Dual-Frequency Impedance Transformer Using Coupled-Line for Ultra-High Transforming Ratio

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Abstract. In this paper, a new type of dual-frequency impedance transformer is presented for ultra-high transforming ratio. The proposed configuration consists of parallel coupled-line, series transmission lines and short-ended stubs. The even and odd-mode analysis is applied to obtain the design equations and hence to provide an accurate solution. Three examples of the dual-frequency transformer with load impedance of 500, 1000 and 1500 Ω are designed to study the matching capability and bandwidth property. To prove the frequency agility of the proposed network, three prototypes of dual-frequency impedance transformer with transforming ratio of 10, 20 and 30 are fabricated and tested. The measured return loss is greater than 15 dB at two operating frequencies for all the prototypes. Also, the bandwidth is obtained more than 60 MHz at each frequency band for all the prototypes. The measured return loss is found in good agreement with the circuit and full-wave simulations.

Keywords
Impedance transformer, dual-frequency, ultra-high transforming ratio, coupled-line, transmission line

1. Introduction

Impedance transformers (ITs)/matching networks (MNs) are indispensable components in the design of various RF/Microwave systems to provide advantages such as maximum power transfer, high linearity, and low noise level. Therefore, it is frequently used in many applications such as power combining/splitting, baluns, couplers, antennas and amplifiers. Many researchers focused on the development of ITs with ultra-high transforming ratio (UHTR) for wideband [1–11] and multi-frequency operations [12–23]. Various techniques such as a transmission line and coupled-line section [1], a parallel coupled-line and shunt open-stub [2], multi-section quarter-wavelength line [3] and two-section transmission lines [4] have been employed to develop wideband ITs. These transformers achieved excellent broadband response but it have been designed for smaller impedance transforming ratios. To overcome this issue, some techniques have been applied to design wideband ITs with high/ultra-high transforming ratios [5–11]. In [5], a coupled-line section of electrical length λ/8 has been used to design an IT with transforming ratio of 3.4. A broadband matching network with transforming ratio of 3.4 has been developed by employing a coupled three-line section [6]. An IT has been designed using a modified coupled-line structure for the transforming ratio of 5 [7]. In [8], a broadband IT with transforming ratio of 5 based on the 4:1 Guanella-transformer has been demonstrated. In [9], a matching network for transforming ratio of 5 has been designed using coupled-line and shunt open-stub. Two-section parallel coupled-lines have been employed to design a UHTR IT in [10]. This UHTR IT achieves a transforming ratio of 10 with a fractional bandwidth of 8.27%. In [11], two UHTR ITs have been developed based on the short-ended coupled-line sections for a transforming ratio of 20.

With the appearance of dual-frequency RF/Microwave systems, several ideas have been implemented for the design of dual-band impedance transformer (DBIT) [12–23]. In [12], transmission lines and shunt open-stubs have been employed to design a DBIT with transmission zero. The T-type network has been used to develop a dual-frequency impedance transformer (DFIT) to match frequency dependent complex loads in [13]. A π-model DBIT has been presented to match two arbitrary complex loads [14]. In [15], a DBIT has been designed using T-shaped coupled-line to transfer two complex load to a real source. Different transmission lines are utilized to transfer two unequal complex loads in [16]. A DBIT has been designed to match reflection coefficients seen by the active devices to a 50 Ω source. This DBIT has been utilized for the design of dual-band low noise amplifier [17]. In [18], a matching network operating at two different frequencies has been presented using load-healing concept to improve load range. A DBIT based on π-model has been proposed for the synthesis of power amplifiers in [19]. Two-section with one-third wavelength has been employed to develop a DBIT for a frequency and its first harmonic [20], in which, the design equations are not exact. Hence, an exact analytical treatment of the two-section has been presented in [21]. Also, an extension of the two-section transformer has been analyzed to obtain ex-
2. Analysis of the Proposed Network

Figure 1 shows the topology of the proposed dual-frequency impedance transformer, which is decomposed into even-half and odd-half network as depicted in Fig. 2 and 3, respectively. The proposed structure consists of one parallel 

coupled-line (Z_e, Z_o), two series transmission lines (Z_1) and 

short-circuited stubs (Z_2) with electrical lengths of \( \theta_c, \theta_1 \) and 

