Optimization of Tone Reservation-Based PAPR Reduction for OFDM Systems

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Submitted July 16, 2015 / Accepted April 27, 2017

Abstract. *A major drawback of orthogonal frequency division multiplexing (OFDM) is its high peak-to-average power ratio (PAPR). Tone reservation (TR) is one of the numerous methods for reducing the PAPR. Two parameters, the weight factor and the clipping ratio have significant impact on the PAPR reduction capability of TR methods. In this paper we thoroughly analyze the effect of these parameters. Based on these investigations two novel schemes, the globally and locally improved clipping ratio are introduced. Both methods rely on the feature that the CR can be adjusted between iterations. The main advantage of the globally improved clipping ratio is that, after offline preprocessing, it improves the PAPR reduction capabilities of the conventional TR scheme with same real-time computation complexity. The locally improved method can further enhance PAPR reduction capabilites on the cost of computation time.*

Keywords

OFDM, tone reservation, PAPR reduction, clipping ratio, optimization

1. Introduction

Orthogonal frequency division multiplexing (OFDM) is a well-known multicarrier modulation scheme for high speed digital communication. It is used in many wireless communications applications, and it is applied in various standards (e.g. DVB, WLAN, LTE). The amplitude of OFDM signals has Gaussian distribution which results in a high dynamic range and consequently a high peak-to-average power ratio (PAPR). If an OFDM signal is applied at the input of an amplifier, the circuit may be driven past its linear operating range, causing nonlinear distortions [1] which can degrade the overall system performance e.g., adjacent channel leakage ratio, and error vector magnitude.

Multiple PAPR reduction techniques have been introduced to reduce the probability of operating in the nonlinear domain such as clipping and filtering [2], interleaving [3], partial transmit sequence (PTS) [4], Tone reservation (TR) [5] and active constellation extension (ACE) [6]. When employing any of these techniques, it is important to examine whether they impair the transmission quality by introducing distortions, reducing the data rate, increasing the bit-error ratio (BER) or the necessary transmit power or call for additional information to be transmitted along the data. ACE and TR are currently the only PAPR reduction techniques having been adopted in an established standard, the terrestrial second generation DVB standard (DVB-T2) [7], [8]. In this paper the optimization of the TR technique regarding its PAPR reduction and BER improving capabilities will be investigated.

The TR method uses a pre-determined set of non data carrying subcarriers (reserved "tones"), that are auxiliary signals added to the main signal, helping to reduce the overall PAPR of the OFDM symbol. Two methods are presented in the literature for TR [9]: the kernel-based TR [8], [10], and the clipping-based TR [11], [12]. Both methods yield suboptimal performance for TR-based PAPR reduction. Several methods have been employed for determining the optimal modulation symbols of the TR subcarriers [5], [13], but many parameters of the TR method have not yet been dealt with in the literature, while still having tremendous impact on the effectiveness of the PAPR reduction. This paper aims to increase the effectiveness by proposing optimal values for the clipping ratio (CR), the weight factor μ and the number of iterations. While it is known that the CR has a large influence on the overall PAPR reduction performance of TR, the optimal value of the CR during the iterations still remains an open question. To address this issue, we propose a method to determine the CR that is globally and also locally applicable to each OFDM symbol. To evaluate the effectiveness of the determined CR values on each TR variant, a theoretical lower bound for the PAPR is given.

The paper is organized as follows. In Sec. 2 an overview of the most important parameters of the OFDM signal is provided and the PAPR metric is introduced. In Sec. 3 the basic idea of the TR method is presented; also the optimum solution using quadratic programming and the two suboptimal TR techniques are described in detail. Section 4 introduces the proposed novel approaches for the improvement of these methods. Simulation results are presented in Sec. 5.

