Spectrum Sensing Based on Higher Order Cumulants and Kurtosis Statistics Tests in Cognitive Radio

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Abstract. *In this paper, new algorithms for spectrum sensing in cognitive radio based on higher order cumulants and kurtosis are proposed. The cumulants represent statistical signal processing based on pattern recognition for signals of different structure, and has low implementation complexity. Kurtosis statistics are a well-known technique for testing the Gaussianity feature of the signals. Under the assumption that a detected signal can be modelled according to an autoregressive model, noise variance is estimated from that noisy signal. The simulation results show that spectrum sensing algorithms based on the estimated normalised values of joint higher order cumulants (of fourth and sixth orders) and kurtosis are reliable for a wide range of signal-to-noise ratio environments. In order to improve performances of the spectrum sensing, the combination of these statistics tests into unique one statistic test is proposed. Simulation results have verified improvement of the performances.*

Keywords

Cognitive radio, cumulants, energy detector, kurtosis, noise variance estimation, spectrum sensing

1. Introduction

Cognitive radio (CR) is an intelligent wireless communications system capable of (re)configuring its radio-system parameters [1]. The CR is aware of its radio's operating environment, and will adapt its transmission parameters to statistical variations in the existing radio environment accordingly, by adjusting and tuning the operating parameters (waveform, protocol, frequency band, modulation mode, transmit power, etc.) in real-time. Spectral awareness, sensing spectrum availability with high reliability, and the ability to modify transmission parameters rapidly and autonomously are the most important components of the cognitive radio implementation concept [2], [3]

The non-coherent energy detector (ED) is the most popular and most commonly used scheme applied from among all spectrum sensing (SS) methods [4]. The main reason is its implementation simplicity, low cost, lower sensing time, wideband signal sensing, and lack of any requirement of any a prior knowledge about the primary user (PU) signal [5]. Signal detection is realized by comparing the energy detector output with the decision threshold. In practical systems, even noise power can be measured, but it is not stationary, which makes it difficult to achieve highly performances in signal detection with a fixed threshold. In order to improve reliability of detection, CR as an environment-aware system should promptly react to changes in the operating environment through a dynamical adaptation of the decision threshold accordingly. In [6], energy-based spectrum sensing in dynamic scenarios is studied under constraints imposed on the expected performances, and an iterative gradient-based algorithm for the SS threshold adaptation is proposed. Assuming noise variance must be estimated from the noisy signal received by CR, a closed-form solution for optimal decision SS threshold determination and modification in dynamic electromagnetic environment is proposed and analysed in [7].

Even though better performances are achieved by proposed threshold adaptations in dynamic scenario, the energy detector has many disadvantages, such as the high sensitivity of the threshold level to variations of the noise level, high sensing time in low SNR environments, inability to distinguish primary and secondary user signals (interference), inability to differentiate signals and noise, and problems in detecting spread spectrum signals. The CR's aim is to discover as many relevant data as possible about the received signals (potentially PU signals), in order to function properly in that kind of radio environment. The aim of this paper is to provide more useful relevant information about the environment to CR, by using additional signal processing. This will enable CR to react adequately, in the sense of reconfiguring the transmission parameters accordingly. The assumption is that usage of a statistically based approach in signal processing will bring additional quality to the SS process and enable differentiation between the signals as well as between the signals and noise. This will lead to improvement of the energy detector's performance.

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In the literature, it can be found a lot of statistically based SS methods. In [8] high order cumulants based SS and power recognition detector for hybrid interweave-underlay spectrum access is proposed. Leveraging high order cumulants for SS and power recognition in CR is exploited in [9]. The proposed solutions can eliminate the adverse impact of the noise power uncertainty. Multicumulants SS technique to detect signal blindly is proposed in [10]. SS algorithm based on kurtosis values estimation of the received signals is described in [11]. To set up detection threshold properly, in the early detection phase, noise should be measured and buffered [11]. The performance of a SS algorithm based on Jarque-Bera (JB) test (which is used to verify the adherence to a Gaussian distribution) under impacts of Rayleigh fading is evaluated in [12]. A modified JB test for SS subject to Rayleigh fading is presented in [13]. It was shown that the detection probability performance outperformed the original JB test [13]. There are many Goodness of Fit (GoF) tests based SS proposed in literature. One of the most important are Kolmogorov-Smirnov (it is not recommended when relevant parameters are estimated; it is performed when the mean and variance of the signal are known) and Anderson-Darling test (to decide whether the received samples are drawn from the noise Cumulative Distribution Function (CDF) or an alternative) [13], [14]. Likelihood GoF test based on Pearson Chi-square χ^2 test is proposed in [15]. It is known that, if the random variable (received signal) is normally distributed, the received energy is chi-squared distributed with 2 degree of freedom [15]. SS algorithm based on skewness and kurtosis (also known as GoF HOS testing (GHOST)) is proposed in [16].