\( \theta_2 \), respectively. The dual-frequency structure exhibits same 

characteristics as that of a quarter-wavelength transmission 

line (90°) with impedance of \( Z_X \) at two arbitrarily chosen 

frequencies. The necessary design equations can be derived 

using even-odd mode analysis.

\[
\begin{align*}
Y_{even} &= j \frac{Y_e}{f_1} \left( \frac{\tan \theta_c - Y_2 \cot \theta_2 + Y_1 \tan \theta_1}{Y_1} \right) \tan \theta_1, \\
Y_{odd} &= j \frac{Y_e}{f_2} \left( \frac{\tan \theta_c - Y_2 \cot \theta_2 + Y_1 \tan \theta_1}{Y_1} \right) \tan \theta_1.
\end{align*}
\]

As shown in Fig. 2, the proposed even half-network is equivalent to an open-ended stub having characteristic impedance of \( Z_X \) with electrical length of ±45°. The input admittance \( (Y_{even}) \) of the proposed even half-network can be written as:

\[
Y_{even} = j \frac{Y_e}{f_1} \left( \frac{\tan \theta_c - Y_2 \cot \theta_2 + Y_1 \tan \theta_1}{Y_1} \right) \tan \theta_1.
\]

The input admittance of the proposed odd half-network \( (Y_{odd}) \) of the proposed odd half-network can be written as:

\[
Y_{odd} = j \frac{Y_e}{f_2} \left( \frac{\tan \theta_c - Y_2 \cot \theta_2 + Y_1 \tan \theta_1}{Y_1} \right) \tan \theta_1.
\]

The equivalent admittance (even) is given as:

\[
Y_X = Y_{even} = j \frac{Y_e}{f_1} \left( \frac{\tan \theta_c - Y_2 \cot \theta_2 + Y_1 \tan \theta_1}{Y_1} \right) \tan \theta_1.
\]

By combining equations (4) and (5), the equivalent admittance is given as:

\[
Y_X = Y_{odd} = j \frac{Y_e}{f_2} \left( \frac{\tan \theta_c - Y_2 \cot \theta_2 + Y_1 \tan \theta_1}{Y_1} \right) \tan \theta_1.
\]
Putting $\theta_C = \theta_1 = \theta_2 = \theta$ in equation (3), the equivalent admittance at two operating frequencies $f_1$ and $f_2$ is given in equations (7) and (8), respectively:

$$\text{eq. 7}$$

$$Y_X|_{\omega f_1} = jY_1 \frac{Y_1 \tan \theta_{f_1} - Y_2 \cot \theta_{f_1} + Y_1 \tan \theta_{f_1}}{Y_1 - \left[ Y_2 \tan \theta_{f_1} - Y_2 \cot \theta_{f_1} \right] \tan \theta_{f_1}}$$

$$\text{eq. 8}$$

$$Y_X|_{\omega f_2} = jY_1 \frac{Y_1 \tan \theta_{f_2} - Y_2 \cot \theta_{f_2} + Y_1 \tan \theta_{f_2}}{Y_1 - \left[ Y_2 \tan \theta_{f_2} - Y_2 \cot \theta_{f_2} \right] \tan \theta_{f_2}}$$

where $\theta_{f_1}$ and $\theta_{f_2}$ are electrical length of the transmission lines at $f_1$ and $f_2$, respectively. The magnitude of $Y_X$ is equal to two operating frequencies, which can be clearly expressed as:

$$\text{eq. 9}$$

$$Y_X|_{\omega f_1} = Y_X|_{\omega f_2}$$

Solving equation (9), the required condition for dual-frequency characteristic can be stated as:

$$\tan \theta_{f_1} = -\tan \theta_{f_2}$$

The general solution of the equation (10) can be written as:

$$\theta_{f_1} + \theta_{f_2} = n\pi$$

where $n = 1, 2, 3, \ldots$, and assuming the condition that

$$\frac{\theta_{f_2}}{\theta_{f_1}} = \frac{f_2}{f_1}$$

we obtained the electrical lengths $\theta_C = \theta_1 = \theta_2 = \theta$ as follows:

$$\text{eq. 11}$$

$$\theta = \frac{n\pi}{1 + \frac{f_2}{f_1}}$$

From equations (3) and (6), the even and odd-mode admittances of the coupled-line are given as:

$$\text{eq. 12}$$

$$Y_e = \frac{Y_{1X} + Y_{2X} \cot \theta_2 \tan \theta_1 + Y_1 Y_2 \cot \theta_2 - Y_2^2 \tan \theta_1}{\left[ Y_1 + Y_X \tan \theta_1 \right] \tan \theta_C}$$

$$\text{eq. 13}$$

$$Y_o = \frac{Y_{1X} + Y_{2X} \cot \theta_2 \tan \theta_1 - Y_1 Y_2 \cot \theta_2 + Y_1^2 \tan \theta_1}{\left[ Y_1 - Y_X \tan \theta_1 \right] \cot \theta_C}$$

Based on the above analysis, a design guideline to develop ultra-high transforming ratio dual-frequency impedance transformer is summarized as follows:

1. Select the two different operating frequencies $f_1$ and $f_2$.

2. Once the desired frequencies are chosen, using equation (13), the electrical lengths ($\theta_C$, $\theta_1$ and $\theta_2$) can be calculated. For compactness, the value of $n$ starts from 1.

3. The impedance transforming ratio is calculated as $r = Z_{\text{load}}/Z_{\text{source}}$. Where $Z_S$ and $Z_L$ are source and load impedances, respectively. For $Z_S = 50 \Omega$, calculate the value of $Z_X = 1/Y_X = 50\sqrt{7} \Omega$.

4. The characteristic impedances $Z_1$ and $Z_2$ are assumed to be independent (free) variables, as we have two equations (14)–(15) and four unknowns ($Z_e$, $Z_o$, $Z_1$, and $Z_2$). Suitable values of $Z_1$ and $Z_2$ can be chosen to achieve ultra-high transforming ratio.

5. Based on the values obtained in step four, the parameters $Z_e$ and $Z_o$ are calculated from equations (14) and (15), respectively. Using microstrip line calculator, the physical length, width and spacing of the coupled-line are calculated based on the values of $Z_e$, $Z_o$ and $\theta_C$.

6. The physical parameters are determined at the first operating frequency $f_1$.

By following the above detailed synthesis procedure, the proposed structure can be designed to work as an impedance transformer for ultra-high transforming ratios at two operating frequencies. As the electrical lengths $\theta_1$, $\theta_2$ and $\theta_C$ are calculated based on the arbitrarily chosen operating frequencies, the transcendental equations (14) and (15) are used to compute $Z_e$ and $Z_o$. As we have two equations and four unknowns, $Z_1$ and $Z_2$ are assumed to be independent (free) variables. Hence, suitable values of $Z_1$ and $Z_2$ can be chosen to achieve large impedance transforming ratio. The value of $Z_X = 1/Y_X = 50\sqrt{7} \Omega$ is chosen by the designer. Therefore, it is clear that the parameters $Z_1$ and $Z_2$ can be varied, or $Z_1$ ($Z_2$) can be varied for a fixed value of $Z_2$ ($Z_1$). Moreover, the range of realizable frequency ratios can be determined by applying different combinations of $Z_e$ and $Z_o$. To obtain the frequency ratio range ($f_2/f_1$) of the proposed DBIT, $Z_e$ and $Z_o$ are plotted as a function of $Z_1$ for the different values of $Z_2$ and $p$ for a load of 1000 $\Omega$ as depicted in Fig. 4. From the plot, the range of realizable frequency ratio ($f_2/f_1$) is calculated as 2.4 – 3.8 for the load impedances of 1000 $\Omega$. Similarly, the design curves of $Z_e$ and $Z_o$ can be plotted for different loads. The range of realizable frequency ratios for the loads of 500 $\Omega$ and 1500 $\Omega$ are computed as 2.5 – 4.5 and 2.4 – 3.4, respectively.