2. Signal Model and PAPR Metric

2.1 OFDM Signal Model

OFDM systems are based on orthogonal, independent subcarriers, where the modulation can be easily realized with a simple N-point inverse fast Fourier transform (IFFT) operation, which is a summation of modulated complex harmonics [14] that can be expressed as follows:

$$x_n = \frac{1}{\sqrt{LN}} \sum_{k=0}^{LN-1} X_k e^{j2\pi \frac{kn}{LN}}, \quad 0 \le n \le LN-1 \qquad (1)$$

where the subcarrier index is denoted by k, the oversampling factor is denoted by L, while the index of the discrete sample and the complex modulation value of the k^{th} subcarrier is n and X_k, respectively.

2.2 Peak-to-Average Power Ratio

An often used metric to describe signal dynamics is PAPR. To calculate the PAPR of an OFDM symbol the following expression is used [14]:

$$\text{PAPR} = 10\log_{10}\left(\frac{\max|x_n|^2}{\frac{1}{LN}\sum\limits_n |x_n|^2}\right), \quad 0 \le n \le LN-1. \quad (2)$$

The complementary cumulative distribution function (CCDF) is often used to visualize a the distribution of a random variable, in this case the PAPR value, when PAPR_0 is known. This function can be expressed as follows [14]:

$$\text{P}(\text{PAPR} > \text{PAPR}_0) = 1 - \text{P}(\text{PAPR}_0). \qquad (3)$$

Increasing the number of subcarriers (N) results in a higher probability that the PAPR of the symbols exceeds a predefined PAPR_0 value. The high signal peak corresponding to the high PAPR value degrades the overall performance. The main goal is therefore to reduce the PAPR as much as possible.

3. Tone Reservation-Based Algorithms

This section describes optimal and suboptimal TR methods to determine the transmit signal components on the reserved carriers. The most significant advantage of TR is that the receiver structure (compared to the conventional OFDM receiver) can remain unaltered as the data carriers remain intact.

3.1 Lower Bound on the PAPR

A mathematical model can be set up to formulate the PAPR reduction using the TR method. Based on this model an optimization problem can be solved resulting in the maximal achievable PAPR reduction for a given OFDM symbol achiveable by the TR method. This result can be used as a reference for heuristic methods to reveal how much room for improvement is still available [15].

The PAPR reduced signal can be expressed as the sum of the original OFDM symbol and an auxiliary signal:

$$a_n = x_n + r_n, \qquad (4)$$

where x_n is the original OFDM symbol, r_n is the auxiliary signal used to reduce the PAPR and a_n is the PAPR reduced OFDM symbol. According to (2) PAPR can be decreased by minimizing the maximal power – and thus the numerator of (2) – of an OFDM symbol. This statement can be formulated mathematically as an optimization problem. Let E be the upper bound of the signal's instantaneous power $|a_n|^2$, then the minimization can be written as:

$$\text{minimize } E \text{ subject to } a_n = x_n + r_n, |a_n|^2 < E. \qquad (5)$$

Since the instantenous signal power is a quadratic function of the signal samples, the result is a quadratically constrained program (QCP).

The auxiliary signal r_n has to fulfill the general criteria for the TR method, which are formulated in frequency domain. Thus to add the TR constraints to the mathematical model the IDFT matrix F^{-1} is introduced. In TR only a subset of carriers is used to reduce the PAPR value. Therefore the F^{-1} matrix is of dimension $N_{PRC} \times LN$, where N_{PRC} is the number of the peak reduction carrier (PRC) positions (i.e. the number of reserved tones).The model extended with the criteria for TR method is then the following:

$$\text{minimize } E \text{ subject to } \begin{bmatrix} x & F^{-1} & -I \end{bmatrix} \begin{bmatrix} 1 \\ R \\ a \end{bmatrix} = 0,$$
$$E \ge |a_n|^2, \ -\infty \le R_k \le \infty, k \in \text{PRC} \qquad (6)$$

where R is the vector of reserved carriers in the frequency domain i.e. $r = F^{-1}R$, thus the above matrix multiplication is equivalent to (4). The only parameter in this program is practically the vector R. Thus by solving the program the minimal value of E is found for which the TR constraints are satisfied. We are using the interior point method of the MOSEK 7 toolbox to solve the optimization problem.

