# A Novel Mega-stable Chaotic Circuit

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**Abstract.** *In recent years designing new multistable chaotic oscillators has been of noticeable interest. A multistable system is a double-edged sword which can have many benefits in some applications while in some other situations they can be even dangerous. In this paper, we introduce a new multistable two-dimensional oscillator. The forced version of this new oscillator can exhibit chaotic solutions which makes it much more exciting. Also, another scarce feature of this system is the complex basins of attraction for the infinite coexisting attractors. Some initial conditions can escape the whirlpools of nearby attractors and settle down in faraway destinations. The dynamical properties of this new system are investigated by the help of equilibria analysis, bifurcation diagram, Lyapunov exponents' spectrum, and the plot of basins of attraction. The feasibility of the proposed system is also verified through circuit implementation.*

## Keywords

Multistability, chaotic oscillators, basin of attraction, coexisting attractors

## 1. Introduction

Multistability is a critical phenomenon in dynamical systems [1]. Sometimes multistability is undesirable. That is the case when it is essential for us to have the system in one specific attractor, but due to even small changes in parameters or external disturbance, the system's state exits from that attractor and fall into another coexisting attractor. On the other hand, it allows adaptability in the system's administration without modifying parameters. It would be possible with the appropriate control tactics to induce switching between different coexisting states [1], [2].

Recently there has been growing attention in finding chaotic systems with special qualities. Systems with no equilibrium [3], [4], with stable equilibria [5], [6], with curves of equilibria [7–9], with surface of equilibria [10–12], with multi-scroll attractors [13], with hidden attractors [14], [15], with amplitude control [16], [17], with simplest form , having hyperchaos [18–20], having fractional order form [21–23], with topological horseshoes [24], [25], and with extreme multistability [26–29], are examples of them. Another major category of chaotic systems includes periodically-forced nonlinear oscillators [30]. Almost all conventional chaotic systems are systems with a finite number of fixed points [30]. Recent researches have laid a platform to formulate systems with an infinite number of equilibrium points [31], [32] Local features of such equilibrium points may or may not influence the global response of the chaotic system. A very recent category of chaotic systems are systems with mega-stability. Mega-stability is the coexistence of a countable infinity of nested attractors in a dynamical system [33]. In [34], mega-stability found in a quasi-periodically forced system exhibiting. A new mega-stable nonlinear oscillator with infinite islands of self-excited and hidden attractors reported in [35]. A new oscillator with infinite coexisting asymmetric attractors introduced in [31]. Some other such examples have been proposed recently [36–39].

In this paper, based on the systems in [40] we propose a revised oscillator with an infinite number of coexisting limit cycles. Interestingly, the forced version of this new oscillator can display chaotic solutions. In this oscillator, many of the initial limit cycles vanish while some new limit cycles and strange attractors are born (depending on the parameters). Also, another infrequent feature of this system is the complex basins of attraction for the infinite coexisting attractors. Some initial conditions can avoid the whirlpools of nearby attractors and settle down in faraway destinations. In the next section, the new oscillator is introduced and investigated. In Sec. 3, the forced version of this oscillator is presented, and its dynamical properties are examined by the help of the bifurcation diagram, Lyapunov exponents' spectrum, and the plot of basins of attraction. Also with a circuit implementation, its feasibility for possible engineering application is displayed in Sec. 4. Finally, discussion and conclusion are given in Sec. 5. It should be noted that we have used MATLAB for all the simulations. Attractors detected manually by trial and error. Limit cycles were distinguished from chaotic attractors by the help of Lyapunov exponents, using Wolf algorithm [41].

## 2. The New Oscillator

Consider system (1),

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -\cos(ax) + by\cos(x). \end{aligned} \tag{1}$$

This system is a modification of system (2) [40]:

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -x + y\cos(x). \end{aligned} \tag{2}$$

The number of equilibrium points in system (1) is infinite. They are located in $(\frac{(2k-1)\pi}{2a}, 0)$ where $k$ is an arbitrary integer number.

