Performance of Cell-Free Systems with Channel Reciprocity Errors

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Abstract. *Cell-free systems are characterized by the ab*sence of a cell-based spatial subdivision. In these systems a large number of access points may serve each user, which contribute to improve signal transmission conditions. In this context, it is important to obtain equations that describe the behavior of the system, as a function of its main parameters. Such equations become more complete when more effects are taken into account. One of these effects is the loss of channel reciprocity due to radiofrequency (RF) mismatch. This paper proposes the introduction of a multiplicative model for the reciprocity errors resulting from RF mismatch in all devices of a cell-free model. Additionally, it also proposes the use of different levels of mismatch for each device. The main contribution of this work is an analytical expression for the downlink achievable rates in the presence of multiplicative reciprocity errors due to RF mismatch. Based on it, one can compute the approximate value of the achievable rates. The analytical expression is used in scenarios with and without line-of-sight. It is shown that the analytical expression is very close when there is line-of-sight, as it provides achievable rate values closer to that obtained by using Monte Carlo simulation.

Keywords

Achievable rates, cell-free systems, channel reciprocity, line-of-sight, RF mismatch

1. Introduction

In recent years, research on co-located massive Multiple Input Multiple Output (MIMO) systems has divided attention with those in which radiobase units (RAUs) are distributed across the coverage area [1,2]. A distributed system variant is the cell-free system [3], in which there is no cell-based spatial division. Another feature of these systems is a large number of access points (AP) that normally serve a lesser amount of user equipment in comparison with that served in traditional systems [4].

Two remarkable properties of massive MIMO systems, favorable propagation and channel hardening, were studied by Chen and Bjornson in the context of cell-free systems [5]. The authors concluded that the value of the exponent of geometric losses and the number of antennas per AP are determinant for the occurrence of channel hardening, even with a smaller number of AP in the system. In the study of cell-free systems, the use of carrier frequency around 1.9 GHz, with a 20 MHz narrowband, is usual [3, 4, 6–8]. Despite this, Alonzo and Buzzi [9] opted to compare conventional cell-free systems with those using user-centric virtual cells under millimeter waves, with a carrier frequency of 73 GHz and a 200 MHz bandwidth.

With the advancement of the study of cell-free communication systems, an effort has also been made to obtain equations that allow the computation of parameters such as achievable rates, spectral efficiency and energy efficiency [6, 10]. In addition, such equations can be used in the development of optimization methods, for example. Ngo et al. [11] derived an analytical expression for the uplink and downlink achievable rate on cell-free systems. This expression was successfully used for the development of optimization methods in the allocation of pilot sequences and power control, since they contributed to raise the rates, concentrating them around a higher median. It proved to be an advantage even over small-cell systems.

Therefore, the possibility of expanding the cell-free system model was devised, considering hardware effects in the form of multiplicative reciprocity errors, in addition to reaching an expression that could be used to arrive at an approximate performance of the system. The complete model relies on obtaining the statistics of the channel estimators in the presence of such hardware effects. The entire procedure is detailed in this work.

1.1 Related Works

A study on the impact of different levels of hardware impairments on cell-free systems is presented in [12] and [13]. In the first paper, closed expressions for the uplink spectral efficiency and energy efficiency were obtained considering the presence of hardware impairments. Additionally, the variation of these metrics with the number of access points, considering different scaling factors, was also analyzed. Hardware impairments were characterized by using quality coefficients that range from 0 to 1, and additive distortion noise. The authors adapted the Linear Minimum Mean Square Error (LMMSE) estimator to take into account such imperfections. In [13], downlink spectral efficiency expressions were obtained under these conditions. These expressions were used to develop a max-min power control optimization algorithm, which proved to be superior to heuristic methods. The results obtained in that work show the importance of considering hardware effects.

One of the aspects studied in recent years is the absence of channel reciprocity. It affects the performance of the system, since, in the Time Division Duplexing (TDD) mode, the estimated channel tends to be used for various purposes [14]. A possible cause of non-reciprocity is the occurrence of radiofrequency (RF) mismatch [15]. Assuming that the transmitting and receiving RF gains affect the signals in different ways, the estimated uplink channel will not match the downlink channel.