3.2 Suboptimal TR Methods

There are multiple existing approaches on how to implement computationally efficient schemes which apply TR constraints. A thorough overview of the fundamentals of TR methods are presented in [9], whereas recent studies show advances in both kernel based [8], [10], and clipping-based TR [11], [12]. A brief summary of both schemes is given in the following subsections. This summary is nescessary to formulate the fundamental equations and steps of the TR scheme in order to introduce our improved method.

3.2.1 Kernel-Based TR (TR-K)

First, a reference kernel vector is created, denoted by $\mathbf{p_n}$, that is an impulse-like function defined as:

$$\mathbf{p_n} = \frac{\sqrt{N_{\mathrm{FFT}}}}{N_{\mathrm{PRC}}} \mathrm{IFFT}(\mathbf{P_k}) \tag{7}$$

where $N_{\mathrm{FFT}} = NL$ is the FFT size. The length of the vector $\mathbf{P_k}$ is N_{FFT}, where the PRC positions are set to one, and all other values are set to zero. Ideally the kernel would consist of a single peak and all the other samples would be zero, but due to the limited bandwidth this cannot be achieved.

In every iteration the maximum amplitude and its position among the NL values has to be found:

$$A^i = \max_n |x_n^i|, \tag{8}$$

$$m^i = \arg\max_n |x_n^i| \tag{9}$$

where x_n^i is the n^{th} element of the vector x^i, A^i and m^i are the maximum amplitude and its position during the i^{th} iteration, respectively. Subsequently, the maximal amplitude of p_n kernel vector is circularly shifted to the same position as m^i. Then the kernel is scaled and phase rotated so that adding it to the original input reduces the power of the peak to a previously determined target clipping level. The modified kernel is added to x, then the PAPR value is calculated. These steps can be described as follows:

$$x^{i+1} = x^i - \alpha^i p_n(m^i), \tag{10}$$

$$\alpha^i = \frac{x^i(m^i)}{A^i}(A^i - A_{\max}) \tag{11}$$

where $p_n(m^i)$ is the circularly shifted kernel and $A_{\max}$ denotes the clipping amplitude. If the calculated PAPR value reaches the previously determined limit or an iteration number is exceeded, the algorithm is terminated. Otherwise the steps are repeated. The transmitted signal after the i^{th} iteration can be expressed as

$$x^i = x + \sum_{k=1}^{i} \alpha^k p_n(m^k). \tag{12}$$

3.2.2 Clipping-Based TR (TR-C)

In this method, first a clipping is performed on the x_n signal in the following way:

$$y_n = \begin{cases} x_n, & \text{if } |x_n| < A_{\max}, \\ A_{\max}\, \mathrm{e}^{\mathrm{j}\varphi(x_n)}, & \text{if } |x_n| > A_{\max} \end{cases} \tag{13}$$

where y_n is the clipped signal and $\varphi(x_n)$ is the phase of the signal. The maximum magnitude of the clipped signal is defined by the clipping ratio which can be calculated as

$$\mathrm{CR_{dB}} = 10\log_{10}\left(\frac{A_{\max}^2}{\frac{1}{LN}\sum_n |x_n|^2}\right). \tag{14}$$

The correction term c_n is calculated by subtracting the clipped signal from the original signal:

$${c_n}^i = {x_n}^i - {y_n}^i. \tag{15}$$

The correction term in the frequency domain is obtained by performing an FFT on the time domain correction term i.e., $C_k = \mathrm{FFT}(c_n)$.

To fulfill the requirements of the TR method, the changes are only applied at the PRC positions, all other values are set to zero.

$$\hat{C}_k = \begin{cases} C_k, & k \in PRC, \\ 0, & k \notin PRC. \end{cases} \tag{16}$$

Every iteration results in a $\hat{C}_k$ vector, and applying an IFFT on that, a time-domain signal $\hat{c}_n$ is obtained. At the end of every iteration, this clipped and filtered correction signal is added to the original ${x_n}^i$ input vector.

$${x_n}^{i+1} = {x_n}^i + \mu\hat{c}_n \tag{17}$$

where μ is a weight factor. The maximum number of iterations and μ are the parameters of this technique.