Considering $a = 0.3$ and $b = -0.1$ (these values have been selected to find chaotic solutions in the forced version of system (1), which will be discussed in next sections), we focus on system (3):

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -\cos(0.3x) - 0.1y\cos(x). \end{aligned} \tag{3}$$

The Jacobian of the above system in its equilibria is:

$$J = \begin{bmatrix} 0 & 1 \\ 0.3\sin(0.3x) + 0.1y\sin(x) & -0.1\cos(x) \end{bmatrix} \xrightarrow{(x,y)=\left(\frac{5(2k-1)\pi}{3},0\right)}$$

$$\begin{aligned} J &= \begin{bmatrix} 0 & 1 \\ 0.3\sin\left(\frac{(2k-1)\pi}{2}\right) & -0.1\cos\left(\frac{5(2k-1)\pi}{3}\right) \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 \\ 0.3(-1)^{k+1} & -0.1\cos\left(\frac{5(2k-1)\pi}{3}\right) \end{bmatrix}. \end{aligned} \tag{4}$$

So, the eigenvalues will be:

$$\begin{aligned} &|\lambda I - J| = 0 \rightarrow \\ &\begin{vmatrix} \lambda & -1 \\ -0.3(-1)^{k+1} & \lambda + 0.1\cos\left(\frac{5(2k-1)\pi}{3}\right) \end{vmatrix} = 0 \rightarrow \\ &\lambda^2 + 0.1\cos\left(\frac{5(2k-1)\pi}{3}\right)\lambda - 0.3(-1)^{k+1} = 0 \rightarrow \\ &\lambda_{1,2} = \frac{-0.1\cos\left(\frac{5(2k-1)\pi}{3}\right) \pm \sqrt{\left(0.1\cos\left(\frac{5(2k-1)\pi}{3}\right)\right)^2 + 1.2(-1)^{k+1}}}{2}. \end{aligned} \tag{5}$$

Thus, any equilibrium in which both Real$(\lambda_{1,2}) < 0$ is stable. Otherwise the equilibrium is unstable.

Figure 1 is a plot of trajectories in system (3) for 51 different initial conditions located on the $x$-axis (from $x = -50$ to $x = +50$ with steps equal to 2). The stable equilibrium points are shown by blue circles, while unstable equilibrium points are given by red crosses. Each trajectory is plotted for 1000 seconds. The first 800 seconds of each trajectory is plotted with green dots, showing the transient parts of trajectories. The last 200 seconds of each trajectory is plotted with thick black lines, showing the steady state of that trajectory.

It can be seen that six limit cycles and four stable equilibria coexist in the shown area (note that they are examples of an infinite number of attractors around the $x$-axis). The basin of attraction for these attractors can be seen in Fig. 2.

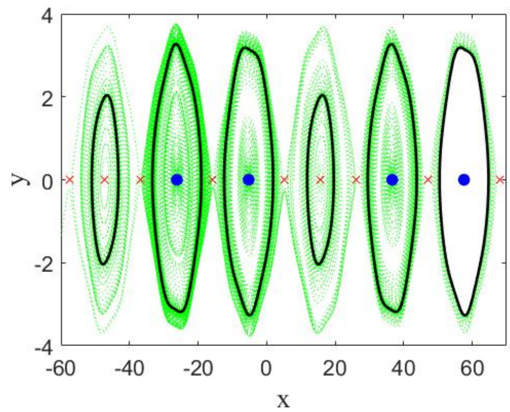


**Fig. 1.** Trajectories in system (3) for 51 initial conditions located on the $x$-axis (from $x = -50$ to $x = +50$ with steps equal to 2). The stable equilibrium points are shown by blue circles, while unstable equilibrium points are displayed by red crosses. The transient parts of trajectories are shown by green dots. The attractors outside this frame are not shown. The total of 10 attractors (6 limit cycles and 4 stable equilibria can be observed in this plot). For more details see the text.

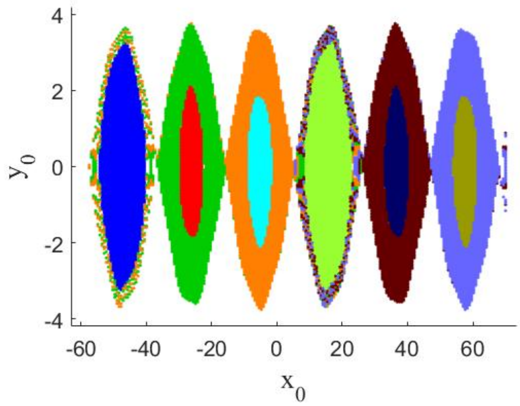


**Fig. 2.** Basin of attraction plot for the 6 limit cycle attractors and 4 stable equilibria shown in Fig.1. Initial conditions in the white area go to attractors outside this frame. This figure is obtained by a grid of $200 \times 200$ initial conditions. The color of each smooth area belongs to the attractor inside it.