The influence of the radiofrequency mismatch, that is, gain mismatches of the transceiver radiofrequency circuits (mixers, amplifiers and analog to digital converters, for example), on simplified communication systems, was analyzed by Wei et al [16]. They also proposed the use of antenna calibration to overcome channel non-reciprocity and derived ergodic sum rates for evaluating the impact of calibration error on system performance. A similar study was conducted by De Mi et al [17] to obtain approximate expressions for the downlink signal-to-interference-plus-noise ratio (SINR) when considering Zero-Forcing (ZF) and Maximum-Ratio Transmission (MRT) precoding schemes. In that work, the analysis was performed considering only small-scale fading, white additive Gaussian noise and an estimation error represented by an coefficient that ranges from 0 to 1 by a complex Gaussian random variable (r.v.) with zero mean and unitary variance. In that paper, it is assumed that amplitude and phase of the device response coefficients vary jointly with the small-scale fading. Their study did not consider neither the occurrence of large-scale fading nor the influence of mismatch on channel estimation. The impact of RF mismatch depends on environmental conditions, such as humidity and temperature [18].

Another example of how the introduction of RF mismatch into models can be advantageous is the robust estimation error precoder proposed by Chen et al [19]. It is demonstrated that, in the presence of RF mismatch, the usage of a modified MMSE precoder has increased downlink sum rates. In that work, the authors also present an analytical expression for sum rates. The authors considered that the RF mismatch varies jointly with the large-scale fading.

Spectral efficiency of cell-free massive MIMO systems in which some links present line-of-sight (LoS) was studied by Ozdogan et al [7]. They tested the system performance under two channel estimation methods: MMSE and Least-Squares (LS). They also derived a spectral efficiency analytical expression for that scenario, when operating in uplink mode. In that paper, the equalization method considered was the maximum-ratio combining (MRC). They observed, as in systems with no line-of-sight, that MMSE estimators outperform LS. Shortly thereafter, the same authors expanded the work, including downlink mode and the LMMSE estimator [20]. Downlink mode was studied in coherent and non-coherent precoding schemes. An important result of this work is that when the phase of transmission is unknown, when MMSE estimation is used, the error is narrowly associated to the pilot contamination due to sequences length.

The development of an SINR equation for a cell-free system under no line-of-sight (NLoS) conditions and with fast varying RF mismatch only in the access points is presented in [21], while the performance analysis of the scenario with no NLoS and slow varying mismatch only in the access points is presented in [22]. The importance of improving the model is justified, because optimization methods are based on analytical expressions. In particular, when each device has an error level, neglecting the existence of mismatch can harm even more the optimization procedures. The objective of this work is to extend the cell-free model with reciprocity errors, performing analyzes and obtaining expressions different from those presented in the cited works. The proposal, in the present work, is to use multiplicative models to represent reciprocity errors, modeled by Truncated Gaussian r.v., in both UEs and APs; also, are obtained analytical expressions for the downlink SINR and for channel estimators variance in such a scenario, considering different levels for reciprocity errors.

1.2 Key Contributions

- In this work, the cell-free model is extended to take into account the reciprocity errors resulting from RF mismatch in both ends of a link, i.e. user equipment and access points. Using similar approach of [17] and [23], errors due hardware effects are include as multiplicative complex coefficients, instead the approach used to model hardware impairments in [12] and [13], based on additive terms and hardware quality coefficients. As done in [17] and [24], the amplitude and phase of reciprocity errors are modeled as truncated Gaussian random variables instead of uniform r.v., because the latter has been pointed as unrealistic.
- · Analytical expressions are obtained for the downlink

achievable rate of this system, with line-of-sight and with NLoS, for scenarios in which the amplitude and phase of mismatch coefficients vary jointly with the small-scale (fast varying) and large-scale (slow varying) fading.

The remaining of this paper is organized as follows. In Sec. 2, the features of the model used to simulate the cellfree system are presented, including the fading and downlink signal model as well as the RF mismatch. In Sec. 3, the characteristics of the estimators are studied. Sections 4 and 5 are dedicated to the analytical expressions of the achievable downlink rates, considering the mismatches of fast and slow variation, respectively. In Sec. 6, the results of the simulations are presented and discussed. Finally, Section 7 is devoted to the conclusions of this work.

2. System Model

In order to verify the validity of the obtained expressions, a cell-free system needs to be modeled. The model uses Monte Carlo simulations to compute the rates provided by analytical expressions. A cell-free system can have more than one antenna per access point and per user equipment [25,26]. In this research, there are M single-antenna access points (AP) and K single-antenna user equipment (UE) randomly distributed in a square region with area $d \times d$. All access points are connected to the same central processing unity (CPU) via the backhaul. This system is illustrated in Fig. 1. Although the system is not divided into cells, in order to simulate the system, only access points and user equipment located within the square region are considered. No spatial correlation model was considered for shadowing, because, from the results obtained by Hien et al [11], it can be seen that it only worsens the performance of the system, reducing its the achievable rates.

