A Distributed Compressed Sensing-based Algorithm for the Joint Recovery of Signal Ensemble

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Abstract. *This paper considers sparsity-aware adaptive compressed sensing acquisition and the joint reconstruction of intra- and inter-correlated signals in the wireless sensor networks via distributed compressed sensing. Due to the different sparsity order of the finite-length signals, we develop an adaptive sensing framework based on the sparsity order, in which sensor readings are sampled according to its own sparsity order measure. On the decoder side, utilizing a distributed compressive sensing scheme, a joint reconstruction method is proposed to recover signal ensemble even in imperfect data communication. Moreover, we explore that by adapting the sampling rate of the sensed signals, not only the whole required number of measurements is reduced, but also the reconstruction performance is significantly improved. Numerical experiments verify that our proposed algorithm achieves higher reconstruction accuracy with a smaller number of required transmission, and with lower complexity as compared to those of the state of the art CS methods.*

Keywords

Sparsity measure, sparsity-aware distributed compressed sensing, compressive sensing

1. Introduction

The advances in the field of telecommunication and newly developed applications have increased the need for deploying distributed wireless sensor networks (WSNs), which of multiple sensors for monitoring a specific phenomenon both in the time and space of an area of interest. There are three main challenges in WSNs, i.e., network lifetime, computational ability and bandwidth constraints [1]. In this respect, the theory of distributed compressive sensing (DCS) has been used to exploit inter- and intra-signal correlations [2]. In a typical DCS setting with a joint sparsity model (JSM), each sensor compresses its signal independently by projecting the signal onto an incoherent basis and transmitting the compressed information to the fusion center (FC). Under the right conditions, the FC can jointly reconstruct all the signals by knowing that the measured signals of each sensor are individually sparse in some basis.

In applications with limited computation and complexity capabilities, compressing the transmitted signals, as much as possible is of great importance. Suppose N is the length of the original signal satisfying Nyquist rate needed to sample a signal x, M is the CS measurement samples, and k is the sparsity order of the original signal. the compressed sensing measures only M data samples, which is $M = O(k\log(N/k))$, where $k << N$ is the order of the signal sparsity [3]. Practically, obtaining the required number of M data points needs some prior knowledge of the signal, which is not applicable in this case. So the upper bound on sparsity order is used which may causes high number of measurements. Although many natural signals of interest (i.e., smooth and piecewise smooth 1-D and 2-D signals with bounded variations) are not exactly sparse, norm $|.|_0$ may not be the desired measure of sparsity. Therefore, we derive an appropriate sparsity measure which utilizes the efficient GINI index (GI) introduced in [4].

The present paper proposes a framework for adaptively compressing the signals of a WSN and reducing the amount of the transmitted data as much as possible. Hence, an adaptive CS encoding procedure is considered at each time instant $\tau \geq 1$, where the measurements are taken with respect to the current sensors' readings $x(\tau)$. Since, the sensed signal at each time slot τ may has different sparsity pattern, we propose to set the size of the measurement matrix based on the sparsity order of the signal at the current time slot itself. We will show that by adapting the sampling rate, the required number of measurements would be allocated efficiently in such a way that we achieve significant saving with respect to the amount of information that must be transmitted by each sensor. In spite of the adaptive CS encoding scheme at the sensors, a joint reconstruction algorithm is proposed to exploit both intra- and inter-signal correlations at the decoder, which supposes an intrinsic shared part exists between the signals of the sensors. To do so, we develop a centralized DCS-based algorithm for reconstructing the ensemble of sparse signals in presence of additive white Gaussian noise (AWGN). In this regard, we address the problem of the reconstruction ensemble of signals for sensor nodes investigating a typical CS reconstruction [5], JSM model [2], and a proposed model.

The rest of this paper is organized as follows: in Related Works section, some state of the art related works are reviewed in brief. In System Model section, the whole scenario is presented in detail to adaptively measure and reconstruct the sparse signal ensemble. In Experimental Results, the performance of the proposed method is compared with the state of the art algorithms. Finally, the paper concludes in Conclusion section.

2. Related Works

In wireless sensor networks, sensors have limited computation capability and energy resources without assistance of an y established infrastructures, so many studies in the literature are conducted considering these limitations for various applications [6–8]. In [9], influences of compressive sensing parameters in compression of a common set of artificial signal on nodes' lifetime is partially discussed. Also in [10], by adjusting sampling rate, a sparse generated matrix is proposed to maintain an acceptable signal reconstruction performance. Differently from our work, the authors just rely on least absolute shrinkage and selection operator (LASSO) for reconstruction instead of addressing the problem of investigating different optimized reconstruction schemes. In [11], the usage of a weighted form of the basis pursuit is studied, and even though the energy consumption in generating the random projection matrix on the node itself is taken into consideration. Nevertheless, the aim of the paper is quite different from ours: the authors detect a specific event characterized by a well-defined frequency, and this makes it easier to train the reconstruction algorithm to detect the specified event; whereas in our approach, we address the reconstruction without an y priors about the signal ensemble characteristics.

For the purposes of joint processing of the ensemble of multiple sensor arrays an acoustic bearing estimation problem is addressed in [12]. Differently from our proposed method, the algorithm is highly specific for the application. In this paper, a more general scheme is investigated for the joint reconstruction of a large number of signals obtained from tens or hundreds of nodes. In order to obtain a better sparsifying matrix during the reconstruction phase a PCA based algorithm is proposed in [13]. The authors aim at exploiting the spatial an temporal correlation characteristics to enhance the reconstruction side proposing an approach using jointly CS and PCA whereas the sensor side, where the compression takes place, is totally neglected and much more investigated in our paper. A joint-sparse recovery from multiple measurements based on extension of the single measurement vector method is presented in [14]. In the paper authors do not investigate the performance of the algorithm against channel variations, focusing on the reconstruction of signals in lossless transmissions. Additionally, a blind reconstruction approach is proposed in [15] in order to efficiently reconstruct an unknown number of noisy components.