A blind SS algorithm, which is designed for realistic operating conditions, is proposed in this paper. This novel blind SS algorithm is based on different statistical tests - higher order cumulants and kurtosis. The primary motive to develop higher order cumulants-based SS algorithms is simplicity of implementation, which plays significant role in practical applications. Besides they are not numerically complex methods, very important aspect of proposed cumulant-based SS algorithms, represents the possibility to respond to signal quality degradation in a realistic channel model (multipath propagation channel with unknown channel impulse response). That is enabled by using approach with multipath channel estimation algorithm based on fourth order moments to estimate and compensate channel's impact on SS process. Robustness to signal degradation was even improved by combination (without affecting level of the numerical complexity) of the proposed algorithms into unique one algorithm, especially in low SNR. Under the assumption that received signal can be modelled as an autoregressive signal, noice variance is estimated. Unlike [11], the proposed SS algorithm based on kurtosis is using standard deviation of kurtosis values as a threshold for signal detection, which maintaining reasonable complexity. As proposed algorithms do not require any a priori information about radio environment, the lack of knowledge about noise and PU's signal parameters becomes an irrelevant problem. Also, by using statistical processing of the detected signals, differentiation between signals and noise, as well as between signals among themselves, is enabled, and more information to CR about the radio environment is provided.

It is considered that the PU transmits signals which are modulated with some M-QAM (Quadrature Amplitude Modulation) modulation technique from the set of potential M-QAM modulation techniques. The first step of CR is to estimate noise variance. This is done under the assumption that received signals can be modelled as an autoregressive signal. This estimated noise variance is then further used for estimation of the other relevant statistical parameters for the SS process - normalised higher order joint cumulants (of the fourth and sixth orders) and kurtosis. Finally, a combinational logic circuit is proposed in order to combine these three statistically based SS algorithms into one and to improve results, especially in low SNR conditions.

The performances of the proposed algorithms are analysed and tested in a real-world communication channel (channel with multipath propagation and AWGN). The objective of this paper is to evaluate the potential practical usage, so performance evaluations are conducted, as much as possible, under realistic operating conditions. That is why during simulation, a multipath channel with unknown impulse response, is used, and all relevant channel parameters are estimated by using a dedicated estimation algorithm. The performance of the proposed algorithms were compared to modified JB test based SS algorithm [13] and to energy detector.

The paper is organized as follows: system and general problem descriptions are presented in Sec. 2. Higher order cumulants determination is described in Sec. 3, and kurtosis statistics in Sec. 4. SS algorithms with noise estimation for realistic propagation conditions are elaborated in Sec. 5. Simulation results are shown in Sec. 6. The conclusion is presented in Sec. 7.

2. System Description

The licensed user, known as PU, is transmitting signal $x(n)$ as an array of length $N (n = 0, 1, \ldots, N-1)$ complex symbols, where each array member represents the M-QAM symbol. Parameter M will take values randomly from the set $\{4, 16, 64\}$ with equal probability, each of the symbols having equiprobable occurrence. When modulation type is chosen from this set by PU, each of N symbols in that array will be uniformly modulated by using that modulation technique.

The SU is without license, and must be invisible in radio spectrum for the PU, because interference must be avoided. Any activity of the SU (CR) should not cause noticeable interference or other negative impact on the PU's performances. In order to analyse a realistic scenario, a real-world channel with multipath propagation, which is modelled as a Rayleigh fading channel, is considered. The transmitted signal is corrupted by noise and multipath propagation, and at the CR it is a significantly different signal than the original signal sent by the PU. Despite degradation, it is the only available signal for SS. The CR should process that received signal and make decisions about spectrum vacancy accordingly. The system model is schematically described in the block diagram shown in Fig. 1.

After the CR processing along with the analog to digital conversion (ADC), the received signal $c(n)$ can be expressed as

$$c(n) = x(n) \otimes h(n) + \omega(n) \tag{1}$$

with $x(n)$ being the active radio signal generated by the PU, $h(n)$ is a communication channel in a form of filter, and $\omega(n)$ the AWGN, corrupting the active signal with zero mean and variance σ_ω^2 and "$\otimes$" as a convolution operator. Signal $x(n)$ has a mean zero, with variance σ_x^2 [17].