In this model, the system is narrowband and operates in TDD mode. Assuming channel reciprocity, uplink and downlink channel gains are equal in a coherence interval. It is assumed that large-scale fading gain varies every 40 small-scale fading coherence intervals [4], and that the fading coefficient remains constant in a coherence interval. Each coherence interval can be dedicated to four steps: uplink and downlink channel estimation and uplink and downlink data transmission [14]. Here, channel estimation in the downlink was not considered. The gain between the k-th user equipment and the *m*-th access point is given by $v_{mk} = h_{mk} e^{j\varphi_{mk}} + g_{mk}$, in which φ_{mk} is a random variable uniformly distributed between π and $-\pi$, and $g_{mk} = h_{mk}\sqrt{\beta_{mk}}$ represents the no line-of-sight components [27]. The symbols h_{mk} and β_{mk} are, respectively, the small-scale and large-scale fading coefficients. The small-scale fading coefficient is characterized by a complex Gaussian r.v. with zero mean and unit variance. Therefore, g_{mk} can be described by a Gaussian r.v. with zero mean and variance β_{mk} . Thus, the term v_{mk} represents the specular component of the signal. For the LoS condition,



 κ -factor is given by $10^{1.3-0.003d_{mk}}$, in which d_{mk} is the distance between *k*-th UE and *m*-th AP [28]. In this scenario, the large-scale fading coefficient and the magnitude of the specular component are given, respectively, by [20, 28]:

$$\beta_{\rm mk} = \frac{PL_{\rm mk}}{1 + \kappa_{\rm mk}},\tag{1}$$

$$\overline{h}_{\rm mk} = \sqrt{\frac{\kappa_{\rm mk}}{1 + \kappa_{\rm mk}}} \sqrt{PL_{\rm mk}}.$$
(2)

For the no line-of-sight condition, $\kappa_{mk} = 0$. The path-loss COST231 Walfish-Ikegami model, presented in the 3rd Generation Partnership Project (3GPP) Report [28] consists of two equations, which can be used to model the large-scale fading. The first corresponds to the condition in which there is line-of-sight (LoS) and the second when there is no line-of-sight (NLoS). In the first condition, the shadowing standard deviation (σ_s) is 4 dB and, considering 1.9 GHz as carrier frequency, the path-loss is given by:

$$PL_{\rm mk} = 10^{(-34.53 - 38\log_{10}(d_{\rm mk}) + \sigma_{\rm s} z_{\rm mk})/10},\tag{3}$$

in which d_{mk} is the distance between the *k*-th user equipment and the *m*-th access point. The shadowing coefficient, z_{mk} , is characterized by a random variable with Log-normal distribution [27,29]. When there is no line-of-sight, the shadowing standard deviation is 10 dB, and the path-loss is given by:

$$PL_{\rm mk} = 10^{(-30.18 - 26\log_{10}(d_{\rm mk}) + \sigma_{\rm s} z_{\rm mk})/10}.$$
 (4)

In order to obtain an expression for its SINR, the downlink signal received by the k-th UE is defined by:

$$y_{k} = \sqrt{P_{d}} \sum_{m=1}^{M} \sum_{i=1}^{K} v_{mk} h_{bt,m} h_{ur,k} (\hat{v}_{mi}^{UL})^{*} \eta_{mi}^{1/2} q_{i} + n_{k}, \quad (5)$$

in which n_k is the additive white Gaussian noise in the *k*-th user equipment. In (5), P_d is the normalized downlink transmission power, given by P_d^{cf}/P_n , in which P_n is the noise power given by Bk_BT_0F . In this expression, *F* is the noise factor, T_0 is the absolute temperature, *B* is the bandwidth and k_B is the Boltzmann constant. Since *NF* is the noise figure, the noise factor is given by $F = 10^{NF/10}$. In this research,

as in [6], the bandwidth is 20 MHz, the noise figure is 9 dB, and 290 K is the environment temperature.

Considering the occurrence of RF mismatch at the access point of transmitters and user equipment receivers, the downlink received signal of the *k*-th UE can be rewritten as:

$$y_{k} = \sqrt{P_{d}} h_{ur,k} \mathbf{v}_{k}^{T} \left(\mathbf{H}_{bt} \boldsymbol{\eta}_{k}^{1/2} \mathbf{w}_{k} q_{k} + \sum_{i \neq k}^{K} \mathbf{H}_{bt} \boldsymbol{\eta}_{i}^{1/2} \mathbf{w}_{i} q_{i} \right) + n_{k}$$