3. System Model

Suppose that J sensors are distributed at a number of outdoor locations measuring an event such as temperature, pressure, wind speed, etc. throughout the day. Let $\boldsymbol{X}(\tau) \in R^{N\times J}$ consist of L consecutive readings of all J sensors at time slot $\tau \in \{t-L+1, ..., t\}$. As the joint sparsity model in [2], all signals share a common component $\boldsymbol{x}_{\mathrm{c}}(\tau) \in R^N$ such that $\boldsymbol{x}_j(\tau) = \boldsymbol{x}_{\mathrm{c}}(\tau) + \boldsymbol{x}_{\mathrm{in}_j}(\tau)$, $(j \in 1, 2, ..., J)$, where $\boldsymbol{x}_{\mathrm{in}_j}(\tau) \in R^N$ is the innovation part of each signal $\boldsymbol{x}_j(\tau)$. There is a dictionary $D \in R^{N\times K}$ that sparsely represents the signal $(\boldsymbol{x}_j(\tau) = D\boldsymbol{\alpha}_j(\tau) = D(\boldsymbol{\alpha}_{\mathrm{c}}(\tau) + \boldsymbol{\alpha}_{\mathrm{in}_j}(\tau)))$ where $|\boldsymbol{\alpha}_{\mathrm{c}}(\tau)|_{\ell_0} = k_{\mathrm{c}}$ and $\left|\boldsymbol{\alpha}_{\mathrm{in}_j(\tau)}\right|_{\ell_0} = k_j$.

Each sensor measured its signal $\boldsymbol{y}_j(\tau) = \Phi_j(\tau)\boldsymbol{x}_j(\tau)$ by using an individual measurement matrix $\Phi_j(\tau) \in R^{w_j\times N}$. The samples of the sensed signal $\boldsymbol{y}_j(\tau) \in R^{w_j}$ transmitted to FC and consequently were recovered via a conventional digital communication transceiver module. Obviously, $\boldsymbol{y}_j(\tau)$ is the combination of two parts: the common part $\boldsymbol{y}_{\mathrm{c}_j}(\tau) \in R^{w_j}$ and the innovation part $\boldsymbol{y}_{\mathrm{in}_j}(\tau) \in R^{w_j}$, which can be written as:

$$\boldsymbol{y}_{\mathrm{c}_j}(\tau) = \Phi_j(\tau)\boldsymbol{x}_{\mathrm{c}}(\tau), \;\; \boldsymbol{y}_{\mathrm{in}_j}(\tau) = \Phi_j(\tau)\boldsymbol{x}_{\mathrm{in}_j}(\tau). \quad (1)$$

Therefore, it is possible to state that $\boldsymbol{r}_j(\tau) = \boldsymbol{r}_{\mathrm{c}_j}(\tau) + \boldsymbol{r}_{\mathrm{in}_j}(\tau)$, where $\boldsymbol{r}_{\mathrm{c}_j}(\tau)$ and $\boldsymbol{r}_{\mathrm{in}_j}(\tau)$ are the jth received signals in the FC corresponding to the common and innovation parts of the transmitted one. The first and simplest idea to independently recover the signals is as follows:

$$\hat{\alpha}_j(\tau) = \min_{\acute{\alpha}_j(\tau)} \left|\acute{\alpha}_j(\tau)\right|_{0 \text{ or } 1},$$
$$\text{s.t. } \left|\boldsymbol{r}_j(\tau) - \Phi_j(\tau)D\acute{\alpha}_j(\tau)\right|_2 \le \epsilon \quad (2)$$

where the reconstructed signals are attained by $\hat{\boldsymbol{x}}_j(\tau) = D\hat{\alpha}_j(\tau)$.

But it is clear that recovering the samples by solving (2) can only be useful when there is no difference between $\boldsymbol{r}_j$ and $\boldsymbol{y}_j$. This destructive difference can be caused by inaccurate reconstruction resulting from compressive sensing, which makes this recovery method impractical, especially for scenarios in which symbol errors occur.

3.1 Designing Adaptive Measurement Matrix

In this subsection, we structurally design an efficient measurement matrix for each captured signal at time instant τ. Because each signal has a different sparsity pattern, the choice of the number of sensing measurements depends on the sparsity of the signal. Hence, we first set the size of the measurement matrix for each sensed signal based on the sparsity order of the signal itself. Therefore, the measurement matrix jth sensor Φ_j is replaced with $\Phi_j(\tau)$ for each time instant $\{t-L+1, ..., t\}$ which is an $M_\tau \times N_\tau$ orthonormal matrix with $M_\tau = \mathrm{c}k_\tau \log(N_\tau/k_\tau)$ for some constant c, where k_τ is the sparsity order of the signal at τth time instant.

In signal representation, practical sparsity is defined based on different favorable properties, such as the number of non-zero coefficients in the signal representation, relative distribution of the energy among the coefficients, and many other attributes. The authors [4] examine and compare quantitatively several sparsity measures, and their findings show that the Gini index (GI) is the only measure that has all the defined properties. Hence, for the GI; given a vector $\boldsymbol{x} = [x_1, ..., x_N]$, with its elements re-ordered from smallest to largest, $x_{[k]}$ for $k = 1, 2, ..., N$, where $x_{[1]} \leq x_{[2]} \leq \ldots \leq x_{[N]}$, then there is the following:

$$\mathrm{GI}(\boldsymbol{x}) = 1 - 2\sum_{k=1}^{N} \frac{x_{[k]}}{|\boldsymbol{x}|_1}\left(\frac{N-k+0.5}{N}\right) \quad (3)$$

where $|\boldsymbol{x}|_1$ is the l_1 norm of $\boldsymbol{x}$. Because the GI is a normalized index between 0 and 1 for an y vector, 0 is given for the least sparse signal with all the coefficients having an equal amount of energy, and 1 is given for the most sparse one with all the energy in just one coefficient. As a consequence, for different sparsity measures, the number of samples assigned to the captured signal $\boldsymbol{x}_j(\tau)$ is as follows:

$$M_{\boldsymbol{x}_j(\tau)} = \left(\frac{1 - \mathrm{GI}(\boldsymbol{x}_j(\tau))}{\sum\limits_{k=t-L+1}^{\tau}\left(1 - \mathrm{GI}(\boldsymbol{x}_j(k))\right)}\right) \times M. \quad (4)$$

The number of measurements assigned for each signal is proportional to the GI index of the signal itself; because the GI does not determine the exact sparsity order. Note that via the adaptive measuring, all the preceding GI estimates from time slots $\{t-L+1, ..., t-1\}$ implicitly affect the number of samples assigned to the signal at time slot t.