The binary hypothesis is performed at any given time instant n in order to estimate occupancy of the frequency band, where H_0 and H_1 denote hypothesis of the absence and the presence of the PU signal, respectively, as formulated by

$$H_0 : c(n) = \omega(n), \tag{2}$$

$$H_1 : c(n) = x(n) \otimes h(n) + \omega(n). \tag{3}$$

The SS algorithm in this system is based on usage of the statistical signal processing techniques for recognition of the feature of interest present in modulated signals. Higher order statistics (HOS) are used - normalised joint cumulants of the fourth and sixth orders and normalized kurtosis. Theoretically, higher order cumulants (orders larger than two) of a normal distribution are zero (AWGN will follow normal distribution) [18], [19], as well as excess kurtosis of AWGN signals [20]. Those values are not zero for PU signals, so the idea is to use these as simple test statistics and decision criteria for SS purposes. In addition, fourth and sixth order joint cumulants have different theoretically expected values for various modulated signals, which could be used, not only for simple SS decisions, but for additional signal classification (Automatic Modulation Classification, AMC).

The performance of the spectrum sensing can be measured through the probability of successful detection of active PU, P_d, and the probability of a false alarm for presence of active PU, P_fa.

The designed SS system is blind because CR does not have information about the characteristics of the PU signals (power, modulation type,. . .) and communication channel (noise variance, impulse response). The first step is to estimate noise variance, then to use it for estimation of the other relevant HOS statistics elements for further SS implementation.

3. Cumulants

The proposed SS system is based on a specific cumulants - so called joint cumulants of several random variables of the fourth and sixth orders, [21], [22]. In this paper, it is considered that all six (for sixth order cumulants) and four (for fourth order cumulants) variables used for calculations are the same, whereby the number of used conjugated values of those variables are three (sixth order cumulants) and two (fourth order cumulants). This is not the only possible combination of variables and its conjugates for higher order joint cumulants. But, in this paper, the terms fourth and sixth order cumulants will have that meaning.

Let us begin by observing the transmitted signal - array of symbols $x(n)$ - and let it be a random variable x with mean zero. The second order joint cumulants, written as $C_{21,x}$, are defined as

$$C_{21,x} = E(|x|^2) \tag{4}$$

where $E()$ is a mathematical expectation, which is realized as average value over observed sensing interval.

According to the joint cumulant generating formula [23], the sixth order joint cumulants, labelled as $C_{63,x} = (x, x, x, x^*, x^*, x^*)$, where x^* denotes conjugate value of x, is defined as [21]

$$\begin{aligned} C_{63,x} &= E(|x|^6) - 9E(|x|^4)E(|x|^2) \\ &\quad + 12\left|E(x^2)\right|^2 E(|x|^2) + 12E^3(|x|^2). \end{aligned} \tag{5}$$

The normalised sixth order cumulants, $\hat{C}_{63,x}$, are defined as [21]

$$\hat{C}_{63,x} = \frac{C_{63,x}}{(C_{21,x})^3}. \tag{6}$$

The fourth order joint cumulants, labelled as $C_{42,x} = (x, x, x^*, x^*)$ are defined as [21]

$$C_{42,x} = E(\left|x^4\right|) - \left|E(x^2)\right|^2 - 2E^2(\left|x^2\right|). \tag{7}$$

The normalised fourth order joint cumulants, $\hat{C}_{42,x}$ are obtained as [21]

$$\hat{C}_{42,x} = \frac{C_{42,x}}{(C_{21,x})^2}. \tag{8}$$

The normalised sixth and fourth order joint cumulants theoretical numerical values for different modulation types, considered during algorithm performances testing, and for AWGN, are shown in Tab. 1.

Modulation type:	**QPSK**	**16-QAM**	**64-QAM**	**AWGN**
$\hat{C}_{63,x}$	4	2.08	1.797	0
$\hat{C}_{42,x}$	–1	0.68	–0.6191	0

Tab. 1. Theoretical normalized 6th and 4th order joint cumulants numerical values for different modulation types.

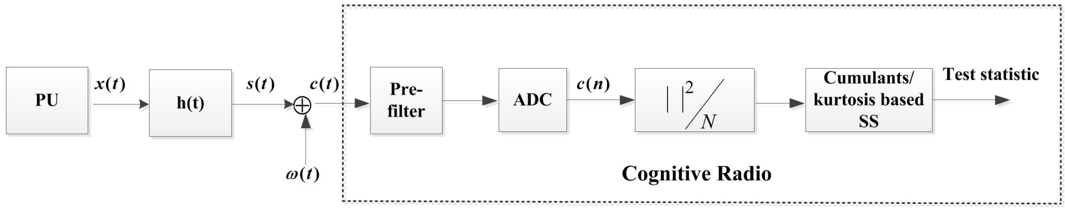

Fig. 1. System model of SS based on statistics tests.

As it can be seen from Tab. 1, the values are different for different modulation techniques. For noise, modelled as AWGN, all joint cumulants of order larger than two are equal to zero. This fact can be used for SS purposes. In addition, an AMC can be performed, but it is out of the scope of this paper. Typically, borders for differentiation are set in the middle of the adjacent intervals, but it can be subject to optimization.