4. Optimization of the TR Methods

As shown in the previous section, both TR-C and TR-K methods depend on $\mathrm{CR_{dB}}$, furthermore regarding the TR-C method the weight factor μ in (17) can also be adjusted. The goal is to find out what values of these parameters lead to the highest PAPR reduction. The proposed methods exploit the ability that $\mathrm{CR_{dB}}$ can be updated in each iteration. Thus we reformulate the method in (11) for TR-K as:

$$\alpha^i = \frac{x^i(m^i)}{A^i}(A^i - A_{\max}^i). \tag{18}$$

Note that in this case the maximum amplitude $A_{\max}^i$ is different in each iteration. The formula for TR-C are modified in similar manner. Thus the clipping ratio for TR-C becomes:

$$\mathrm{CR_{dB}^i} = 10\log_{10}\left(\frac{\left(A_{\max}^i\right)^2}{\frac{1}{LN}\sum_n |x_n|^2}\right). \tag{19}$$

It follows that the clipped signal is then defined as

$$y_n = \begin{cases} x_n, & \text{if } |x_n| < A_{\max}^i, \\ A_{\max}^i\, \mathrm{e}^{\mathrm{j}\varphi(x_n)}, & \text{if } |x_n| > A_{\max}^i \end{cases} \tag{20}$$

where the maximum amplitude $A_{\max}^i$ is then also iteration dependent.

In Fig. 1 and Fig. 2 the block diagrams are shown for the modified TR-K and TR-C schemes, respectively. The additional step with respect to the original TR schemes is highlighted with a dashed frame in the figures. In the following we discuss three approaches how to determine the CR.

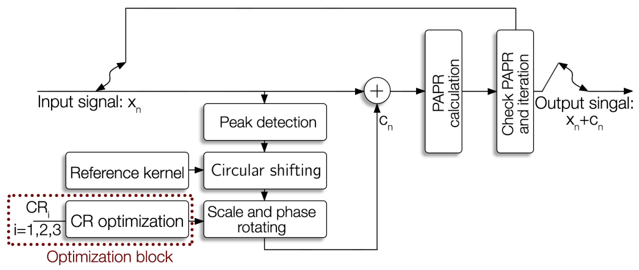

Fig. 1. Block diagram of the proposed kernel based TR.

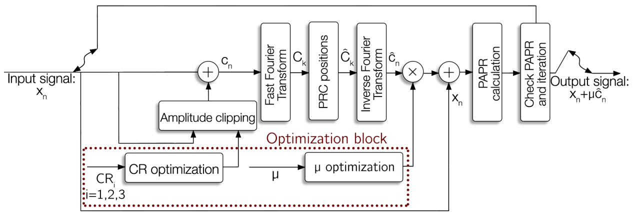

Fig. 2. Block diagram of the proposed clipping based TR.

4.1 Constant CR

The most straightworfard method is keeping the CR constant throughout the iterations. To choose the apropriate CR a set of randomly generated OFDM symbols are used as a training sequence. Then the CCDF function of the PAPR is monitored at a certain probability in each iteration. The choice for constant CR is then the one which results in the highest PAPR reduction after the predefined number of iterations.

4.2 The Globally Improved CR

In this case the ability to alter the CR in every iteration is exploited. A set of randomly generated OFDM symbols is used as a training sequence, and the CCDF function of the PAPR of these symbols is monitored. For each iteration the TR algorithm is applied for various CRs. After the predefined number of iterations is reached the CR trajectory leading to the highest PAPR reduction is chosen. The benefit of this process is that once it is performed offline for a set of OFDM parameters (e.g., number of carriers, modulation alphabet, number of reserved tones, etc.), it can be used with any OFDM symbol. However, this method will yield a "statistically optimal" CR trajectory, since it is optimal for the training symbol set. Nevertheless, after the offline optimization is done, the computational complexity of the scheme will be equal with that of the constant CR.