3.2 Deploying a Weighting Matrix

In this subsection, the received signal of jth sensor node at the FC $\boldsymbol{r}_j(\tau) \in R^{w_j}$ is modeled as $\boldsymbol{r}_j(\tau) = \boldsymbol{y}_j(\tau) \odot \boldsymbol{\xi}_j(\tau) + \boldsymbol{n}_j(\tau)$. In which $\odot$, $\boldsymbol{\xi}_j(\tau) \in R^{w_j}$ and $\boldsymbol{n}_j(\tau) \in R^{w_j}$ show the circular convolution operator, disturbance filter, and additive noise, respectively. These errors occur during transmission of the signals. Suppose that the FC receives the common and innovation parts of the transmitted sensor signal separately; we try to estimate the filters by modeling the received common signal at time slot τ as $\boldsymbol{r}_{\mathrm{c}_j}(\tau) = \boldsymbol{Y}_{\mathrm{c}_j}(\tau)\boldsymbol{\xi}_j(\tau) + \boldsymbol{n}_j(\tau)$, where $\boldsymbol{Y}_{\mathrm{c}}(\tau) \in R^{w_j \times w_j}$ is the circulant matrix of $\boldsymbol{y}_{\mathrm{c}_j}(\tau) \in R^{w_j}$.

Supposing that the $\boldsymbol{y}_{\mathrm{c}_j}(\tau)$ signals are known by the FC, it is equivalent knowing $\boldsymbol{\alpha}_{\mathrm{c}}(\tau)$ and $\Phi_j(\tau)$s. Therefore, the estimated impulse response of the destructive filter $\hat{\boldsymbol{\xi}}_j(\tau) \in R^{w_j}$ can be achieved by solving the optimization problem, as follows:

$$\hat{\boldsymbol{\xi}}_j(\tau) = \min_{\acute{\xi}_j(\tau)} \left|\boldsymbol{r}_{\mathrm{c}_j}(\tau) - \boldsymbol{Y}_{\mathrm{c}_j}(\tau)\acute{\boldsymbol{\xi}}_j(\tau)\right|_2. \quad (5)$$

Now, the circulant matrix of the estimated weighting filters $\hat{\boldsymbol{\xi}}_j(\tau)$ is attained by the following:

$$\bar{\Xi}_j(\tau) = \begin{bmatrix} \hat{\xi}_j(1) & \hat{\xi}_j(w_j) & \cdots & \hat{\xi}_j(2) \\ \hat{\xi}_j(2) & \hat{\xi}_j(1) & \cdots & \hat{\xi}_j(3) \\ \hat{\xi}_j(3) & \hat{\xi}_j(2) & \cdots & \hat{\xi}_j(4) \\ \vdots & \vdots & \vdots & \vdots \\ \hat{\xi}_j(w_j) & \hat{\xi}_j(w_j-1) & \cdots & \hat{\xi}_j(1) \end{bmatrix}. \quad (6)$$

3.3 DCS-Based Reconstruction Algorithm

From the two earlier subsections and inspired from JSM models [2], we propose a way to jointly recover the signal ensemble of the sensor nodes, as follows:

$$R(\tau) = \bar{\boldsymbol{\Xi}}(\tau)\,\Phi(\tau)\,\Psi\boldsymbol{\alpha}(\tau) + \boldsymbol{n}(\tau), \quad (7)$$

$$R(\tau) = \begin{bmatrix} \boldsymbol{r}_1(\tau) \\ \boldsymbol{r}_2(\tau) \\ \vdots \\ \boldsymbol{r}_J(\tau) \end{bmatrix}, \boldsymbol{n}(\tau) = \begin{bmatrix} \boldsymbol{n}_1(\tau) \\ \boldsymbol{n}_2(\tau) \\ \vdots \\ \boldsymbol{n}_J(\tau) \end{bmatrix}, \boldsymbol{\alpha}(\tau) = \begin{bmatrix} \boldsymbol{\alpha}_{\mathrm{c}}(\tau) \\ \boldsymbol{\alpha}_{\mathrm{in}_1}(\tau) \\ \vdots \\ \boldsymbol{\alpha}_{\mathrm{in}_J}(\tau) \end{bmatrix}, \quad (8)$$

$$\bar{\boldsymbol{\Xi}}(\tau) = \begin{bmatrix} \bar{\Xi}_1(\tau) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \bar{\Xi}_J(\tau) \end{bmatrix}, \quad (9)$$

$$\Phi(\tau) = \begin{bmatrix} \Phi_1(\tau) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \Phi_J(\tau) \end{bmatrix}, \quad (10)$$

$$\Psi = \begin{bmatrix} D & D & 0 & \cdots & 0 \\ D & 0 & D & \cdots & \vdots \\ \vdots & & & \ddots & 0 \\ D & 0 & \cdots & 0 & D \end{bmatrix} \quad (11)$$

where $R(\tau) \in R^{W}$, $\boldsymbol{n}(\tau) \in R^{W}$, $\boldsymbol{\alpha}(\tau) \in R^{K(J+1)}$, $\bar{\boldsymbol{\Xi}}(\tau) \in R^{W\times W}$, $\boldsymbol{\Phi}(\tau) \in R^{W\times NJ}$, $\boldsymbol{\Psi} \in R^{JN\times K(J+1)}$ and $W = \sum\limits_{j=1}^{J} w_j$. To recover the desired signals, first $\hat{\boldsymbol{\alpha}}(\tau) = \begin{bmatrix} \hat{\alpha}_c^{\mathrm{T}}(\tau) & \hat{\alpha}_{\mathrm{in}_1}^{\mathrm{T}}(\tau) & \cdots & \hat{\alpha}_{\mathrm{in}_J}^{\mathrm{T}}(\tau) \end{bmatrix}^{\mathrm{T}}$ is computed by solving the optimization problem as follows:

$$\hat{\alpha}(\tau) = \min_{\acute{\alpha}(\tau)} |\acute{\alpha}(\tau)|_{0 \text{ or } 1},$$
$$\text{s.t. } \left|R(\tau) - \bar{\boldsymbol{\Xi}}(\tau)\,\Phi(\tau)\,\Psi\boldsymbol{\alpha}(\tau)\right|_2 \leq \epsilon. \quad (12)$$

