An Optimal Analysis in Wireless Powered Full-duplex Relaying Network

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Abstract. Wireless-powered cellular networks (WPCNs) are currently being investigated to exploit the reliability and improve battery lifetime of mobile users. This paper investigates the energy harvesting structure of the full-duplex relaying networks. By using the time switching based relaying (TSR) protocol and Amplify-and-Forward (AF) model in delay-limited transmission scheme, we propose the closed-form expression of the outage probability and then calculate the optimal throughput. An important result can be taken obviously that the time fraction in TSR, the position of relay, the noise as well as the energy conversation impacting on the outage probability as well as the optimal throughput. By Monte Carlo simulation, the numerical results indicate an effective relaying strategy in full-duplex cooperative systems. Finally, we provide fundamental design guidelines for selecting time fraction in TSR that satisfies the requirements of a practical relaying system.

Keywords

Energy harvesting, full-duplex, one way relaying network, time switching-based protocol, throughput, amplify-and-forward

1. Introduction

The use of the regenerated energy in the next generation wireless systems such as 5G cellular network will bring about the considerable profits for the continuous operation in the wireless devices. Together with the other power resources such as solar, wind energy, the wireless power which are drawn from the radio-frequency (RF) signals departed by the surrounding transmitters can be replaced for the normal wired energy networks. Such energy harvested from the natural environment is a promising approach to maintain the life time of the energy constrained wireless nodes [1].

The authors in [2] considered simultaneous wireless power and information transfer (SWPIT) which is applied TS-based two-way relaying (TS-TWR) protocol. They investigated an optimization problem under total power constraint to examine the system throughput limit with data rate fairness.

In such that cases, a view of SWPIT networks with a special concentration on the hardware performance of the rectifying circuits and the real technics which get the SWPIT in the areas of the time, the space, the energy, the antennas as well as the profits of a potentiality synthesis of the SWPIT techniques in advanced information systems in the context of the source distributed and cooperative knowledge wireless systems are presented about in [3–5]. Moreover, a cognitive wireless network with one primary user (PU) and one secondary user (SU) and their transmitters run in TSR is considered. The SU, which collects the power solely from the surrounding wireless information, pursues a save-then-transmit protocol. In that way, a scenario, the SUs maximal cooperation strategic method, called, the maximal decision (in order to decide to work with the PU or not) and the maximal act (to consume how much time on the power collection and to distribute how much energy for the cooperative relay) is studied by the previous works as in [6–10].

On the other hand, an amplify-and-forward (AF) relaying system, in that, the power constrained relay node which collected the power from the receiving RF signal, utilized that collected power to transmit the resource communication to the target is studied. Relyed on the time switching and power splitting receiving structures, two relaying protocols i) the time switching-based relaying (TSR) protocol and ii) the power splitting-based relaying (PSR) protocol are suggested, offer collecting the power and support the communication link to the relay. Pressed by model based on the canonical cognitive radio (CR) system form, an unusual way of the radio systems existing at the same time in which the short-energy proceeded in a second system, named, the sec-
ondary transmitters (STs), collects the peripheral RF power from transmittances by the operation transceiver near around a main system, named, primary transmitters (PTs), whereas opportuneuly penetrating the spectrum authorized to the first system is suggested. A random-geometry pattern in that PTs and STs are allocated as the independence homogeneous Poisson point processes (HPPPs) and contacted with their planned transceivers at the unchanged spaces is examined in [11–17].

In fact, the authors in [18] indicated that the full-duplex mode is an attractive choice for the fixed relays provided that the loop interference power is maintained at a tolerable level. In addition, the authors in [19] presented the power allocation strategy to maximize the sum-rate of FD-two-way relaying system under the realistic residual self-interference (RSI).

Recently, a few research trends have been conducted in FD relay system in context of the SWIPT scheme. In [20], the throughput is analyzed for three relay control schemes, including the maximum relay, optimal relay, and target relay. Analytical expressions for the outage probability and ergodic capacity are also presented for these considered relay control schemes. However, no work related to the optimal throughput has considered for the application of the one-way FD relaying in RF energy harvesting systems.

Therefore, in this paper, we analyze the outage probability and the throughput of full-duplex relaying with the novel ability of the simultaneous energy harvesting and information transfer. Based on this analytical expressions, the best model for optimal energy efficiency are given.