## 3. The Forced Chaotic Oscillator

By adding a periodic forcing function to system (3), a new oscillator is introduced:

$$\begin{aligned}\dot{x} &= y,\\ \dot{y} &= -\cos(0.3x) - 0.1y\cos(x) + A\sin(\omega t).\end{aligned} \tag{6}$$

We are interested in finding chaotic solutions in this system. Different combinations of $(A, \omega)$ may result in chaotic solutions. Selecting $\omega = 0.5$, we choose $A$ as the bifurcation parameter. Figure 3 shows the bifurcation diagram and Figure 4 shows the Lyapunov exponents of system (6) versus $A$. We can see that in $A$ between zero and approximately 0.04, the dynamical behavior of system (6) is attracting torus (two zero and one negative Lyapunov exponents [30]). After that, suddenly the solution is converging to a stable equilibrium (one zero and two negative Lyapunov exponents). In the area after $A = 0.07$ some chaotic solutions are born containing periodic windows. Some period doubling route to chaos can be observed in those areas.

Choosing $A = 0.1$ (and $\omega = 0.5$) from the chaotic region, we continue our analysis. Figure 6 shows impressive features of system (6). This is a plot of coexisting attractors for the same initial conditions used in plotting Fig. 1. For a better demonstration of this figure, we have zoomed some parts of it and shown them in Fig. 7. Also, the time-series and attractor for initial condition $(1, 1)$ are plotted in Fig. 5. In Figs. 6 and 7, it can be observed that the initial limit cycles are now replaced by some coexisting strange attractors and new limit cycles. It is interesting that some new limit cycles are the results of merging old limit cycles. Also, a unique phenomenon can be detected in Fig. 6. Some initial conditions are not get trapped in nearby attractors. Instead, they travel far away and settle down in an unexpected attractor.

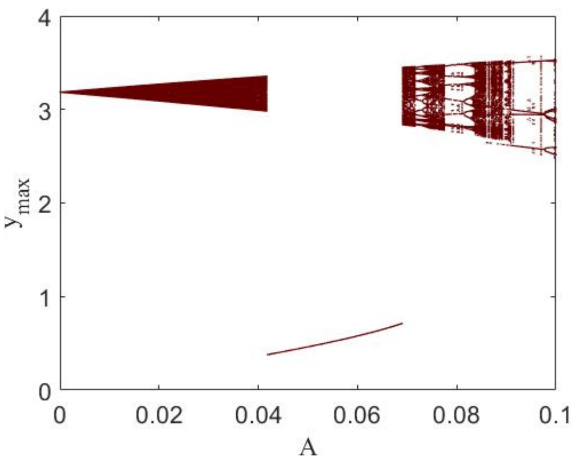


**Fig. 3.** Bifurcation diagram when changing parameter $A$ in system (6) with $\omega = 0.5$. The initial conditions for every value of $A$ were $(0, 0)$.