Then, $\hat{\boldsymbol{x}}_j(\tau)$ is computed by following:

$$\hat{\boldsymbol{x}}_j(\tau) = D(\hat{\boldsymbol{\alpha}}_{\mathrm{c}}(\tau) + \hat{\boldsymbol{\alpha}}_{\mathrm{in}_j}(\tau)) \quad (13)$$

where $\hat{\boldsymbol{\alpha}}_{\mathrm{c}}(\tau)$ and $\hat{\boldsymbol{\alpha}}_{\mathrm{in}_j}(\tau)$s are within the $\hat{\alpha}(\tau)$ vector.

3.4 Modifying DCS-based Reconstruction Algorithm

In this subsection, we are going to improve the reconstruction accuracy and time consumed by the proposed method. In this sense, assuming that FC knows the common part $\boldsymbol{\alpha}_{\mathrm{c}}(\tau)$ of the transmitted signals for each time slot τ, then $\boldsymbol{\alpha}_{\mathrm{c}}(\tau)$ is omitted from the reconstruction of (12) by the following mathematical ways:

$$R(\tau) = \left[\bar{\Xi}(\tau)\,\Phi(\tau)\,\Psi\right]\begin{bmatrix}\boldsymbol{\alpha}_{\mathrm{c}}(\tau)\\ \boldsymbol{\alpha}_{\mathrm{in}_1}(\tau)\\ \vdots\\ \boldsymbol{\alpha}_{\mathrm{in}_J}(\tau)\end{bmatrix} + \boldsymbol{n}(\tau). \quad (14)$$

Equation (14) can be rewritten as follows:

$$\begin{aligned} R(\tau) &= \left[A(\tau)\,\|H(\tau)\right]\begin{bmatrix}\boldsymbol{\alpha}_{\mathrm{c}}(\tau)\\ --\\ \boldsymbol{\alpha}_{\mathrm{in}_1}(\tau)\\ \vdots\\ \boldsymbol{\alpha}_{\mathrm{in}_J}(\tau)\end{bmatrix} + \boldsymbol{n}(\tau)\\ &= A(\tau)\,\boldsymbol{\alpha}_{\mathrm{c}}(\tau) + H(\tau)\,\boldsymbol{\alpha}_I(\tau) + \boldsymbol{n}(\tau) \quad (15)\end{aligned}$$

where $A(\tau)$ and $H(\tau)$ are combination matrix joining of the different involved parameters and $\boldsymbol{\alpha}_I(\tau) = \left[\ \boldsymbol{\alpha}_{\mathrm{in}_1}(\tau)^{\mathrm{T}}\ \boldsymbol{\alpha}_{\mathrm{in}_2}(\tau)^{\mathrm{T}}\ \cdots\ \boldsymbol{\alpha}_{\mathrm{in}_J}(\tau)^{\mathrm{T}}\ \right]^{\mathrm{T}}$.

Then, if we define $R_{\mathrm{in}}(\tau) = R(\tau) - A(\tau)\,\boldsymbol{\alpha}_{\mathrm{c}}(\tau)$, the recovery formula is modified to find just the innovation parts of the signals, as follows:

$$\begin{aligned} \hat{\boldsymbol{\alpha}}_I(\tau) &= \min_{\acute{\boldsymbol{\alpha}}_I(\tau)} |\acute{\boldsymbol{\alpha}}_I(\tau)|_{0 \text{ or } 1},\\ \text{s.t. } & |R_{\mathrm{in}}(\tau) - H(\tau)\,\acute{\boldsymbol{\alpha}}_I(\tau)|_2 \le \epsilon. \quad (16)\end{aligned}$$

Consequently, the signal of each sensor $\hat{\boldsymbol{x}}_j(\tau)$ is reconstructed by the following formula:

$$\hat{\boldsymbol{x}}_j(\tau) = D(\boldsymbol{\alpha}_{\mathrm{c}}(\tau) + \hat{\boldsymbol{\alpha}}_{\mathrm{in}_j}(\tau)) \quad (17)$$

where $\hat{\boldsymbol{\alpha}}_{\mathrm{in}_j}(\tau)$s are within the computed $\hat{\boldsymbol{\alpha}}_I(\tau)$ vector.

3.5 Suggestion on Optimum $\boldsymbol{\alpha}_{\mathrm{c}}(\tau)$ for Fast Recovery and More Accuracy

In the earlier subsection, we proposed an efficient reconstruction algorithm without involving the common parameters $\boldsymbol{\alpha}_{\mathrm{c}}(\tau)$ in the recovery process. It seems that finding the optimum vector for $\boldsymbol{\alpha}_{\mathrm{c}}(\tau)$ can bring more improvements. Because our proposed CS decoder is based on the JSM model, the first and simplest approach to find the optimum $\boldsymbol{\alpha}_{\mathrm{c}_{\mathrm{opt}}}(\tau)$ is to solve the JSM-based optimization problem as follows:

$$\boldsymbol{\alpha}_{\mathrm{opt}}(\tau) = \min_{\acute{\boldsymbol{\alpha}}_{\mathrm{opt}}(\tau)} \left|\acute{\boldsymbol{\alpha}}_{\mathrm{opt}}(\tau)\right|_{0 \text{ or } 1}, \ \text{s.t. } \hat{X}(\tau) = \Psi\acute{\boldsymbol{\alpha}}_{\mathrm{opt}}(\tau) \quad (18)$$