$$\tag{6}$$

in which \mathbf{H}_{bt} is the diagonal matrix [24] of the response coefficients ($h_{bt,m}$) of all access point transmitters, $h_{ur,k}$ is the receiver response of the *k*-th user equipment, \mathbf{v}_i is the channel vector between all access points and the *i*-th user equipment, q_i is the signal destined to the *i*-th user equipment, \mathbf{w}_i is the *i*-th user equipment precoding vector and $\boldsymbol{\eta}_i$ is the diagonal matrix of power control coefficients corresponding to the *i*-th user equipment, whose elements depend only on *m*. In this work, only the MRT precoding method is used. Therefore, the power control coefficient between the *m*-th AP and the *k*-th UE is given by [21]:

$$\eta_{\rm mk} = \frac{1}{\sum_{i=1}^{K} \gamma_{\rm mi}^{UL}} \tag{7}$$

and $w_{\rm mi} = (\hat{v}_{\rm mi}^{UL})^*$. Since $\hat{v}_{\rm mi} = v_{\rm mi}^{UL} + \tilde{v}_{\rm mi}^{UL}$, and these r.v. are zero mean and independent, (6) can be rewritten as:

$$y_k = \sqrt{P_d} y_{k,s} + \sqrt{P_d} y_{k,e} + \sqrt{P_d} \sum_{i \neq k}^K y_{k,i} + n_k, \qquad (8)$$

in which:

$$y_{\mathbf{k},\mathbf{s}} = h_{\mathrm{ur},\mathbf{k}} \mathbf{v}_k^{\mathrm{T}} \mathbf{H}_{\mathrm{bt}} \boldsymbol{\eta}_k^{1/2} (\mathbf{v}_k^{UL})^* q_k, \qquad (9)$$

$$y_{k,e} = h_{ur,k} \mathbf{v}_k^{\mathrm{T}} \mathbf{H}_{bt} \boldsymbol{\eta}_k^{1/2} (\tilde{\mathbf{v}}_k^{UL})^* q_k, \qquad (10)$$

$$y_{k,i} = h_{ur,k} \mathbf{v}_k^{\mathrm{T}} \mathbf{H}_{\mathrm{bt}} \boldsymbol{\eta}_i^{1/2} \mathbf{w}_i q_i.$$
(11)

In the computation of SINR, the channel estimation error, $y_{k,e}$, will be considered as part of the noise, along with $y_{k,i}$ and n_k .

The parameters of the Gaussian function (used to generate the truncated Gaussian r.v. that represent magnitude and phase of the device response coefficients) are expressed, respectively, by $(\alpha_g; \sigma_g^2; [a_g, b_g])$ and $(\alpha_f; \sigma_f^2; [a_f, b_f])$, in which α is the Gaussian mean, σ^2 is its variance, *a* and *b* are the truncation limits. In [30], it is mentioned that, after antenna calibration, residual reciprocity errors (amplitude and phase) remain constant for all subcarriers. The reciprocity error levels adopted in this research are the same as in [17], as well as the equation used to calculate the phase error related coefficient, given by:

$$\rho = \frac{\operatorname{erf}\left(\frac{b_f - \alpha_f}{\sqrt{2\sigma_f^2}} - j\frac{\sigma_f}{\sqrt{2}}\right) - \operatorname{erf}\left(\frac{a_f - \alpha_f}{\sqrt{2\sigma_f^2}} - j\frac{\sigma_f}{\sqrt{2}}\right)}{\operatorname{erf}\left(\frac{b_f - \alpha_f}{\sqrt{2\sigma_f^2}}\right) - \operatorname{erf}\left(\frac{a_f - \alpha_f}{\sqrt{2\sigma_f^2}}\right)}.$$
 (12)

This coefficient is the mean of $exp(j\theta)$, in which θ is the phase of the reciprocity error.

After deriving these equations, the next step is to introduce the impact of reciprocity errors resulting from the mismatch on the channel estimation.

3. Channel Estimation

In order to generate the estimated channel coefficients that are used in the precoding vector and in achievable rate calculation, some statistical parameters associated with its estimators are necessary. For channel training based on the transmission of pilot sequences, its symbols must also be known by the receiver [31]. In channel estimation phase, the pilot signal received by the *m*-th AP in τ estimation coherence intervals is projected onto the pilot sequence $\phi_k^{\rm H}$ [32]. This projection results in:

$$\tilde{y}_{P,mk} = \sqrt{P_p \tau} \sum_{l=1}^{K} v_{ml} h_{ut,l} h_{br,m} \boldsymbol{\phi}_k^{H} \boldsymbol{\phi}_l + \boldsymbol{\phi}_k^{H} \mathbf{n}_m^{P}$$
(13)