Our contribution in this paper is to find out the optimal time fraction $\alpha_{\text{max}}$ value to obtain maximum signal-to-noise ratio (SNR), from this result, we can give an optimal model for the energy harvesting full-duplex relaying network and prove that model satisfies for the optimal deployment in future cellular networks.

The remainder of this paper is organized as follows. Section 2 describes the system model of the EH enabled FD one-way relaying network. In Sec. 3, the outage probability and throughput in delay-limited transmission mode are analyzed. The simulation results are presented in Sec. 4. Finally, the conclusion of Sec. 5 is drawn in this paper.

2. System Model

Let us consider a wireless full-duplex relaying network with the AF protocols system illustrated as in Fig. 1, in which the destination node D can be received signal at long distance thanks to help of relay node. The system consists of three nodes, the resource node S is denoted by S and one relay node is denoted by R. The relay node has two antennas, one of them is responsible for signal transmission and the other is responsible for the signal reception. The cooperative relay is assumed to be an energy constrained device so that it must harvest energy from the source, and then use harvested energy to amplify and forward the source information to the destination node. We also assume that the direct link between source and destination node does not exist due to the deep shadowing effect or too long distance of direct link. We denote $h$, $g$, $f$ as channel at the first hop, the second hop and self-interference channel.

In this system model, the interference cancellation structure is adapted to eliminate the self-interference. Due to limit of circuits, the self-interference is unable to be cancelled entirely, a certain amount of its remains. The residual self-interference channel at R is defined by $f$. Let $d_{j}$, $j = 1, 2$ defined the gap of S–R link and R–D link, respectively and $h$, $g$ defined the channel coefficients of S–R link and R–D link, respectively. The model used in this study is the TSR protocol. The key parameters of protocol are depicted in Fig. 2.

Based on the TSR protocol, the information process is made into two stages. In the first stage, the energy is transferred from the source to the relay with a duration of $\alpha T$, $(0 < \alpha < 1)$ and in the second stage, the remaining time, $(1 - \alpha) T$ is employed to transfer information, in which $\alpha$ is the time switching coefficient and $T$ is the time for the examined signal frame. In the energy harvesting phase, the received signal at the relay can be given by

$$y_R = \frac{P_S}{d_1^m} h x_S + n_R$$

where $P_S$ is the transmitted power from the source, $x_S$ is transmit signal, $n_R$ is the additive white Gaussian noise with zero-mean and variance of $\sigma_n^2$. Regarding the wireless received power, the harvested energy at the relay is given

$$E_h = \Omega \alpha T \frac{P_S|h|^2}{d_1^m}$$

where $\Omega$ is energy conversion efficiency. In the information transfer stage, assuming that the source node transmits the respective signal $x_S$ to R and R retransmits signal $x_R$ to the destination node, $x_j$, $j = S, R$. It is assumed that the unit energy and zero mean, i.e, $E[|x_j|^2] = 1$ and $E[x_j] = 0$, in which $E[.]$ is expectation function. Therefore, the received
signal at the relay under the self-interference constraint is given as
\[ y_R = \frac{P_5}{d_1^m} \alpha h + f x_R + n_R \] (3)
where \( x_R \) is the transmit signal at \( R \). We suppose \( R \) receives \( y_R \) and in the next time slot, \( R \) uses that harvested energy to information processing for received signal \( y_R \). Hence, using magnification of the prior received signal, \( x_R \), is illustrated as
\[ x_R = K \sqrt{P_R y_R} \] (4)
where \( K \) is the magnification coefficient of \( R \).

Based on principle of the AF relaying model at \( R \), the magnification factor is computed by
\[ K = \frac{P_5}{d_1^m} |h|^2 + P_R |f|^2 + \sigma_n^2 \] (5).

It is worth noting that the harvested power assists the operation for the next hop transfer, \( P_R \) is taken by
\[ P_R = \frac{E_R}{(1-\alpha)T} = \varphi P_S |h|^2 \] (6)
where \( \varphi \) is defined as \( \varphi = \frac{a_1}{\sigma} \).