where $\hat{X}(\tau) = \left[\ \hat{\boldsymbol{x}}_1(\tau)^{\mathrm{T}}\ \hat{\boldsymbol{x}}_2(\tau)^{\mathrm{T}}\ \cdots\ \hat{\boldsymbol{x}}_J(\tau)^{\mathrm{T}}\ \right]^{\mathrm{T}}$ is the concatenated vector of the recovered sensors signals via (17), $\boldsymbol{\alpha}_{\mathrm{opt}}(\tau)$ is equal to $\left[\ \boldsymbol{\alpha}_{\mathrm{c}_{\mathrm{opt}}}(\tau)^{\mathrm{T}}\ \boldsymbol{\alpha}_{\mathrm{in}_1}(\tau)^{\mathrm{T}}\ \cdots\ \boldsymbol{\alpha}_{\mathrm{in}_J}(\tau)^{\mathrm{T}}\ \right]^{\mathrm{T}}$ in which the desired optimum common component $\boldsymbol{\alpha}_{\mathrm{c}_{\mathrm{opt}(\tau)}}$ is located. Since, the optimization constraints in (18) is the maximum sparsity of the innovations, regardless to the sparsity level of the common components ($\|\boldsymbol{\alpha}_{\mathrm{c}}(\tau)\|_0$), the greater sparsity level for the innovations ($\boldsymbol{\alpha}_I(\tau)$) brings efficient reconstruction performance in terms of the consuming solution time and recovery accuracy. Actually, the idea behind the proposed criterion for more improvement is when (18) is independent from the sparsity of the ($\|\boldsymbol{\alpha}_{\mathrm{c}}(\tau)\|_0$), the common component can be omitted. Therefore, the optimization problem is turned into the following:

$$\boldsymbol{\alpha}_{\mathrm{opt}}(\tau) = \min_{\acute{\boldsymbol{\alpha}}_{\mathrm{opt}}(\tau)} \sum_{j=1}^{J} \left|\acute{\boldsymbol{\alpha}}_{\mathrm{in}_{j_{\mathrm{opt}}}}(\tau)\right| \ \text{s.t. } \hat{X}(\tau) = \Psi\acute{\boldsymbol{\alpha}}_{\mathrm{opt}}(\tau) \quad (19)$$

where the optimum common component $\boldsymbol{\alpha}_{\mathrm{c}_{\mathrm{opt}}}(\tau)$ is within the obtained optimum $\boldsymbol{\alpha}_{\mathrm{opt}}(\tau)$. Due to existence of the similar support between the common and innovation parts, this criterion compresses the energy of the signals as much as possible into the common part of the vector $\boldsymbol{\alpha}_{\mathrm{opt}}(\tau)$ and brings the maximum sparsity for the innovation parts.

3.6 Algorithm Summary and Computational Complexity

The proposed sparsity-aware adaptive compressed data acquisition method with joint signal ensemble recovery based on DCS regularization is summarized in Algorithm 1. At each time slot $\tau \ge 1$, the fusion center periodically gathers the adaptively compressed sensing measures by acquiring the reading from a subset of the sensor nodes (Step *I.*). Then, the decoder solves the weighted joint recovery problem (12) via the basis pursuit denoising (BPDN) [5], resulting in an estimate of $\mathbf{X}(\tau)$ as $\hat{\mathbf{X}}(\tau)$. Once $\hat{\alpha}(\tau)$ has been recovered (12) and the sensors signals $\hat{X}(\tau)$ are reconstructed (13), its J signals are used in the two-step modification $\boldsymbol{\alpha}(\tau)$ as $\boldsymbol{\alpha}_{\mathrm{opt}}(\tau)$ in (18) and (19), respectively (Step *III.*). Then, $\boldsymbol{\alpha}_{\mathrm{opt}}(\tau)$ is used to update the estimates for $\mathbf{x}_1(\tau), \mathbf{x}_2(\tau), \ldots, \mathbf{x}_J(\tau)$ as $\{\hat{\mathbf{x}}_1(\tau), \hat{\mathbf{x}}_2(\tau), \ldots, \hat{\mathbf{x}}_J(\tau)\}$ in 17, i.e., the estimates for sensor data are built based on the most recently obtained estimates for $\boldsymbol{\alpha}_{\mathrm{opt}}(\tau)$ given by (18) and (19). Finally, the estimated optimum common component is back once for L time slots for all sensors to modify their sensed data in advance of transmission to FC. The assumption is that the l_1 of difference signal $\left\|\mathbf{x}_j(\tau) - \mathbf{x}_j(\tau-1)\right\|_{l_1}$ would be small, meaning that the measured signal of the sensors are changing slowly over time.

Algorithm 1 Sparsity Aware Adaptive Compressed Data Acquisition With Jointly DCS Recovery