4. Kurtosis

Kurtosis is a well-known statistical concept, also known as fourth standardized moment. As random variables c are complex numbers (c is an array of corrugated symbols by AWGN and fading, expressed as a complex numbers corresponding to modifications of its constellation diagram values), real-valued kurtosis, denoted as $\mathrm{kurt}(c)$, is of an interest to be determined. It can be obtained as [24], [25]

$$\mathrm{kurt}(c) = E(|c^4|) - |E(c^2)|^2 - 2(E(|c^2|))^2. \qquad (9)$$

The real-valued normalised kurtosis of complex random c, denoted as $\hat{\mathrm{kurt}}(c)$, is defined as

$$\hat{\mathrm{kurt}}(c) = \frac{\mathrm{kurt}(c)}{(E(|x^2|))^2} = \frac{E(|c^4|)}{(E(|c^2|))^2} - \frac{|E(c^2)|^2}{(E(|c^2|))^2} - 2 \qquad (10)$$

where $E()$ is a mathematical expectation, determined as average value over observed number of samples.

Bearing in mind relations between moments and cumulants, the fourth order cumulants with zero lag are unnormalised kurtosis. Considering the joint fourth order cumulants used in this paper, they obviously correspond to kurtosis. The idea is to apply this fact and to perform a kurtosis-based SS in addition to one based on fourth order cumulants, in order to improve system reliability. For kurtosis-based SS purposes, the following rule assumption has been used: detected signal c under hypothesis H_1 is non-Gaussian (having in mind the fading nature of communication channel considered in this paper), and Gaussian under H_0.

If a detected signal c has normal distribution, $\hat{\mathrm{kurt}}(c)$ will converge to zero (in an ideal case for infinitely large number of samples). As in reality the number of samples is of finite number, standard deviation must be taken into consideration, to decide if the kurtosis value of random variables can be treated as zero. In a case where random variables follow normal distribution, normalised kurtosis value will be placed inside that standard deviation range at around zero. That range of values is called Standard Error of Kurtosis (SEK), and it can be determined as [26]

$$SEK = 2\left(\sqrt{\frac{6N_{\mathrm{cr}}(N_{\mathrm{cr}}-1)}{(N_{\mathrm{cr}}-2)(N_{\mathrm{cr}}+1)(N_{\mathrm{cr}}+3)}}\right)\left(\sqrt{\frac{(N_{\mathrm{cr}}^2-1)}{(N_{\mathrm{cr}}-3)(N_{\mathrm{cr}}+5)}}\right) \qquad (11)$$

where N_{cr} is the number of samples CR uses for SS, representing an observation interval equivalent to the sensing time.

This feature can be employed as a Gaussianity test to determine if the received signal (random variables) can be classified as one that follows normal distribution. In the other words, if the $\hat{\mathrm{kurt}}(c)$ of received signal c is inside this range as determined by the (11), the received signal will be treated as AWGN. Otherwise, CR will classify that detected signal as a signal sent by the PU.

5. SS Algorithms in a Rayleigh Fading Channel

The SS algorithm in this system is based on usage of the statistical signal processing - a pattern recognition method for the signals of the different structure, and kurtosis. One of the main motivations to use these elements of signal statistical processing is implementation simplicity. In the pattern recognition systems, two subsystems are formed - one is for "feature of interest analysis" and the other is for classification according to the feature values and comparison with predefined values. Typically, features of interest are moments or cumulants. Cumulants are favourised owing to some characteristics giving them advantage in this type of applications. Higher order statistics is applied in this system, concretely normalized fourth and sixth order joint cumulants, and normalized kurtosis.

Normalised values are necessary, as the signal power at the receiver (CR) is not known a priori, and thus to avoid potential problems with different signal power levels for the same modulation type signals, as well as for the different modulation types.

Noise is modelled as AWGN, with zero mean and variance σ_ω^2. SS system is blind and designed for real-world operating systems, where no information about noise conditions is available. Besides, noise variance is not known a priori, and is not stationary due to ambient interference, temperature changes, etc. AWGN channel impact is expressed through variance σ_ω^2 and CR, with environment awareness feature, must estimate it during the observation time.

It is assumed that CR can find an appropriate rational model to represent detected signals by using second-order statistical relationships [27]. Under the assumption that the signal detected by CR can be represented according to the auto-regressive (AR) model, noise power, equal to noise variance $\sigma_\omega^2(n)$, is estimated from that noisy signal. It is realised by using the AR model of the order p for detected the noisy signal. The parameters of the AR model are estimated by using the least squares procedure for solving the set of Yule-Walker equations [7], [28].