Next, we get the received signal at destination as
\[ y_D = \frac{g}{\sqrt{d_2^m}} y_R + n_D \] (7)
where \( n_D \) is Gaussian noise at destination. Substituting (4), (5), (6) into (7), we compute the receiving signal at destination node as
\[ y_D = P_5 |h|^2 |g|^2 (|f|^2)^{-1} (d_1^m)^{-1} (d_2^m)^{-1} \]
\[ + P_5 |h|^2 (|f|^2)^{-1} (d_1^m)^{-1} \sigma_n^2 (P_R)^{-1} \]
\[ + P_5 |g|^2 (d_2^m)^{-1} + \sigma_n^2. \] (8)

In the above equations, the instantaneous signal-to-interference-and-noise (SINR) at destination node is determined as
\[ \gamma_D = \frac{E[|\text{signal}|^2]}{E[|\text{noise}|^2] + E[|\text{RSI}|^2]} \] (9)
By simple replacement, we get new formula as
\[ \gamma_D = \frac{\gamma_{SD} \alpha (1-\alpha)}{(1-\alpha)^2 + \Omega^2 \alpha^2 \gamma_{LYSD} + \sigma_n^2 \gamma_{LI}} (1-\alpha) \] (10)
with
\[ a_1 = \gamma_{SD} \Omega \]
\[ a_2 = 1 + \Omega^2 \gamma_{LYSD} - \sigma_n^2 \gamma_{LI} \]
\[ a_3 = 2 + \sigma_n^2 \gamma_{LI} \] (11)
and
\[ \gamma_{SD} = \frac{P_5 |h|^2 |g|^2}{d_1^m d_2^m \sigma_n^2} = \text{SINR} \frac{|h|^2 |g|^2}{d_1^m d_2^m} \] (12)
where \( \text{SINR} = \frac{P_5}{\sigma_n^2}, \gamma_{LI} = \frac{|f|^2}{\sigma_n^2} \).

We find \( \alpha \) with respecting maximal SNR, \( \text{SNR}_{\text{max}} \), subject to \( 0 < \alpha < 1 \), it is noted that \( \gamma_D(\alpha) \) is a concave function of the optimal value \( \alpha \) which can be obtained by resolving the equation
\[ \frac{\partial \gamma_D}{\partial \alpha} = 0 \] (13)
and after some manipulations, (13) results in following values
\[ a_1 = \frac{-1 + \sqrt{1 + a_2^2 + a_3^2}}{a_2 + a_3}, \]
\[ a_2 = \frac{-1 - \sqrt{1 + a_2^2 + a_3^2}}{a_2 + a_3} \] (14)
and there is only \( a_1 \) satisfies the condition \( 0 < \alpha < 1 \).

Replacing \( a_1 \) into (10), we obtain new expression as
\[ \gamma_{D_{\text{max}}} = \frac{C_1 C_2}{A_1 + B_1} \] (15)
where
\[ A_1 = (3 + \Omega^2 \gamma_{LYSD}) A_2, \]
\[ A_2 = (1 + \Omega^2 \gamma_{LYSD} - \sigma_n^2 \gamma_{LI} - 1 + \sqrt{3 + \Omega^2 \gamma_{LYSD}}), \]
\[ B_1 = (2 + \sigma_n^2 \gamma_{LI} - 1 + \sqrt{3 + \Omega^2 \gamma_{LYSD}}), \]
\[ B_2 = 3 + \Omega^2 \gamma_{LYSD}, \]
\[ C_1 = \gamma_{SD} \Omega (1 - 1 + \sqrt{3 + \Omega^2 \gamma_{LYSD}}), \]
\[ C_2 = 4 + \Omega^2 \gamma_{LYSD} - \sqrt{3 + \Omega^2 \gamma_{LYSD}}. \]

It is assumed that the channel gains \( |h|^2, |g|^2 \) are independent and identically distributed (i.i.d.) exponential.

3. Outage Probability and Throughput Analysis

In this section, we derive the outage probability and the throughput of full-duplex one-way relaying with energy harvesting and information transfer. Based on that analytical expressions, the outage probability and the throughput of model are given and we can compare and evaluate them. Whereby, we can see that only time fraction parameter \( \alpha_{\text{max}} \) is been given the best performance in terms of maximal SNR and partial maximum throughput.

3.1 Outage Probability Analysis

The outage probability of FD relaying network is computed as
where $R_0$ is target transmission rate and $Z = 2^{R_0} - 1$.