Parameters: c, L, ϵ , $\alpha_{\rm c}$ and $n_j(\tau)$
Initializations
a: Set $\tau = t - L + 1$,
b: Obtain $[\mathbf{x}_1(\tau), \mathbf{x}_2(\tau), \ldots, \mathbf{x}_J(\tau)]$, and form $\mathbf{X}(\tau)$
c: $\alpha_{\rm c} = \alpha_{\rm c_{opt}}$ and compute $\boldsymbol{x}_{\rm c}(\tau) = D\alpha_{\rm c}$
I. Adaptive CS Measurements
a: Solve (3) and (4) to obtain $\boldsymbol{GI}(\boldsymbol{x}_j(\tau))$ and $M_{\boldsymbol{x}_j(\tau)}$, respectively.
b: Compute $\boldsymbol{y}_{{\rm c}_j}(\tau) = \Phi_j(\tau)\boldsymbol{x}_{\rm c}(\tau)$ and $\boldsymbol{y}_{{\rm in}_j}(\tau) = \Phi_j(\tau)(\mathbf{x}_j(\tau) - D\alpha_{\rm c})$.
c: Deliver the adaptive CS measurements $\boldsymbol{y}_{{\rm c}_j}(\tau)$ and $\boldsymbol{y}_{{\rm in}_j}(\tau)$ in (1) to Fusion Center utilizing two different CDMA codes.
II. DCS based Jointly Signal Reconstruction
a: Estimate the weighting filters $\hat{\boldsymbol{\xi}}_j$ via solving optimization problem in (5) and, then construct $\bar{\Xi}(\tau)$.
b: Construct $R(\tau)$, $\Phi(\tau)$ and $\Psi\hat{\alpha}(\tau)$.
c: Solve (12) to obtain $\hat{\alpha}(\tau) = \left[\begin{array}{ccc} \hat{\alpha}_{\rm c}^{\rm T}(\tau) & \ldots & \hat{\alpha}_{{\rm in}_J}^{\rm T}(\tau) \end{array}\right]^{\rm T}$.
d: Reconstruct $\hat{\mathbf{X}}(\tau) = \left[\begin{array}{cccc} \hat{\boldsymbol{x}}_1(\tau)^{\rm T} & \hat{\boldsymbol{x}}_2(\tau)^{\rm T} & \ldots & \hat{\boldsymbol{x}}_J(\tau)^{\rm T} \end{array}\right]^{\rm T}$ by (12)
III. Modifying the Reconstruction Algorithm
a: Solve (18) and (19) to obtain $\boldsymbol{\alpha}_{\rm opt}(\tau)$.
b: Reconstruct the updated estimates for $\mathbf{X}(\tau)$ as $\hat{\mathbf{X}}(\tau)$.
c: Send the optimum components $\alpha_{\rm c_{opt}}$ to the sensor nodes.
d: Set $\tau := \tau + 1$, and go to **Initializations**.

Hence, we are going to an alyze the complexity of the proposed algorithm. From the encoder viewpoint, the proposed adaptive CS algorithm involves sensing, sparsity measuring, and adaptive sampling. Suppose that the total length of sensed signal $\boldsymbol{x}_j(\tau)$ at time instant τ is N, then total number of CS samples is $M_{\boldsymbol{x}_j(\tau)}$. The measuring of the sparsity order takes $O(N)$ operations, and the computational complexity of adaptive sampling is $O\left(M_{\boldsymbol{x}_j(\tau)}N\right)$. Because there are a total of J sensors distributed in the area of interest, the computational complexity for all encoders is $J \times \left(O(N) + O\left(M_{\boldsymbol{x}_j(\tau)}N\right)\right)$. From the decoder viewpoint, the reconstruction complexity depends on the algorithm utilized for the recovery. Here, we have explored three different methods, individual reconstruction by BPDN [5], joint recovery by JSM [2], and the proposed one. According to the mathematical an alysis using the interior point method [16], the reconstruction procedure can be done by $J \times (O(N^3))$, $O\left((J+1)^3 \times N^3\right)$, $O\left(J^3 \times N^3\right)$ operations for BPDN [5], JSM [2], and the proposed algorithm, respectively.

4. Experimental Results

In the following experiments, we consider a single hop wireless sensor network with $J = 25$ sensor nodes and one sink, in which the sensor nodes monitor a phenomenon over $T = 400$ sampling instants, resulting in the data ensemble $\boldsymbol{x}_j \in R^{400}, j = \{1, 2, ..., 25\}$. The sensors are deployed in a divided observation area of $\sqrt{J} \times \sqrt{J}$ grid of square areas, and in each grid, one sensor node is randomly distributed in a uniform manner. The signals from J sensors are generated such that there is a shared common component $\boldsymbol{x}_{\rm c} \in R^{400}$ between them, and each of them are sparse in a random dictionary $D \in R^{400\times512}$ with different sparsity levels (maximum 50-sparse). Consequently, the signals are sensed by different measurement matrices $\Phi_j \in R^{w_j\times400}$ with a Gaussian random set of projections (for simplicity $w_j = M_{\boldsymbol{x}_j}(\tau)$). The sensed samples $\boldsymbol{y}_j \in R^{w_j}, j = \{1, 2, ..., 25\}$ are then sent to the fusion center through a digital transceiver system. The system specifications are binary phase shift keying modulating (BPSK), 1/2 channel encoding, and DS-CDMA with 4-chip's length. Simulations are experimented for 100 frames with different $\boldsymbol{x}_j$s, and the obtained mean results are reported. The CVX Matlab toolbox [17] is used to solve the least square problem in (5). Moreover, other sparse-based optimization problems are solved by the Sparse-Lab [18] Matlab toolbox. For given sensors $J = 25$ and sampling instants $T = 400$, the normalized mean squared error (NMSE) is defined as $\mathrm{NMSE} = \frac{1}{J}\sum_{j=1}^{J}\sum_{n=1}^{\rm T}\left(\frac{\widehat{x}_j(n)}{\|\widehat{x}_j\|_2} - \frac{x_j(n)}{\|x_j\|_2}\right)^2$ where $\widehat{\boldsymbol{x}}_j(\tau)$ denotes the estimate of $\boldsymbol{x}_j(\tau)$. The sensing rate and the compression rate are $\frac{w_j}{T}$ and $1 - \frac{w_j}{T}$, respectively.

Figure 1 shows the average CS recovery error NMSE against varying the number of measurements w_j for individual BPDN recovery, joint recovery by JSM and the proposed algorithm with obtained $\alpha_{\rm opt}$ via (18) and (19). Because individual BPDN recovery neglects the intra- and inter-correlation, its performance is poor against all the other methods. It can be observed that utilizing the intra- and inter-compressibility not only requires less sensor' readings, but it also brings better recovery performance in terms of *NMSE* errors. Figure 1 illustrates that as the number of measurements increases, the performance of individual BPDN recovery and JSM recovery gradually approaches those of the proposed ones, yet the best method is the algorithm with $\alpha_{\rm opt}(\tau)$ in (19). The corresponding reconstruction times versus different measurement numbers for the algorithms is depicted in Fig. 2. The consuming time for the algorithms is close to each other in low number measurement samples, but as the number of samples increases, the execution time of our proposed method gradually changes. Compared to the individual BPDN recovery and joint recovery by JSM, our algorithm requires less reconstruction time at the higher number of measurements.