The noise power estimation is given as [7], [28]

$$\sigma_\omega^2 = \frac{\sum_{j=1}^{p} a_j(\hat{R_c}(j) + \sum_{i=1}^{p} a_i \hat{R_c}(|j-i|))}{\sum_{j=1}^{p} a_j^2} \tag{12}$$

where $\hat{R_c}$ is an estimate of the autocorrelation coefficients of the CR signal $c(n)$, obtained from an overdetermined set of $q > p$ Yule-Walker equations using a least squares procedure [28].

For AWGN, only the first and second order joint cumulants exist (for orders larger than two, it is zero). Due to this characteristic of normal distributed random variables and the additivity feature of cumulants, it is possible based on estimated fourth and sixth order joint cumulants of the random variable c (signal $c(n)$ at the receiver CR) $\hat{C}_{42,c}$, $\hat{C}_{63,c}$, to estimate fourth and sixth order joint cumulants of the random variable x (signal $x(n)$ transmitted by the PU) $\hat{C}_{42,x}$, $\hat{C}_{63,x}$, respectively. It can be derived based on relations

$$\hat{C}_{21,c} = \hat{C}_{21,x} + \sigma_\omega^2(n), \tag{13}$$

$$\hat{C}_{42,c} = \hat{C}_{42,x}, \tag{14}$$

$$\hat{C}_{63,c} = \hat{C}_{63,x}. \tag{15}$$

Finally, by combining (13) and (14), fourth order joint cumulants of the random variable x - $\hat{C}_{42,x}$ can be expressed through the second order joint cumulants - $\hat{C}_{21,c}$ and fourth order joint cumulants - $\hat{C}_{42,c}$ of the received signal (random variable c) as

$$\hat{C}_{42,x} = \frac{C_{42,c}}{(C_{21,c} - \sigma_\omega^2)^2}. \tag{16}$$

Analogously, by combining (13) and (15), sixth order joint cumulants of the random variable x - $\hat{C}_{63,x}$ can be expressed through the second order joint cumulants - $\hat{C}_{21,c}$ and sixth order joint cumulants - $\hat{C}_{63,c}$ of the received signal (random variable c) as

$$\hat{C}_{63,x} = \frac{C_{63,c}}{(C_{21,c} - \sigma_\omega^2)^3}. \tag{17}$$

The proposed SS system is claimed to be designed to solve real-world problems, so evaluation of its performances must include analysis of operations in as realistic as possible scenario environments. The problem of SS in multipath environments is a challenging and complex problem. Besides noise impacts, multipath propagation effects must be taken into consideration. This is done by using a Rayleigh multipath fading channel, which is modelled as a finite impulse filter of length L and coefficients $h(k)$, $k = 0, 1, 2, \dots, L-1$. The received signal is a sum of convolution sum between emitted signal $x(n)$ and impulse channel response $h(k)$, and AWGN

$$c(n) = \sum_{k=0}^{L-1} h(k)x(n-k) + \omega(n). \tag{18}$$

In order to estimate correct values of cumulants, propagation channel impact must be taken into consideration. It is quantised through the coefficient β [29]. The scaling of cumulants values with β is done with purpose to compensate channel effects. In this scenario, determination of the normalized joint cumulants of sixth and fourth order for multipath propagation channel is established as [21]

$$\hat{C}_{63,x} = \frac{1}{\beta}\frac{C_{63,c}}{(C_{21,c} - \sigma_\omega^2)^3}, \beta = \frac{\sum_{k=o}^{L-1} |h(k)|^6}{(\sum_{k=o}^{L-1} |h(k)|^2)^3}, \tag{19}$$

$$\hat{C}_{42,x} = \frac{1}{\beta}\frac{C_{42,c}}{(C_{21,c} - \sigma_\omega^2)^2}, \beta = \frac{\sum_{k=o}^{L-1} |h(k)|^4}{(\sum_{k=o}^{L-1} |h(k)|^2)^2}. \tag{20}$$

The coefficient β is determined by the estimated channel coefficients. The proposed system is performing blind SS, so channel impulse response is not known a priori. CR must estimate channel coefficients and the coefficients β based on the received signal $c(n)$. It is done by using higher order statistical elements, as it was shown in [29], [30]. This approach is much less complex comparing to different types of equalizers. For this purpose, we are using the approach with fourth order moments m_4^c of the received signal for lags τ, ρ and Θ respectively, [21, 29, 30]

$$m_4^c(\tau, \rho, \Theta) = E\left[c(n)c(n+\tau)c(n+\rho)c(n+\Theta)\right]. \tag{21}$$