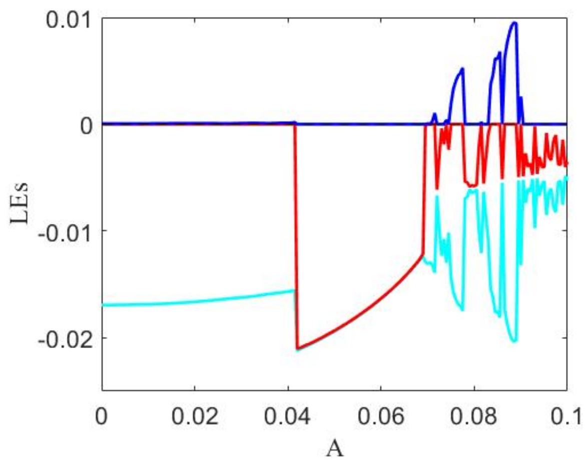


**Fig. 4.** The Lyapunov exponents' spectrum, corresponding to the bifurcation diagram in Fig. 3.

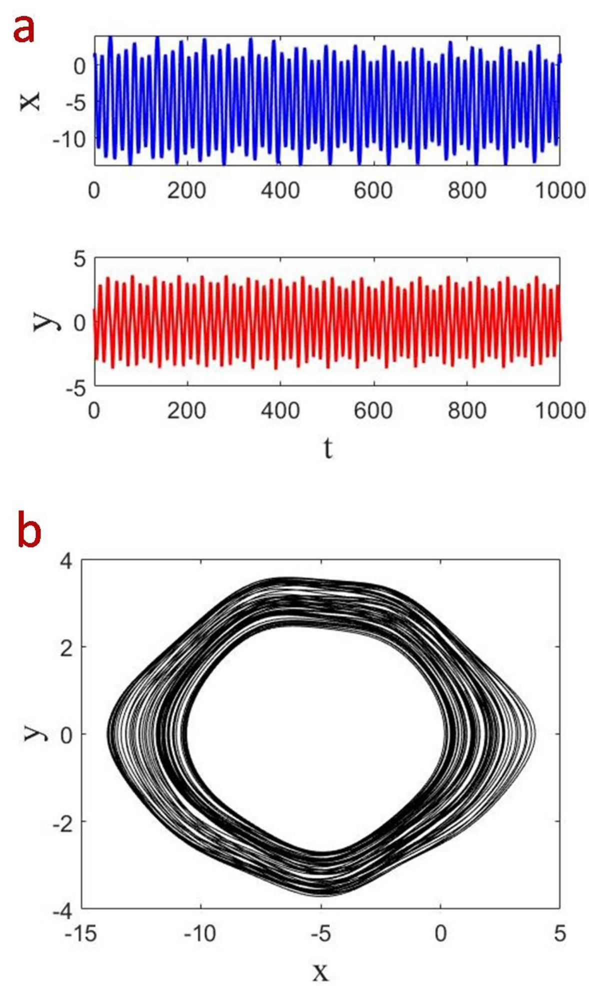


**Fig. 5.** a) Time-series and b) trajectory in system (6) for $A = 0.1$ and $\omega = 0.5$ for the initial conditions $(1, 1)$.