To evaluate the performance of the weighting filters, the sensed measurements are transmitted to the FC through AWGN channels, which cause bit errors. The semi-normalized mean squared error of the signal recovery for different bit error rates (BERs) is shown in Fig. 3 with the number of measurements $w_j = 50$ *to* $90, j = \{1, 2, ..., 25\}$. It can be observed that as expected, higher BERs bring more errors into the methods; however, the proposed method can significantly (almost 10 times better) compensate for destruction and recover signals while producing lower reconstruction errors. To better realize the reconstruction performance of the proposed algorithm, a frame of the obtained signal at jth sensor $(\boldsymbol{x}_j(\tau))$ and its corresponding reconstructed signal $(\hat{\boldsymbol{x}}_j(\tau))$ are presented in Fig. 4. Corresponding to the signal, the original related sparse vectors $\boldsymbol{\alpha}_j(\tau)$s and their recovered ones $\hat{\boldsymbol{\alpha}}_j(\tau)$s by FC are shown in Fig. 5. Figure 6 depicts the execution time of the DCS-based reconstruction problem (16) with respect to the different number of measurements w and sparsity levels k_0. Hence, the dimension of the sparse vector is equal to 64, which is a conventional value in image-processing tasks (patching images by 8×8 blocks). It also shows that the solving time of a CS problem depends on both the sparsity ratio of the target sparse vector $\frac{k_0}{K}$ and measuring ratio of the sensed signals $\frac{w}{K}$ which depends on the type of the application. Figure 7 shows the solving time against different lengths of the sparse vector (K) for three different characteristics of $\frac{k_0}{K}$ and $\frac{w}{K}$. The horizontal axis of the figure is scaled by 64, and each ι value on this axis means that the length of sparse vector $\boldsymbol{\alpha}$ is equal to $\iota \times 64$. As expected, more $\frac{k_0}{K}$ and $\frac{w}{K}$ and also a greater length of the sparse vector (more K) produces a greater solution time. Fig. 7 shows that for each characteristic of $\frac{k_0}{K}$ and $\frac{w}{K}$, there exists a length of sparse vector K, after which, the solution time significantly shoots up with a sharp slope. These points are marked by surrounding circles in Fig. 7. This means that if we want to have an efficient CS-based system, the best length of $\boldsymbol{\alpha}$ should be selected around these points.

The receiver operating characteristic (ROC) curve is an established means which evaluates the sensing system's performance. Figure 8 shows the ROC curves of the different methods. It verify that the proposed method is more accurate than two others. A comparison of the ROC curves in the proposed method with and without using weighting filters for the three different SNRs (indicated by red circles) is depicted in Fig. 9. Using the filters in the proposed scheme creates greater accuracy in signal reconstruction, especially in higher bit error rates (lower SNRs).

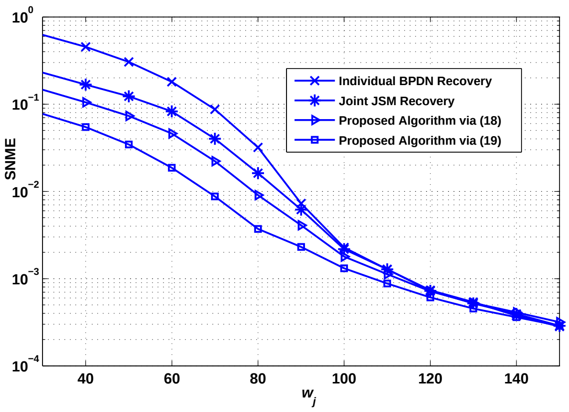


Fig. 1. The CS recovery performance of Individual BPDN recovery , joint recovery by JSM and the proposed algorithm with obtained $\alpha_{\text{opt}}(\tau)$ via (18) and (19)

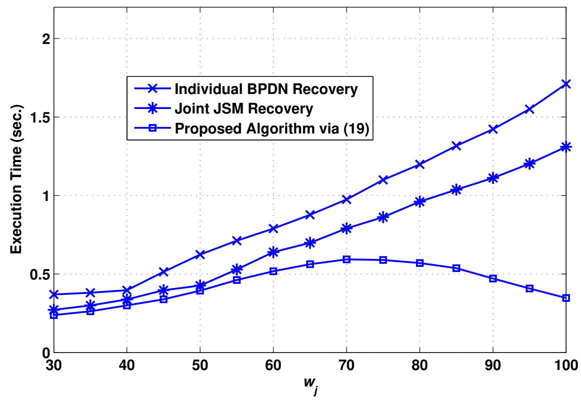


Fig. 2. The reconstruction time for individual BPDN recovery, joint recovery by JSM and the proposed algorithm with obtained $\alpha_{\text{opt}}(\tau)$ via (19).

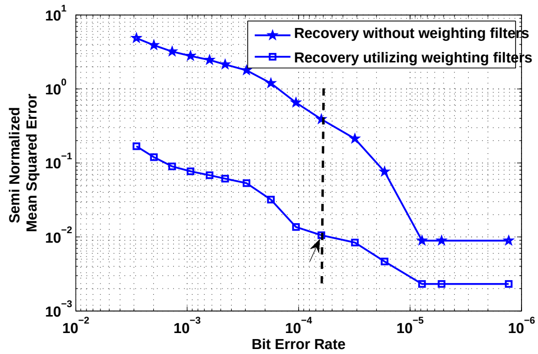
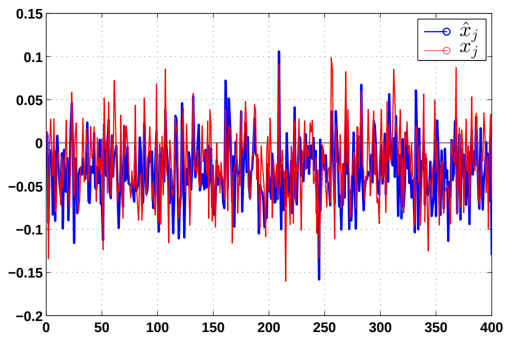
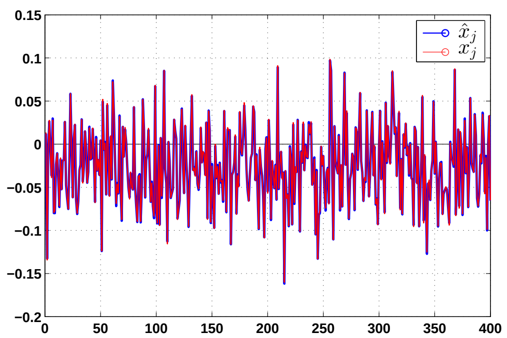