Equation (16) can be rewritten as follows

$$\gamma_{D_{\text{max}}} = \frac{D_1 D_3 (1 - D_2 D_3)}{D_2 (\gamma_0^2 - D_2) D_3 + D_1 D_4 + D_2^2}$$

with

$$\begin{align*}
D_1 &= \sin\theta \left(\frac{|h|^2 |g|^2}{d_1^m d_2^m}\right) \Omega, \\
D_2 &= \sqrt{3 + \Omega^2 \gamma_{11} D_1}, \\
D_3 &= -1 + D_2, \\
D_4 &= 2 + \sigma^2 \Omega \gamma_{11}.
\end{align*}$$

### Proposition
The outage probability of the energy harvesting enabled one way AF full-duplex relay is given as

$$P_{\text{out}} = \Pr \left( \gamma_{D_{\text{max}}} \leq Z \right)$$

$$= 1 - \frac{1}{\varphi Z} \int_0^{\varphi Z} \frac{d^m d^m Z \left( \frac{1}{\varphi} + y \right)}{\lambda_h \lambda_g} \left( \sin\theta_S - \varphi \sin\theta_D Z \right) y K_1 \left( 2 \sqrt{d^m d^m Z \left( \frac{1}{\varphi} + y \right) / (\lambda_h \lambda_g \sin\theta_S - \varphi \sin\theta_D Z) y} \right) \frac{1}{\lambda_f} e^{-\frac{\lambda_f y}{\varphi}} dy$$

where $\lambda_h, \lambda_g, \lambda_f$ are the mean value of the exponential random variables corresponding to $h, g, f$ channels, respectively and $K_1 (x)$ is Bessel function denoted as (8.423.1) in [21].

### Proof
We define $x = |h|^2 |g|^2, y = |f_k|^2$ and we have:

$$P_{\text{out}} = \left\{ \begin{array}{ll}
\Pr \left( x < \frac{d^m d^m Z \left( \frac{1}{\varphi} + y \right)}{\sin\theta_S - \varphi \sin\theta_D Z} \right), & y < \frac{1}{\varphi Z}, \\
1, & y > \frac{1}{\varphi Z}.
\end{array} \right.$$ (19)

Interestingly, the cumulative distribution function of $x$ is computed by

$$F_x (a) = \Pr (x < a)$$

$$= 1 - 2 \sqrt{a / \lambda_h \lambda_g} K_1 \left( 2 \sqrt{a / \lambda_h \lambda_g} \right)$$

and $y$ can be modeled with the probability distribution function $f_y (b) = \left(1 / \lambda_f \right) e^{-b / \lambda_f}$ and the proposition is attained after some simple calculation by hand.

On the other hand, following in [22], we have the DF outage probability given as

$$P_{\text{out}}^{\text{DF}} =$$

$$1 - \left(1 - e^{\frac{-\varphi}{\lambda_f}}\right) \left(2 \sqrt{\frac{\sigma^2 d_1^m d_2^m Z}{\varphi \lambda_s \lambda_d \varphi S}} \right) K_1 \left( 2 \sqrt{\frac{\sigma^2 d_1^m d_2^m Z}{\varphi \lambda_s \lambda_d \varphi S \gamma_{11}}} \right).$$

### 3.2 Optimal Throughput Analysis

In the Proposition, the outage probability of the scheme, when the relay harvests the energy from the resource signal and employs that power to amplify and forward the resource signal to the destination, is a function of the energy harvesting time $\alpha$ and the outage probability is minimum at approximate $\alpha = \alpha_{\text{max}}$ at high value of energy conversion efficiency, whenever $\alpha$ is as smaller than $\alpha_{\text{max}}$, the higher the outage probability is. And the same thing happens when $\alpha > \alpha_{\text{max}}$. In Fig. 4, it is clearly that the throughput of $\alpha \leq 0.1$ is better than that of $\alpha_{\text{max}}$, let get together with above outage probability, of course, $\alpha_{\text{max}}$ is the best value satisfy maximal SNR and remaining acceptable outage probability when we employ this model. As can be seen that the analysis curves provide a strictly agreement with simulation curves as in Fig. 3 and Fig. 4.

$$\tau = (1 - P_o) \frac{R (1 - \alpha) T}{T}.$$ (22)

We propose optimization problem for throughput in delay-limited transmission as

$$\alpha^{*} = \arg \max_{\text{s.t.} \text{SNR}, \text{SNR}_{\text{max}}} \tau.$$ (23)

Unfortunately, it is hard to derive the optimal throughput mathematically due to integral included. However, this value can be obtained numerically by solving the equation $\frac{\partial \tau}{\partial \alpha} = 0$. In addition, we can use either Golden section search method or gradient descent algorithm to find this optimal point. In the following numerical experiments, we find the optimal $\alpha$ based on the constraint of SNR and compare its value with different surrounding values.