3. Advantage of the Proposed Network

The classic Pi-network [19], two-section transformer [22] and Chebyshev transformer [23], in which design equations have no free variable, limit the impedance transforming ratio ($Z_2/Z_5$). To decorate this issue, Tab. 1 depicts the design parameters required to implement a DBIT with transforming ratio of 10 and frequency band ratio of 2.5 for the proposed technique and previously reported schematics. As seen in Tab. 1, the line impedances are beyond the practical realization limits of the microstrip technology (20 $\Omega$ – 150 $\Omega$). In this scenario, the topologies reported in [19], [22] and [23] are not suitable for ultra-high transforming ratio. Hence, we proposed a new DBIT and its mathematical modeling to obtain transforming ratio more than 10. As discussed in Sec. 2, there are two independent (free) variables are available for the proposed topology. Hence, it is more worthwhile...
to illustrate the potential of the transformer using few examples. Therefore, we fabricated and tested three prototypes dual-frequency transformer with transforming ratio of 10, 20 and 30.

4. Design of UHTR-DBIT

Based on the above analysis, three examples of the UHTR-DBIT for real loads of 500 Ω, 1000 Ω and 1500 Ω are considered for design. A single frequency ratio of \( p = 2.7 \) (\( f_1 \) and \( f_2 \) can be arbitrarily chosen) is selected to study the matching property of the proposed DBIT. The design parameters of the DBIT with transforming ratios of 10, 20 and 30 are calculated and listed in Tab. 2. The circuit simulated \( S_1 \) of the DBIT with load impedances of 500 Ω, 1000 Ω and 1500 Ω is shown in Fig. 5. From the graph, it is observed that the return loss is greater than 15 dB at all operating frequencies. This implies the proposed UHTR-DBIT is suitable to transfer very large load impedances to the source impedance of 50 Ω at two arbitrary operating frequencies.

Furthermore, the bandwidth of the multi-frequency transformer is indispensable parameter needs to be focused. In the design of multi-frequency systems, it is required to produce reasonable amount of bandwidth (minimum 50 MHz) for practical applications. In practice, the bandwidth of the transformer increases/decreases for less mismatched load (as
\[ \frac{f_1}{f_2} \quad Z_L \quad \frac{Z_o}{Z_i} \quad Z_1 \quad Z_2 \quad \theta_1 = \theta_2 = \theta_C \]

<table>
<thead>
<tr>
<th>( \frac{f_1}{f_2} )</th>
<th>( Z_L ) (( \Omega ))</th>
<th>( \frac{Z_o}{Z_i} ) (( \Omega ))</th>
<th>( Z_1 ) (( \Omega ))</th>
<th>( Z_2 ) (( \Omega ))</th>
<th>( \theta_1 = \theta_2 = \theta_C ) (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2.7</td>
<td>500</td>
<td>93.24/38.52</td>
<td>40</td>
<td>30</td>
<td>48.64</td>
</tr>
<tr>
<td>1000</td>
<td>121.5/57.3</td>
<td>55</td>
<td>40</td>
<td>48.64</td>
<td></td>
</tr>
<tr>
<td>1500</td>
<td>139.3/77.29</td>
<td>55</td>
<td>40</td>
<td>48.64</td>
<td></td>
</tr>
</tbody>
</table>

Tab. 2. Design parameters of the proposed UHTR-DFIT for a single frequency ratio

\[ Z_L \text{ becomes closer to } Z_S \text{ and more mismatched load (as } Z_L \text{ becomes far to } Z_S \text{), respectively} \[24]. \] Hence, the behavior of the loads in the design of proposed dual-frequency transformer are observed in Fig. 5. From the figure, it is observed that the bandwidth decreases with increasing load. Also, it is seen that the bandwidth of the proposed model is more than 60 MHz considering \( S_{11} < -10 \) dB, which is suitable for most of the applications. Therefore, the proposed DFIT provides flexible solutions for implementation of very large impedance transforming ratio at two different operating frequencies and can be applied to design microwave devices such as amplifiers, power dividers and antennas.