in which \mathbf{n}_{p}^{P} is the noise vector of the *m*-th AP during the samples, P_{p} is the normalized pilot symbol transmission power and $\boldsymbol{\phi}_{k}$ is the pilot sequence used by the *k*-th user in the estimation phase. The normalized pilot symbol transmission power is given by P_{p}^{cf}/P_{n} , in which P_{p}^{cf} is the pilot symbol transmission power. Finally, $h_{br,m}$ and $h_{ut,l}$ are the response coefficients of the *m*-th AP receiver and the *l*-th user equipment, respectively. When the strength of the faded signal is as low or lower than that of the additive noise, the values of the fading coefficients cannot be determined precisely. The same occurs with the reduction of the number of samples of the pilot sequences, as this reduces the precision of the calculated averages. In (13), $v_{ml}^{UL} = v_{ml}h_{br,m}h_{ut,l}$ is the effective uplink channel coefficient.

In this research, the pilot sequences were distributed randomly among users, but avoiding unnecessary repetitions. No optimal sequence allocation method was used. However, pilot contamination [33] has been considered in some scenarios. This effect occurs when two or more equal or non-orthogonal strings are assigned to different devices. A simpler way to avoid such an effect would be to use more sequences, which would reduce the spectral efficiency of the system, since it would reduce the number of samples in the coherence interval dedicated to data transmission [34]. The channel estimation was made before uplink data transmission, using the Discrete Fourier Transform (DFT) coefficients, as it can assume any length, unlike Hadamard sequences [35]. Both sequences would allow the same quality as the estimation. However, due to the fact that the length of the sequences is also their number, and because DFT is more flexible with regard to its length, it was used. Since Φ is a matrix of orthogonal pilot sequences, in which each column (ϕ) corresponds to a sequence, then $\phi_L^{\mu} \phi_1 = 0$.

Among the estimation methods that could be used, Least-Squares (LS) and LMMSE were chosen. In the absence of pilot interference, the Least-Squares estimator can be considered without loss of optimality. However, the LMMSE estimator provides better performance when there is pilot interference [11].

Using the least-squares method [31], the estimated uplink channel is given by: $\tilde{y}_{mk}/\sqrt{P_d}$ [32]. That is,

$$\hat{v}_{mk} = \frac{\sum_{l=1}^{K} \sqrt{P_{p}\tau} v_{ml} h_{ut,l} h_{br,m} \boldsymbol{\phi}_{k}^{H} \boldsymbol{\phi}_{l}}{\sqrt{P_{P}\tau}} + \frac{\boldsymbol{\phi}_{k}^{H} \mathbf{n}_{m}^{P}}{\sqrt{P_{P}\tau}}.$$
 (14)

Since the estimated channel variance is given by $\gamma_{mk} = E\{|\hat{v}_{mk}|^2\}$, when the RF mismatch is fast and slow varying, respectively, it is:

$$\gamma_{\rm mk,a}^f = \sum_{l=1}^K V_{\rm ml} \xi_{\rm t,l} \xi_{\rm r,m} \boldsymbol{\phi}_k^{\rm H} \boldsymbol{\phi}_l + \frac{1}{P_{\rm P} \tau}, \qquad (15)$$

$$\gamma_{\rm mk,a}^{s} = \sum_{l=1}^{K} V_{\rm ml} |h_{\rm ut,l}|^2 |h_{\rm br,m}|^2 \boldsymbol{\phi}_k^{\rm H} \boldsymbol{\phi}_l + \frac{1}{P_{\rm P} \tau}.$$
 (16)

In (15), $V_{\rm ml} = \beta_{\rm ml} + \overline{h}_{\rm ml}^2$, $\xi_{\rm br,m} = \sigma_{\rm br,m}^2 + \alpha_{\rm br,m}^2$ and $\xi_{\rm ut,l} = \sigma_{\rm ut,l}^2 + \alpha_{\rm ut,l}^2$. The terms σ and α are, respectively, the standard deviation and the mean of the truncated Gaussian r.v. that models the magnitude and phase of the device response coefficients.