4.3 The Locally Improved CR

This scheme is essentially the same method as the globally improved CR but instead of using a training sequence, the CR trajectory optimization is carried out for each OFDM symbol to be transmitted independently. This leads to a CR trajectory which is optimal for a given OFDM symbol, however the whole process has to be carried out over and over again (possibly in real time). Thus the computational complexity this scheme is $O(C^I)$, where C is the number of CRs to be tested and I is the number of iterations.

4.4 Weight Factor μ

For the TR-C scheme the choice of the weight factor μ has also significant impact. The search process is the same as in the case of finding the CR. The optimization is done using a set of training symbols and μ is then chosen based on PAPR reduction according to the CCDF curve at a certain probability.

5. Simulation Results

The PAPR reduction capability of the presented TR methods was verified through simulations in MATLAB. The applied parameters are summarized in Table 1. These parameters correspond to the DVB-T2 standard [7].

Name of the parameter	**value**
FFT size	2048
Number of PRCs	18
Position of PRCs	113 124 262 467 479 727 803 862 910 946 980 1201 1322 1342 1396 1397 1562 1565
Maximal allowed amplitude	5
Modulation	16-QAM
Number of carriers (N)	2048
Number of symbols	15000
Oversampling factor (L)	4
Value of μ for the TR-C	-282

Tab. 1. TR simulation parameters.

5.1 Constant Clipping Ratio

First, the PAPR reduction capability of constant CR is investigated. In Fig. 3 and 4 the PAPR value at $10^{-1.8}$ probability is shown with different CR values and iteration numbers using TR-K and TR-C methods, respectively. It can be seen that the CR values which introduce the most significant PAPR reduction for TR-K and TR-C methods are 8.24 dB and 7.75 dB, respectively. Similar results have been presented and summarized in [13].

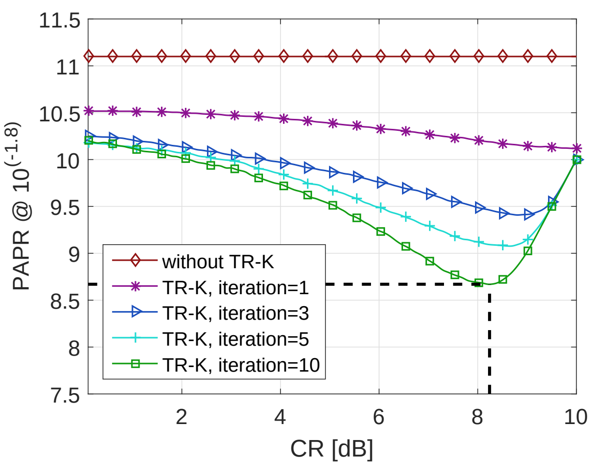

Fig. 3. PAPR reduction performance of the TR-K method in function of the iterations and the CR.

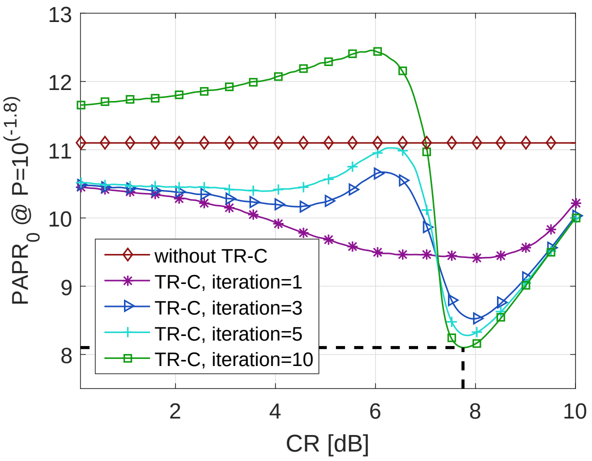

Fig. 4. PAPR reduction performance of the TR-C method in function of the iterations and the CR.