Lags $L-1$, $L-1$ and k represent values τ, ρ and Θ respectively in moments definition [22], and by replacing them, what is obtained as

$$m_4^c(L-1, L-1, k) = E\left[c(n)c(n+L-1)c(n+L-1)c(n+k)\right] \tag{22}$$

where $k = 0, \dots, L-1$. For these lags values, it can be shown that the next relation is valid:

$$m_4^c(L-1, L-1, k) = \gamma_4^x h(0)h(L-1)h(L-1)h(k) \tag{23}$$

where $\gamma_4^x = E\left[x(n)^4\right] = E(x^4)$. According to the moment m_4^c value for $k = 0$, the following relation is established

$$m_4^c(L-1, L-1, 0) = \gamma_4^x h(0)h(L-1)h(L-1)h(0). \quad (24)$$

Finally, the normalised channel coefficients $\hat{h}(k)$ can be calculated as

$$\hat{h}(k) = \frac{h(k)}{h(0)} = \frac{m_4^c(L-1, L-1, k)}{m_4^c(L-1, L-1, 0)}. \quad (25)$$

At the end, by inserting $\hat{h}(k)$ instead of $h(k)$ in equations (19) and (20), channel coefficient β is determined. Then, by using it and estimated noise value from equation (12), the correct values for $\hat{C}_{63,x}$ and $\hat{C}_{42,x}$ can be expected. Finally, considering these values and values from Tab. 1, the SS process can be implemented under a real-world channel conditions.

Finally, SS under a real-world channel conditions is implemented by comparing determined values from equations (19) and (20) with values from Tab. 1 (for cumulants-based SS) and by determining value from equation (10) and comparing it with value from equation (11) (for kurtosis-based).

6. Performance Evaluation

An environment with PU which transmits a signal in a form of an array of M-QAM symbols (M runs values from set $\{4, 16, 64\}$) is now considered. CR only has assumptions about potential types of complex modulation employed by PU, but which one exactly is in use at the moment is not available information. The transmitted signal (PU signal) is modified by passing through a multipath propagation channel (Rayleigh fading channel model used), represented in a form of a filter with finite impulse response. The received signal at CR is additionally corrupted by noise, which is modelled as AWGN. The proposed SS system is blind, and the channel impulse response is not known, and should be estimated on the basis of the received signal, together with noise variance, PU power and the other relevant SS parameters. After receiving the signal, CR should pre-filter it, then sample and convert it to digital, then process it in an adequate way in order to estimate all relevant statistical parameters for SS and to implement test statistics, and finally to make decisions about spectrum vacancy.

The proposed SS algorithm is based on HOS and kurtosis test statistics. Normalized joint cumulants have different values for different modulation type. Typically, border values for differentiation are settled up in the middle of two adjacent intervals in the case of HOS, and within absolute range (SEK value) in the case of kurtosis. For the simulations made in this paper, the array length of PU signals is 10,000 symbols, and CR samples 8,000 of them, which represents SS observation interval. For this chosen value $SEK = 0.054$.

Each of the above-mentioned test statistics can be used independently. The assumption is that the combination of these three different test statistics criteria into unique one would improve SS performance, expressed through probability of successful detection P_d. The idea is to use a combinational logic circuit, which is in fact logical OR circuit. Inputs in circuit would be logical 0/1 from each independent test statistics (hypothesis H_1 is converting to logical 1 and hypothesis H_0 to logical 0). Output C_out from logical circuit would be then converted to the appropriate hypothesis ($C_\mathrm{out} = 0$ to H_0 and $C_\mathrm{out} = 1$ to H_1). If $C_\mathrm{out} = 0$, spectrum is free for usage, otherwise PU is active. Block diagram of SS algorithm based on combination of test statistics is shown in Fig. 2.

The performance of the proposed algorithms was compared to modified JB test based SS algorithm [13] and to energy detector (using fixed threshold). For testing these two algorithms, it was assumed noise variance is a priori known and threshold was determined under assumption P_fa was fixed to 0.1.

6.1 Simulation Results

Performance evaluation of the proposed dynamical SS system was arrived at through 1,000 Monte Carlo simulations in MATLAB software [31]. Comparisons were made possible through probability of successful detection P_d for a different signal to noise environments.