Uplink channel estimation error is given by $\zeta_{mk} = E\{|\tilde{v}_{mk}^{UL}|^2\}$ [20]. Considering again Least-Squares estimation, it can be shown that:

$$\zeta_{\rm mk,a} = E \left\{ \left| v_{\rm mk}^{UL} \right|^2 + \frac{\left| \tilde{y}_{\rm mk} \right|^2}{P_{\rm P} \tau} - \frac{v_{\rm mk}^{UL} \tilde{y}_{\rm mk}^*}{\sqrt{P_{\rm P} \tau}} - \frac{\left(v_{\rm mk}^{UL} \right)^* \tilde{y}_{\rm mk}}{\sqrt{P_{\rm P} \tau}} \right\}.$$
(17)

Therefore, when the RF mismatch is fast varying, estimation error variance is given by:

$$\zeta_{\mathrm{mk,a}}^{f} = \sum_{l=1}^{K} V_{\mathrm{ml}} \xi_{\mathrm{br,m}} \xi_{\mathrm{ut,l}} \boldsymbol{\phi}_{k}^{\mathrm{H}} \boldsymbol{\phi}_{\mathrm{l}} - V_{\mathrm{mk}} \xi_{\mathrm{br,m}} \xi_{\mathrm{ut,k}} + \frac{1}{P_{\mathrm{P}} \tau}.$$
 (18)

On the other hand, when mismatch is slow varying:

...

$$\zeta_{\rm mk,a}^{s} = \gamma_{\rm mk,a}^{s} - V_{\rm mk} |h_{\rm br,m}|^2 |h_{\rm ut,k}|^2.$$
(19)

Based on $\zeta_{mk,a}^s$ and $\zeta_{mk,a}^f$, when no pilot contamination occurs, only the term referring to SNR remains. When P_p or

the sequence length tends to infinity, the estimator behavior resembles to a system with perfect CSI. When this occurs, the uplink channel is perfectly known. In the downlink, using this channel estimate in place of g_{mk} results in precoding errors. The relationship of the mean square error with the transmission power, in estimators based on pilot sequences, was studied by Biguesh et al [31].

Usually, when LMMSE estimator is used to estimate the channel coefficient $g_m k$, estimated channel is given by $c_{\rm mk}\tilde{y}_{\rm mk}$, in which $c_{\rm mk} = \sqrt{\tau P_{\rm p}}V_{\rm mk}/(\tau P_{\rm p}\sum_{l=1}^{K}V_{\rm ml}|\phi_k^{\rm H}\phi_l|^2+1)$ [3,4]. In this work, the pilot signals capture not only the information related to $g_m k$, but that related to $g_{\rm mk}^{UL}$, i.e., the effective uplink channel. In this case, minimization of the mean square error occurs with $\hat{v}_{\rm mk}^{UL} = c_{\rm mk}\tilde{y}_{\rm P,mk}$, in which:

$$r_{\rm mk} = \frac{E\{\tilde{y}_{\rm p,mk}^* v_{\rm mk} h_{\rm ut,k} h_{\rm br,m}\}}{E\{|\tilde{y}_{\rm mk}|^2\}}.$$
 (20)

In $E\{\tilde{y}_{P,mk}^* v_{mk} h_{ut,k} h_{br,m}\}$, due to the fact that the channel coefficients $v_{m,l}$ are zero mean, the summation in $y_{P,mk}$ reduces to $|V_{mk}|^2 |h_{ut,k}|^2 |h_{br,m}|^2$, even with the occurrence of pilot contamination. Then, considering fast varying radiofrequency mismatch, c_{mk} becomes:

$$c_{\rm mk} = \frac{\sqrt{\tau P_{\rm p}} V_{\rm mk} \xi_{\rm br,m} \xi_{\rm ut,k}}{\tau P_{\rm p} \sum_{l=1}^{K} V_{\rm ml} \xi_{\rm ut,l} \xi_{\rm br,m} |\phi_k^{\rm H} \phi_l|^2 + 1}.$$
 (21)

Without this adaptation of c_{mk} , the LMMSE estimator would provide worst results. On the other hand, when radiofrequency mismatch varies slowly:

$$c_{\rm mk} = \frac{\sqrt{\tau P_{\rm p}} V_{\rm mk} |h_{\rm ut,k}|^2 |h_{br,m}|^2}{\tau P_{\rm p} \sum_{l=1}^{K} V_{\rm ml} |h_{\rm br,m}|^2 |h_{ut,l}|^2 |\phi_k^{\rm H} \phi_l|^2 + 1}.$$
 (22)