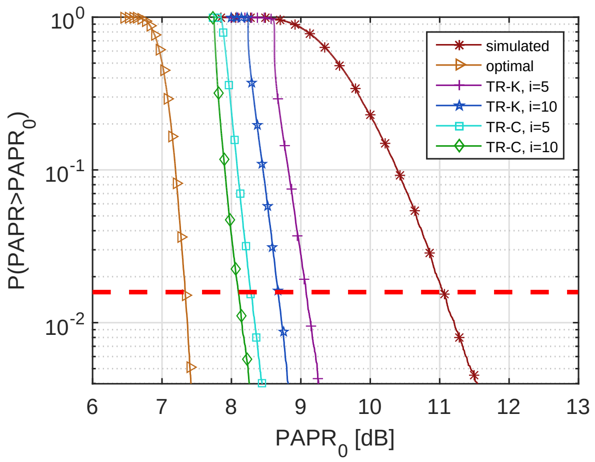

Fig. 5. CCDF of TR methods with different number of iterations using optimized CR, $\mathrm{CR}^{\mathrm{opt}}_{\mathrm{TR-K}} = 8.24$ dB, $\mathrm{CR}^{\mathrm{opt}}_{\mathrm{TR-C}} = 7.75$ dB.

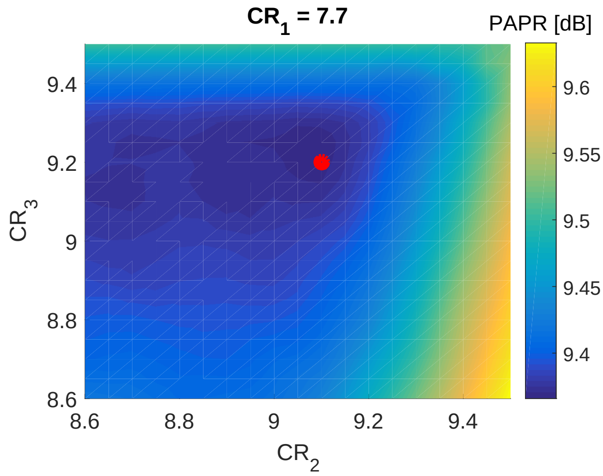

Fig. 6. PAPR of CR trajectories for three iterations with TR-K method.

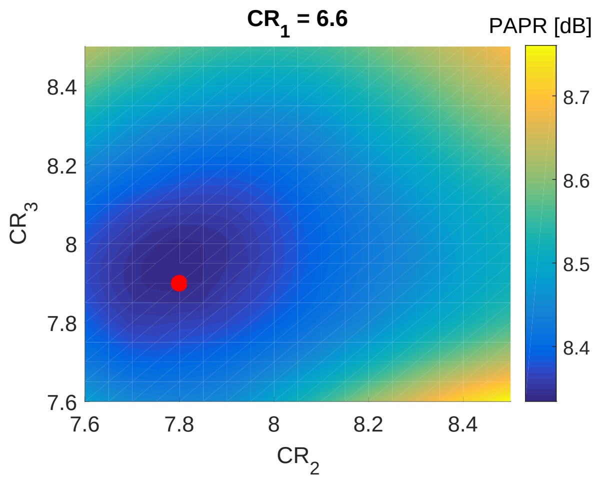

Fig. 7. PAPR of CR trajectories for three iterations with TR-C method.

The resulting CCDF curves are shown in Fig. 5. The optimized CR values were specifically chosen for each method as to have the best performance regarding the PAPR value. In this figure the theoretical lower bound for the achievable PAPR is also indicated, which was determined by solving the optimization program described in Section 3.1. The TR-K technique has lower computational complexity, since the algorithm operates only in time domain however the TR-C has better performance. It can also be seen from Fig. 3 and 4 that the TR-C method has faster convergence.

The maximum allowed energy on the reserved carriers is also an important parameter of the simulations. Allowing higher values is not recommended for practical implementations, so the selection of this parameter is also an essential question. The simulations have shown that the TR-K and TR-C methods are insensitive of this parameter.