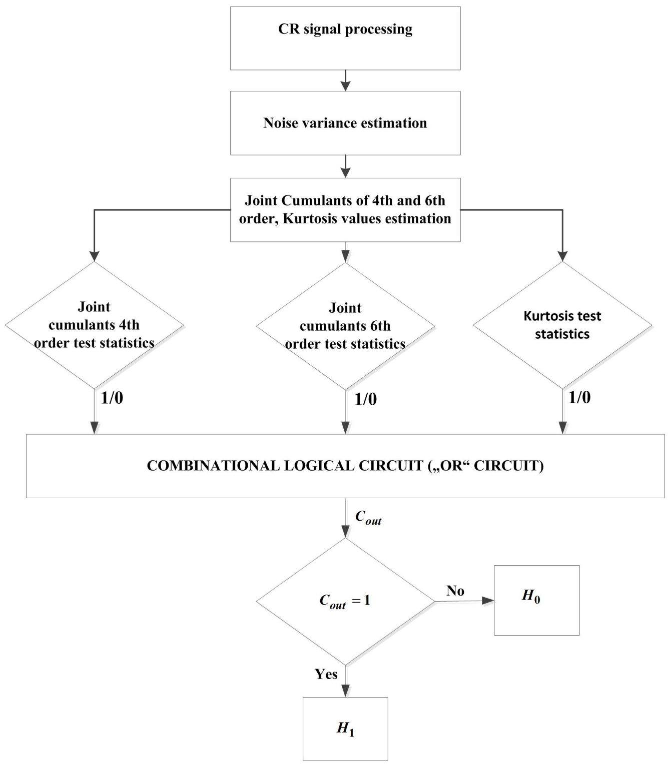

Fig. 2. Block diagram of SS algorithm based on combinational logic of test statistics.

For each Monte Carlo simulation, an array of 10,000 M-QAM symbols are generated (for $M = 4, 16, 64$). Multipath propagation is modelled as a Rayleigh fading channel, a filter with finite impulse response of length $L = 4$ [21], while noise is modelled as AWGN. Filter coefficients and noise variance are not known a priori, and CR should estimate (CR will use 8,000 samples as a time resolution for spectrum sensing) each relevant parameter by adequate processing of the detected signal. The parameters that were used for YW equations are $p = 2, q = 78$ [6]. The probability of successful detection, P_d, of the different modulated PU signals (QPSK, 16-QAM and 64-QAM) in a real-world channel (multipath propagation and AWGN) in a wide SNR range (from $[-25, 20]$ dB) by applying different test statistics is shown in Figs. 3–5. The labels "C42", "C63", "Kur", "Comb", "ED" and "JBm" in all these mentioned figures clearly represent SS test statistics based on normalised joint sixth order cumulants, fourth order cumulants, kurtosis, combination of these three test statistics, energy detector and modified JB test, respectively.

It is evident from Figs. 3–5 that proposed CR in a real-world channel (multipath propagation and AWGN channel) has good SS performances in terms of success in detection of PU signals. It is at about 90% starting from $SNR = -8$ dB, and above that value for algorithms based on kurtosis and combination of the statistics, and $SNR = -8$ dB for algorithm based on fourth order cumulants. For algorithm based on the sixth order joint cumulants maximum value is around 85% for QPSK modulation type. The worse performance in terms of P_d of algorithms based on sixth order cumulants comparing to fourth order cumulants can be explained due to dispersion of estimated cumulants values and positions of expected theoretical values shown in Tab. 1. From $SNR = -5$ dB algorithms using kurtosis and combination of the statistics completed SS process without making wrong decisions, $P_d = 100\%$ (for each PU modulation type - QPSK, 16-QAM, 64-QAM). Generally, the CR objective for SS process is to maximize P_d, and to achieve a value as close as possible to 100%. During the design of this SS system, the aim was to achieve values for P_d equal to or above 90%. This can be used as a reference value to determine the critical SNR values where these kind of SS algorithms can be employed (individually or in combination) to satisfy the condition of recommended value $P_d \geq 90\%$. Also, the initial assumption about improved SS performance of the combination of these three different statistics algorithms when compared with each individual algorithm, was verified for each scenario with QPSK, 16-QAM and 64-QAM PU signals for all range of simulated SNR environment. Always the best results for P_d were achieved by SS algorithm based on combination of test statistics. It is crucial for negative values of SNR, as it significantly improves performance. Regarding other two types of tested SS algorithms, ED achieved $P_d = 90\%$ for $SNR = 0$ dB and $P_d = 100\%$ for $SNR = 3$ dB for each modulation type used. SS algorithm based on modified JB test achieved $P_d = 100\%$ for $SNR = 8$ dB. For the less SNR values, signal was not detected with a big success.

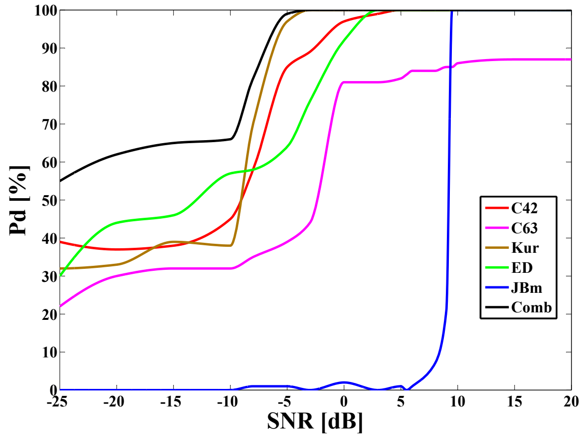

Fig. 3. Different SS algorithms performance P_d comparison, for QPSK PU signals and multipath propagation and AWGN channel.