The variance of the estimated uplink channel is given by $E\{|c_{mk}\tilde{y}_{P,mk}|^2\}$, i.e., $E\{|c_{mk}|^2|\tilde{y}_{P,mk}|^2\}$. Calculating the average, it is obtained, for the cases in which the mismatch is fast and slow, respectively:

$$\gamma_{\rm mk,b}^{f} = \frac{\tau P_{\rm p} V_{\rm mk}^{2} \xi_{\rm ut,k}^{2} \xi_{\rm br,m}^{2}}{\tau P_{\rm p} \sum_{l=1}^{K} V_{\rm ml} \xi_{\rm ut,l} \xi_{\rm br,m} |\phi_{k}^{\rm H} \phi_{l}|^{2} + 1},$$
 (23)

$$\gamma_{\rm mk,b}^{s} = \frac{\tau P_{\rm p} V_{\rm mk}^{2} |h_{\rm ut,k}|^{4} |h_{\rm br,m}|^{4}}{\tau P_{\rm p} \sum_{l=1}^{K} V_{\rm ml} |h_{\rm ut,l}|^{2} |h_{\rm br,m}|^{2} |\phi_{k}^{\rm H} \phi_{l}|^{2} + 1}.$$
 (24)

Finally, as was done in [20], the mean-square error, and, therefore, the variance of the estimation error is given by $E\{|v_{mk}^{UL}|\} - \gamma_{mk}$. Therefore, when the mismatch is fast or slow, respectively:

$$\zeta_{\rm mk,b}^f = \xi_{\rm r,m} \xi_{t,k} V_{\rm mk} - \gamma_{\rm mk,b}^f, \qquad (25)$$

$$\zeta_{\rm mk,b}^{s} = V_{\rm mk} |h_{\rm br,m}|^2 |h_{\rm ut,k}|^2 - \gamma_{\rm mk,b}^{s}.$$
 (26)

By using these parameters it is possible to generate \tilde{v}_{mk} and \hat{v}_{mk} . In addition, the error variance will be necessary for the calculation of theoretical achievable rates.

4. SINR for Fast RF Mismatch

In order to assess the performance of a communication system one of the possible evaluation metrics is the achievable rate. When instantaneous channel coefficients are known, it is given by [4]:

$$r_{d,k} = E\left\{ \log_2 \left(1 + \frac{P_d \left| \sum_{m=1}^{M} \eta_{mk}^{1/2} h_{bt,m} h_{ur,k} v_{mk} \hat{v}_{mk}^{UL*} \right|^2}{P_d \sum_{i \neq k}^{K} \left| \sum_{m=1}^{M} \eta_{mi}^{1/2} v_{mk} h_{bt,m} h_{ur,k} \hat{v}_{mi}^{UL*} \right|^2 + 1} \right) \right\}.$$
(27)

In (27), the mean is calculated over each block of 40 small-scale coherence intervals. Assuming that the large scale fading coefficients are known, the theoretical achievable rate of the k-th user can be obtained from [11]:

$$R_{\rm d,k} = \log_2(1 + \varrho_k), \tag{28}$$

in which ρ_k is the downlink SINR of the *k*-th UE. In order to obtain an analytical expression for the achievable rate, it is necessary to obtain an expression for the SINR. Based on (8), it is given by:

$$\varrho_k = E\left\{\frac{P_{\rm d}P_{\rm k,s}}{P_{\rm d}P_{\rm k,e} + \sum_{i\neq k}^K P_{\rm d}P_{\rm k,i} + P_{\rm N}}\right\},\tag{29}$$

in which $P_N = |n_k|^2$, $P_{k,s}$ is the power of $y_{k,s}$, $P_{k,e}$ is the power of $y_{k,e}$ and $P_{k,i}$ is the power of the interuser interference.

In order to obtain an analytical expression for the SINR, one can approximate (29) as a ratio of the means of its numerator (power of the signal) and denominator (power of noise plus interference), which can lead to loss of accuracy [17] given by:

$$\varrho_k \approx \frac{P_{\rm d} E\{P_{\rm k,s}\}}{P_{\rm d} E\{P_{\rm k,e}\} + P_{\rm d} \sum_{i \neq k}^{K} E\{P_{\rm k,i}\} + 1}.$$
 (30)

First, the strength of the signal of interest is calculated. It is given by:

$$E\{P_{\mathbf{k},\mathbf{s}}\} = E\{h_{\mathrm{ur},\mathbf{k}}\mathbf{v}_{k}^{\mathrm{T}}\mathbf{H}_{\mathrm{bt}}\boldsymbol{\eta}_{k}^{1/2}(\mathbf{v}_{k}^{UL})^{*}|\}^{2}.$$
 (31)

Transforming this equation from vector form to summation form, it is given by:

$$E\{P_{k,s}\} = E\left\{\left|\sum_{m=1}^{M} h_{ur,k} |v_{mk}|^2 h_{bt,m} \eta_{mk}^{1/2} h_{br,m}^* h_{ut,k}^*\right|^2\right\}, \quad (32)$$

which can be rewritten as:

$$E\{P_{k,s}\} = E\left\{\left(\sum_{m=1}^{M} \eta_{mk}^{1/2} |v_{mk}|^2 h_{bt,m} h_{ur,k} h_{br,m}^* h_{ut,k}^*\right) \times \left(\sum_{n=1}^{M} \eta_{nk}^{1/2} |v_{nk}|^2 h_{bt,n}^* h_{ur,k}^* h_{br,n} h_{ut,k}\right)\right\}.$$
 (33)