Fig. 3. The performance of the proposed joint signal reconstruction algorithm against AWGN channel effects for different BERs with and without utilizing weighting filters (9).

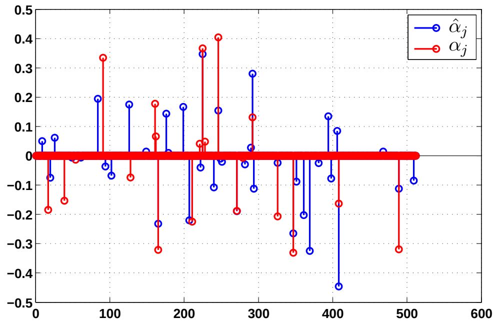


(a) Reconstruction without utilizing the weighting filters (9)

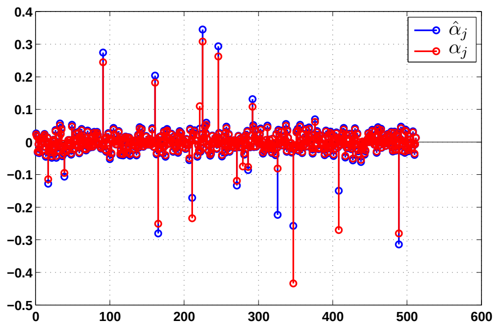


(b) Reconstruction with utilizing the weighting filters (9)

Fig. 4. A frame of obtained signal at jth sensor $(\boldsymbol{x}_j)$ and its corresponding reconstructed signal $(\hat{\boldsymbol{x}}_j)$ in the FC.



(a) Reconstruction without utilizing the weighting filters (9)



(b) Reconstruction with utilizing the weighting filters (9)

Fig. 5. Sparse vectors $\boldsymbol{\alpha}_j(\tau)$ and their reconstructed ones $\hat{\boldsymbol{\alpha}}_j(\tau)$ corresponding to the signal in Fig. 4.

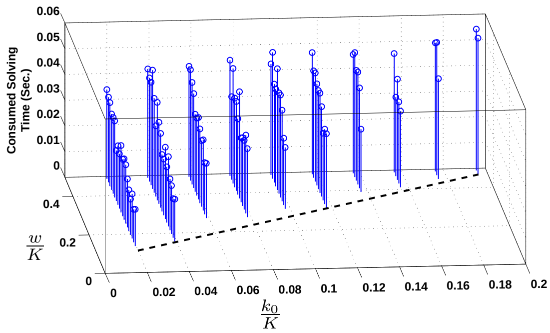


Fig. 6. The DCS-based reconstruction time depends on the both sparsity ratio and measuring ratio.

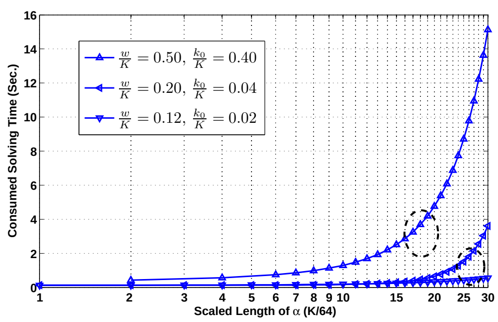


Fig. 7. Execution time of the DCS problem (16) vs. length of sparse vector (K).

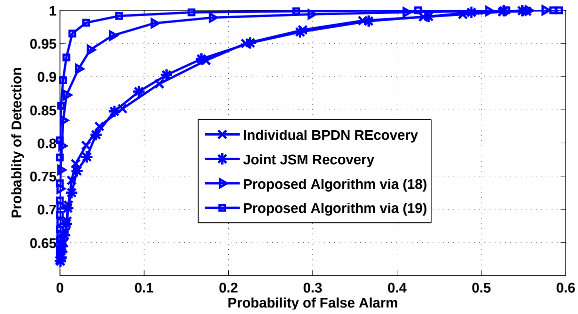


Fig. 8. ROC curves for individual BPDN recovery, joint recovery by JSM and the proposed algorithm with obtained $\alpha_{\text{opt}}(\tau)$ via (18) and (19).

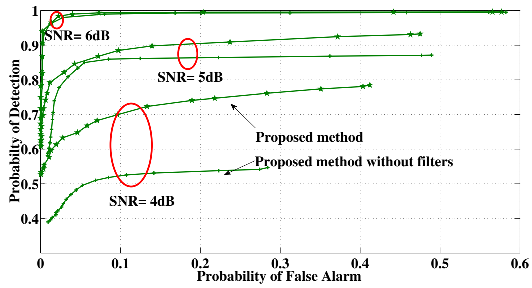


Fig. 9. ROC comparison of the proposed algorithm in both with and without utilizing the weighing filters.

5. Conclusion

In this paper, a distributed adaptive CS-based reconstruction algorithm is proposed for WSN applications. Sensor signals are adaptively measured and then are transmitted to the FC. On the other side, the perturbation of the transmission system is modeled by using disturbance filters between each sensor node and the FC. The estimated filters is embedded into the reconstruction formula to compensate for the errors. The simulation results verify that the proposed recovery algorithm can effectively improve the quality of the reconstructed signal and decrease the execution time.

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