### 4. Numerical Results

In this section, we use the derived analytical results to provide the outage probability, the optimal throughput, the optimal energy harvesting time. We set the source transmission rate $R = 3$ (bps/Hz), and hence the outage SINR threshold is given by $Z = 2^R - 1$. The energy harvesting efficiency is set to be $\Omega = 0.4$ (eliminate Fig. 5, Fig. 6), $\sigma^2 = 0.1$ (exclude Figures 7–10, the path loss exponent is set to be $m = 3$. For simplicity, we set the distance $d_1 = d_2 = 1$ (exclude Fig. 3 and Fig. 4). Also, we set $\lambda_h = \lambda_g = 1; \lambda_f = 0.1$, unless in different special cases.

From Fig. 3, the outage probability of the different scenarios of time is allocated following $\alpha$ values. The outage probability is minimum at approximate $\alpha = \alpha_{\text{max}}$ at high value of energy conversion efficiency, whenever $\alpha$ is as smaller than $\alpha_{\text{max}}$, the higher the outage probability is. And the same thing happens when $\alpha > \alpha_{\text{max}}$. In Fig. 4, it is clearly that the throughput of $\alpha \leq 0.1$ is better than that of $\alpha_{\text{max}}$, let get together with above outage probability, of course, $\alpha_{\text{max}}$ is the best value satisfy maximal SNR and remaining acceptable outage probability when we employ this model. As can be seen that the analysis curves provide a strictly agreement with simulation curves as in Fig. 3 and Fig. 4.
Fig. 3. Outage probability of FD energy-aware relaying network versus $\Omega$.

Fig. 4. Throughput of FD relaying versus $\Omega$.

Fig. 5. Outage probability of FD relaying network versus noise term.

Fig. 6. Throughput of FD relaying versus noise term.

Fig. 7. Outage probability of FD relaying network versus $d_1$.

Fig. 8. Throughput of FD relaying versus $d_1$. 
Next, the similar trend happens in Fig. 9 and Fig. 10, as bigger $\sigma$, as worse the outage probability of two model can be seen. Interestingly, the outage probability of DF model is better than that of AF model slightly and its throughput is better than, too. This is easily to explain because of noise at relay node which has impact on SNR and then results in system performance.

Remark 1: It worth noting that, there are two values of $\alpha$ lead to similar performance of outage and throughput, i.e. $\alpha_{\text{max}}$, $\alpha = 0.1$. In general, when comparing between optimal time switching $\alpha_{\text{max}}$ and $\alpha = 0.1$, it can be seen clearly that with $\alpha$ is smaller than $\alpha_{\text{max}}$, its throughput is better than that of $\alpha_{\text{max}}$, this is so easy to understand because when $\alpha$ is as smaller, as bigger the information quantity is sent to the destination but its outage probability is very high, it means that the operation of system may be stopped at higher level. And the second reason is that $\alpha$ is as smaller, the necessary energy which is harvested by the relay not enough power to transmit signal to the destination node. So, considering all of things, we find one thing that the model has the best performance for both maximal SNR and optimal throughput only with $\alpha_{\text{max}}$.

5. Conclusion

In this paper, we have proposed a full-duplex relay networking with wireless energy harvesting and information transfer protocol, where an energy constrained relay node harvests energy from the received RF signal and uses that harvested the energy to forward the source signal to the destination in delay limited mode. In order to determine the achievable outage probability and throughput, the analytical expressions for the outage probability and throughput following the optimal value of the energy harvesting time in AF scheme of TSR protocol can be found by the simulation. As a result, we can see that the model deployed with $\alpha_{\text{max}}$ becomes the optimal model gotten the best performance.

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References


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