-efficient power allocation scheme for DAS using multiple receive antennas in composite Rayleigh fading channel, and proposed a computationally efficient algorithm to obtain the optimal number of active RAs. Based on this, optimal power allocation is derived, and the corresponding closed-form expression is attained. The results indicate that our scheme has lower complexity than the existing schemes. Moreover, it can obtain the same EE performance as the existing optimal scheme, and provide the optimal active antennas in composite Rayleigh fading channel, and proposed a power allocation scheme for DAS using multiple receive antennas in composite Rayleigh fading channel, and proposed a computationally efficient algorithm to obtain the optimal number of active RAs. Based on this, optimal power allocation is derived, and the corresponding closed-form expression is attained. The results indicate that our scheme has lower complexity than the existing schemes. Moreover, it can obtain the same EE performance as the existing optimal scheme, and provide the optimal active antenna number and power allocation accurately.

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References


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