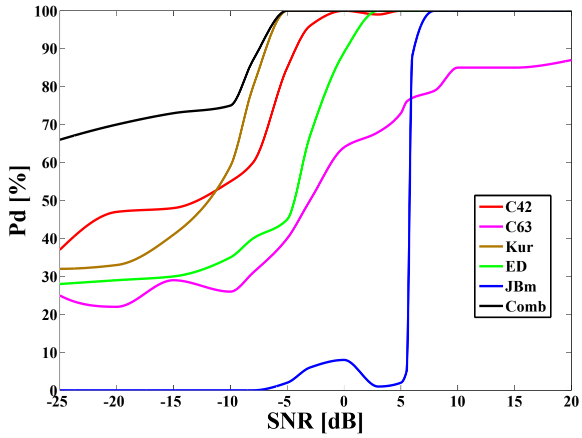

Fig. 4. Different SS algorithms performance P_d comparison, for 16 QAM PU signals and multipath propagation and AWGN channel.

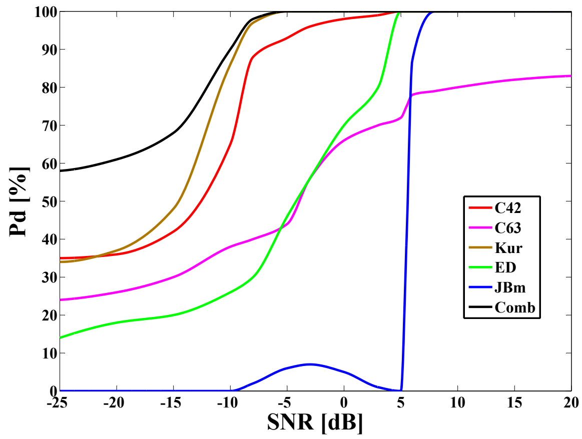

Fig. 5. Different SS algorithms performance P_d comparison, for 64 QAM PU signals and multipath propagation and AWGN channel.

Statistics tests	C42	C63	Kur	Comb
$P_{fa}[\%]$	26.67	20	6.67	20

Tab. 2. Different SS algorithms performance comparison through P_{fa}.

Variance of the sample estimates for higher order statistics elements is drastically increased due to multipath propagation, what reduces SS performances, especially in terms of additional signal processing (i.e. AMC function). The higher the order the larger tends to be the variance in the estimated cumulants. Channel coefficients estimation error will contribute to the increase of a standard deviation of estimated cumulants, what is additionally degrading SS performances. Improvement of the channel estimation algorithm will result in improvement of multipath channel impact compensation, and SS performances in general. Also, noise variance estimation error reduces SS performances of the proposed algorithms. It is smaller for higher SNR values, which implies better noise variance estimation and SS performance. All relevant SS parameter estimation errors can be minimized by increasing the estimation intervals (using larger number of samples for SS), but it will increase the overall SS duration.

For SS performance evaluation, it is of interest to test probability of false alarm P_{fa}. In Tab. 2 the achieved values for P_{fa} during conducted simulations for different performed tests statistics are presented. The column labels "C42", "C63", "Kur", "Comb" clearly represent SS test statistics based on normalised joint sixth order cumulants, fourth order cumulants, kurtosis and combination of these three test statistics, respectively.

Besides maximization of probability of successful detection, P_d, another aim of the SS algorithm is to minimize the probability of false alarms, P_{fa}. It is clear, from Tab. 2, that the best performance in terms of P_{fa} has SS algorithm which uses kurtosis test statistics. Generally, the appropriate SS algorithm can be chosen, based on designing aims, bearing in mind results for P_d and P_{fa} mentioned above in this section.

7. Conclusion

The blind SS algorithms based on tests statistics - higher order joint cumulants (fourth and sixth orders) and kurtosis, were proposed in this paper, as well as algorithm based on the combination of these mentioned algorithms. The biggest advantage of the proposed SS algorithms is that CR does not require any information about PU signals, noise conditions, propagation channels, or power levels (as we are using normalized values) for SS process implementation. According to the conducted performance evaluation tests, achieved results and design objectives, it can be concluded what the appropriate SS algorithms are in terms of the successful detection and false alarm probability in a different SNR environment.

The proposed algorithms have low implementation complexity, as well as acceptable computational order, which makes them even more attractive for potential usage. Bearing in mind that these are blind SS algorithms and that a high level of accuracy in detection of the PU signals was demonstrated by them (especially in the case of the one using combinational logic), they have big potential for wide usage. The price for the algorithm based on combination of the different statistics is the very slight time delay needed for additional processing, but performances are improved.

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