### 5. Fabrication and Measurement

To validate the circuit analysis, three prototypes of the UHTR-DFIT with transforming ratios of 10, 20 and 30 are fabricated. For all these prototypes, different frequency ratios are considered to demonstrate the flexibility of the proposed transformer. The load impedances 500 \( \Omega \), 1000 \( \Omega \), and 1500 \( \Omega \) are constructed by the SMD resistors with corresponding values. Based on the above analysis, the design parameters of the proposed UHTR-DFIT are calculated and listed in Tab. 3. The dimensions of the final layout as shown in Fig. 6 are computed at \( f_1 \) and listed in Tab. 4. A Rogers RT/Duriod substrate having thickness of 0.787, \( \varepsilon_r = 2.33 \) and loss tangent of 0.0012 is used for fabrication. The photograph of the fabricated UHTR-DFITs are shown in Fig. 7. The rectangular dimensions of the DFTIs with transforming ratios of 10, 20 and 30 are 27.11 \( \times \) 48.1 mm\(^2\), 18.4 \( \times \) 32 mm\(^2\) and 33.9 \( \times \) 66.54 mm\(^2\), respectively.

Tab. 3. Design parameters for proposed DFIT with different transforming ratios

<table>
<thead>
<tr>
<th>( r )</th>
<th>( \frac{f_1}{f_2} )</th>
<th>( \frac{Z_o}{Z_i} ) (( \Omega ))</th>
<th>( Z_1 ) (( \Omega ))</th>
<th>( Z_2 ) (( \Omega ))</th>
<th>( \theta_1 = \theta_2 = \theta_C ) (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.9/3.6</td>
<td>161.9/98.5</td>
<td>40</td>
<td>95</td>
<td>36</td>
</tr>
<tr>
<td>20</td>
<td>1.5/5.4</td>
<td>160.8/130.6</td>
<td>55</td>
<td>90</td>
<td>39.13</td>
</tr>
<tr>
<td>30</td>
<td>0.9/2.45</td>
<td>160.5/71</td>
<td>65</td>
<td>50</td>
<td>48.36</td>
</tr>
</tbody>
</table>

Tab. 4. Physical dimensions of the final layout for different transforming ratios

Fig. 5. The circuit simulated magnitude response of the DFIT with transforming ratios of 10, 20 and 30.

Fig. 6. The physical layout of the proposed DFIT.

Fig. 7. Photograph of the fabricated prototypes with transforming ratios of 10, 20 and 30.

Fig. 8. Circuit, full-wave simulated and measured \( S_{11} \) response of the UHTR-DFIT with transforming ratio of 10.
A Rohde & Schwarz ZVL network analyzer is used to measure the return loss characteristics of the fabricated DFTIs. The circuit, full-wave simulated and tested \( S_{11} \) of the fabricated prototypes with transforming ratio of 10, 20 and 30 are shown in Fig. 8, 9 and 10, respectively. From the plots, it is observed that the \( S_{11} \) is well below \(-15\) dB for all the fabricated prototypes. The exact values of the \( S_{11} \) are calculated and shown in Tab. 5. Considering \( S_{11} < -10 \) dB reference level, the bandwidths (in MHz) at \( f_1/f_2 \) of the fabricated DFTIs with transforming ratio of 10, 20 and 30 are calculated and illustrated in Tab. 6. Hence the proposed DFIT is very much useful for very large transforming ratio. Due to the fabrication tolerance and connector loss, a very small deviation between simulated and measured performances has been observed. The proposed dual-frequency technique is simple, easy to fabricate and feasible for many RF/Microwave system applications. A comparison table for frequency bands and transforming ratio of the other reported works and the proposed technique is depicted in Tab. 7.

### 6. Conclusion

In this paper, a novel dual-frequency impedance transformer is designed using coupled-line, series transmission line and shorted stubs for ultra-high transforming ratio. The even-odd mode analysis is applied to derive closed form design equations. After describing analytical derivation and procedure in detail, three prototypes with impedance trans-
forming ratio of 10, 20 and 30 are fabricated, and demonstrated. The measured return loss is better than 15 dB at two operating frequencies for all the prototypes.

References


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Rusan Kumar BARIK was born in Odisha, India. He received the B. Tech degree in Electronics and Tele-Communication Engineering from Biju patnaik University of Technology, Rourkela, India in 2012 and the M. Des degree in Communication Systems Design from Indian Institute of Information Technology Design and Manufacturing, Kancheepuram, India in 2015, where he is currently working towards his PhD degree. His research interests include design of multi-band microwave passive components and low noise amplifier for millimeter wave applications.
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