5.2 Globally Improved Clipping Ratio

This section introduces the idea of globally improved clipping ratio. The difference from constant clipping ratio is that CR is changed from iteration to iteration to find the best trajectory. The PAPR value was investigated for three iterations, and in each iteration the value of CR was swept between 1 and 10 dB with 0.1 dB steps. Here we show the results of the trajectory leading to the lowest PAPR, examined at $10^{-1.8}$ probability of the CCDF curve. In Fig. 6 and 7 PAPR reduction performance for TR-K and TR-C are shown. The first CR is constant, the second and third CR are the x and y axis of the plot, respectively. The achievable minimum PAPR is indicated with a red dot of the surface plot.

It can be seen that both methods reach lower PAPR values using the globally improved CR than by using a constant one. The improved clipping ratio results in an improvement of 0.042 dB for the TR-K and an improvement of 0.188 dB for the TR-C method.

Based on the simulations we propose the clipping trajectories: $\mathrm{CR}^{\mathrm{K,global}}_{1,2,3} = 7.7, 9.1, 9.2$ dB and $\mathrm{CR}^{\mathrm{C,global}}_{1,2,3} = 6.6, 7.8, 7.9$ dB for TR-K and TR-C, respectively.

Please note that the improvement can be further increased using higher iteration numbers; however, the computational complexity grows exponentially.

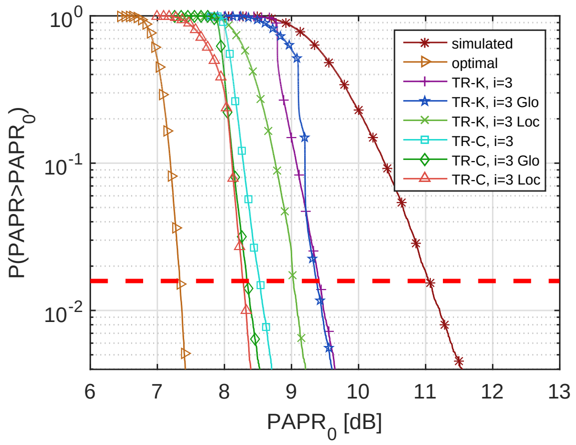

Fig. 8. CCDF comparison of the constant, locally and globally improved CR with TR-K and TR-C algorithms.

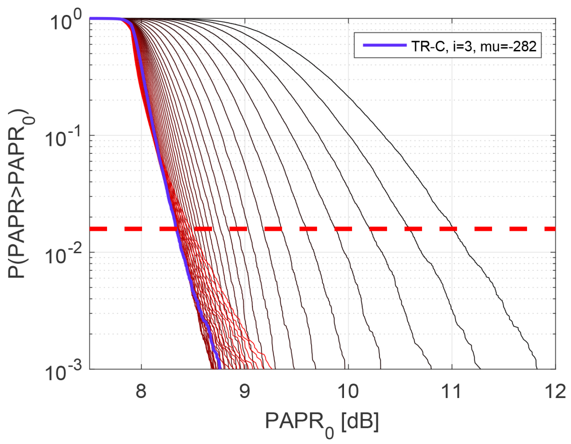

Fig. 9. PAPR curves for different μ values using TR-C algorithm with 3 iterations and adaptive CR.

5.3 Locally Improved Clipping Ratio

Using locally improved clipping ratio the results can be further improved regarding the PAPR value. The results obtained by locally improved TR-K and TR-C methods are shown in Fig. 8. While improvement with the globally improved algorithm is 0.042 dB using the TR-K method, this value with locally improved CR increases to 0.381 dB. Using the TR-C technique an improvement of 0.246 dB can be observed. This method shows that on the cost of computational complexity, the PAPR can be further reduced with a clipping trajectory which is optimized for the given symbol.

5.4 Weight Factor μ

During the optimization different CR and μ values were set using 10 iterations and the PAPR curve was examined after every parameter change. Investigations were also performed with 3 iterations and optimized clipping trajectory and both simulation lead to an optimal value of $\mu_{\text{opt}} = -282$. Its high magnitude can be explained with the fact that the transmitted signal was normalized during mapping. The PAPR curves for different μ values using an optimal clipping trajectory and 3 iterations are shown in Fig. 9.