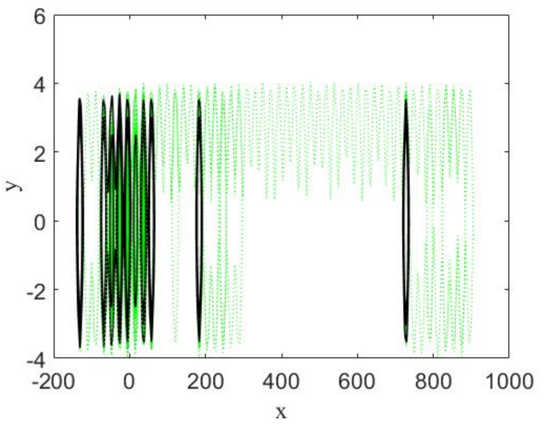


**Fig. 6.** Trajectories in system (6) for $A = 0.1$ and $\omega = 0.5$ for the same initial conditions used in Fig.1.

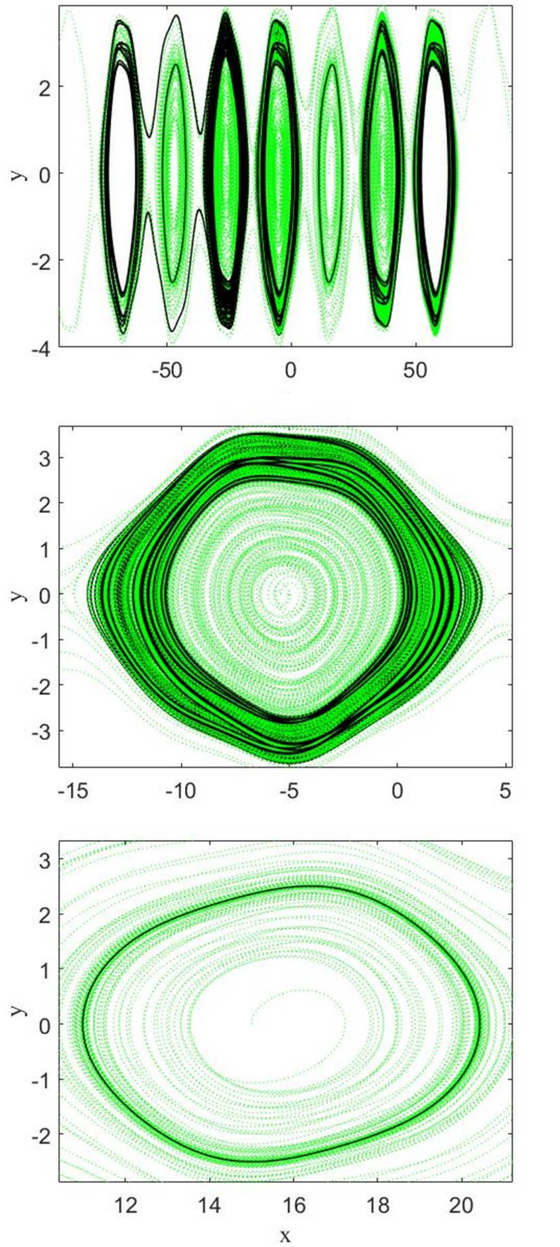


**Fig. 7.** Some zoomed parts of Fig. 6.

# 4. Circuit Design

Previous researches have described general methods in order to implement mathematical models using voltage-mode devices [42] and current-mode active elements [10]. It is possible to implement the forced chaotic oscillator (6) by using a circuit [43–51] as designed in Fig. 8. Two voltages at the capacitors $(C_1, C_2)$ are $X$ and $Y$. In Fig. 8, the value of $0.3X$ has been realized by two inverting amplifiers $(U_4, U_5)$. Integrated circuits $U_1$–$U_6$ are TL084 operational amplifiers. It is noted that we have only presented the cosine transfer functions as two-port lumped circuits in Fig. 8. In fact, the cosine function in the circuit equations can be realized by using trigonometric function generator AD639 [52], [53]. For the designed circuit, we have selected $R_1 = R_2 = R = 10\,\text{k}\Omega$, $R_3 = R_4 = R_5 = 100\,\text{k}\Omega$, $R_6 = 30,\text{k}\Omega$, $C_1 = C_2 = C_3 = C = 10\,\text{nF}$, and $f = 0.795\,\text{kHz}$. PSpice trajectories in Fig. 9 show the circuit's chaos.

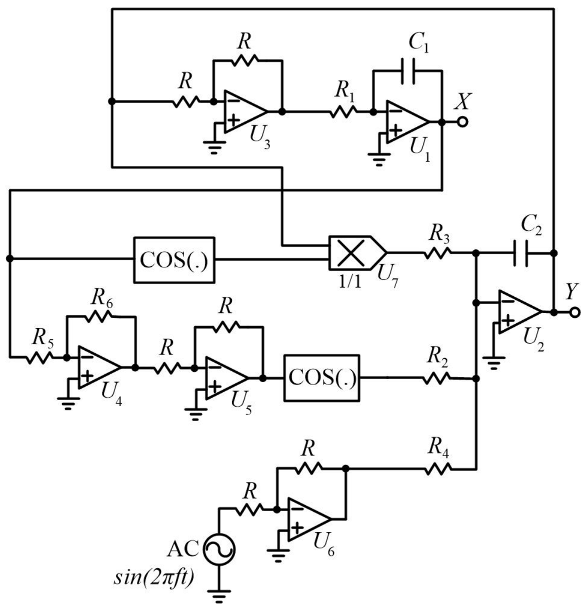


**Fig. 8.** The forced chaotic oscillator (6) emulated in a circuit.

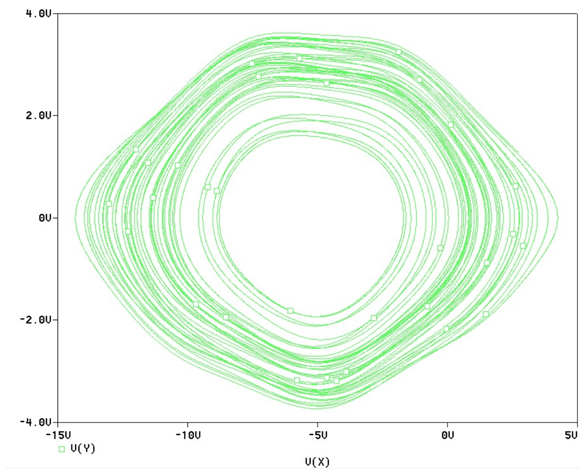


**Fig. 9.** PSpice trajectories displayed in the circuit.

# 5. Conclusion

Introducing rare dynamical oscillators with unusual properties has been a hot topic in nonlinear dynamics recently. In this paper, we designed and investigated a new mega-stable oscillator. This new system had an infinite number of coexisting attractors (limit cycles and stable equilibria). Adding a forcing term to this oscillator a new oscillator obtained which was capable of showing very rich dynamical solutions torus, chaos, and limit cycle. The initial conditions in this system can escape neighboring attractors and settle down in unexpected far destinations. The forced chaotic oscillator was emulated in a circuit. However, we think due to saturation in the elements, it is difficult to obtain other attractors in the circuit results.

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