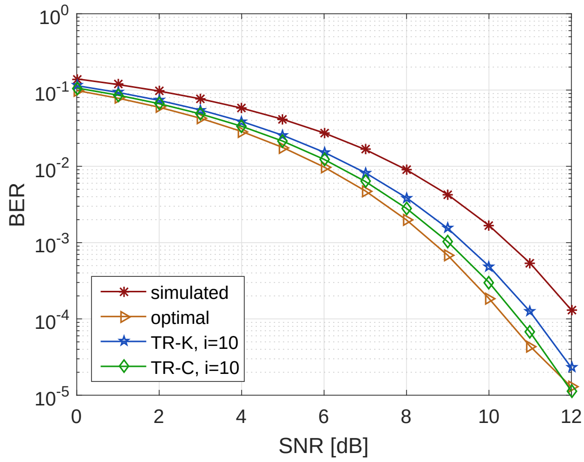

Fig. 10. BER results of optimized constant CR for the different TR methods using 10 iterations.

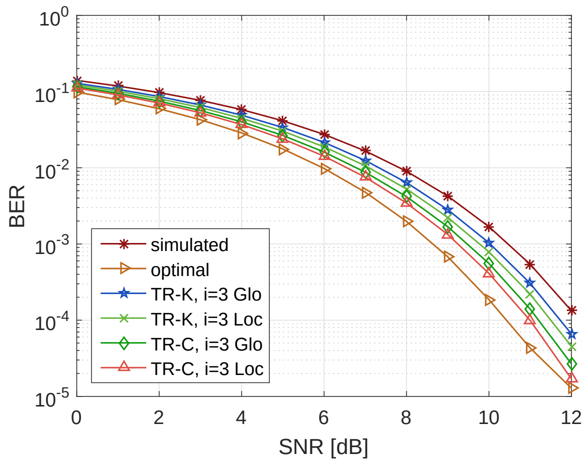

Fig. 11. BER results of TR-K and TR-C (globally and locally improved CR) methods using 3 iterations.

5.5 Bit Error Ratio

Besides the PAPR reduction capabilities of the TR techniques the possible BER improvement is investigated in the presence of additive white Gaussian noise (AWGN) channel. The scenario describes the case of data transmission from point A to B, where the maximum magnitude of the linearly amplifiable signal is fixed. The magnitude of each signal is normalized to this fixed value, however the amplitude of the noise is calculated based on the original OFDM signal power. This demonstrates that the PAPR reduction can enhance transmission performance even in a linear channel.

First we applied constant CR with 10 iterations. The result of the simulation is presented in Fig. 10. It can be seen that both methods are capable of improving the BER. The optimal solution has the most improvement in BER, however it cannot be applied in real-time applications due to its high complexity. It can also be observed that from the two suboptimal TR methods the TR-C has much better BER improvement potential than TR-K.

Next, the BER curves for the globally and locally improved CR values were investigated. In Fig. 11 the BER results with the locally and globally improved CR methods are presented with 3 iterations. It can be seen that the TR-C method outperforms the TR-K method. The gain in BER is directly comparable with Fig. 10 , and it can be concluded that less iterations are required with globally and locally improved methods to achieve the same improvement. Also we have presented for both algorithms that the locally improved method definitely outperforms the globally improved CR.

6. Conclusion

In this paper the kernel based and clipping based PAPR reduction schemes were investigated for OFDM signals. We showed that the efficiency of these methods strongly depend on the choise weight factor and the clipping ratio. We introduced a novel algorithm to enhance the PAPR reduction capabilites, which exploits the feature that the CR can be adjusted between iterations. The two variants of the algorithm are the globally and locally improved clipping, respectively. The introduced schemes were compared with former results and thus the improved PAPR reduction capability was shown via simulations. The results revealed that using the improved clipping trajectory the TR-C method outperforms the TR-K scheme and that the locally improved CR can further enhance the PAPR reduction efficiency on the cost of computation complexity. However, the globally improved CR offers significant reduction gain while the complex computation can be done beforehand on a set of training symbols. Thus for a transmission system using several different OFDM parameter sets (e.g, DVB-T2) the optimization can be done offline and the results can be stored in a lookup table.

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