In (33), the product can be can be splitted into a sum:

$$E\{P_{\mathbf{k},\mathbf{s}}\} = E\left\{\sum_{m=1}^{M} S_1\right\} + E\left\{\sum_{n\neq m}^{M} S_2\right\},\tag{34}$$

in which:

$$S_1 = \eta_{\rm mk} |v_{\rm mk}|^4 |h_{\rm bt,m}|^2 |h_{\rm ur,k}|^2 |h_{\rm br,m}|^2 |h_{\rm ut,k}|^2$$
(35)

$$S_{2} = \eta_{\rm mk}^{1/2} \eta_{\rm nk}^{1/2} |v_{\rm mk}|^{2} |v_{\rm nk}|^{2} h_{\rm bt,m} h_{\rm bt,n}^{*} \times |h_{\rm ur,k}|^{2} h_{\rm br,m}^{*} h_{\rm br,n} |h_{\rm ut,k}|^{2}.$$
(36)

Computing the means in (34), one obtains:

$$E\{P_{k,s}\} = \sum_{m=1}^{M} \eta_{mk} E\{|v_{mk}|^{4}\} h_{br,n} \xi_{ur,k} \xi_{br,m} \xi_{ut,k} + \sum_{n \neq m}^{M} \eta_{mk}^{1/2} \eta_{nk}^{1/2} V_{mk} V_{nk} \nu_{bt,m} \nu_{bt,n}^{*} \xi_{ur,k} \nu_{br,m}^{*} \nu_{br,n} \xi_{ut,k}, \quad (37)$$

in which $v_{br,m} = \alpha_{br,m}\rho_{br,m}$, $v_{bt,m} = \alpha_{bt,m}\rho_{bt,m}$, $v_{br,n} = \alpha_{br,n}\rho_{r,n}$, $v_{bt,n} = \alpha_{t,n}\rho_{bt,n}$, and:

$$E\{|v_{mk}|^{4}\} = \overline{h}_{mk}^{4} + 2\beta_{mk}(\beta_{mk} + \overline{h}_{mk}^{2}).$$
(38)

As mentioned earlier, the power of the estimation error is considered as part of the noise power. Since this signal is defined in (10), its power is given by:

$$E\{P_{k,e}\} = E\left\{\left|h_{ur,k}\mathbf{v}_{k}^{\mathrm{T}}\mathbf{H}_{\mathrm{bt}}\boldsymbol{\eta}_{k}^{1/2}(\tilde{\mathbf{v}}_{k}^{UL})^{*}\right|^{2}\right\},\qquad(39)$$

which, after some manipulation, leads to:

$$E\{P_{k,e}\} = E\left\{\sum_{m=1}^{M} \eta_{mk} |v_{mk}|^{2} |h_{bt,m}|^{2} |h_{ur,k}|^{2} |\tilde{v}_{mk}^{UL}|^{2}\right\} + E\left\{\sum_{n \neq m}^{M} \eta_{mk}^{1/2} \eta_{nk}^{1/2} v_{mk} v_{nk}^{*} h_{bt,m} h_{bt,n}^{*} |h_{ur,k}|^{2} \tilde{v}_{mk}^{UL} (\tilde{v}_{nk}^{UL})^{*}\right\}.$$
 (40)

The second sum of (40) has independent r.v. with zero mean. For this reason, only the first term remains. So, its mean is:

$$E\{P_{k,e}\} = \sum_{m=1}^{M} \eta_{mk} V_{mk} h_{br,n} \xi_{ur,k} \zeta_{mk}^{f}.$$
 (41)

Finally, based on (11), the power of the interuser interference is given by:

$$E\{P_{k,i}\} = E\left\{ \left| \sum_{m=1}^{M} \eta_{mi}^{1/2} v_{mk} v_{mi}^{*} h_{bt,m} h_{ur,k} h_{br,m}^{*} h_{ut,i}^{*} + \sum_{m=1}^{M} \eta_{mi}^{1/2} v_{mk} h_{bt,m} h_{ur,k} (\tilde{v}_{mi}^{UL})^{*} \right|^{2} \right\}.$$
 (42)

Due to the independence of some random variables in these two sums, the modulus can be splitted into a sum of two other modulus, that is:

$$E\{P_{k,i}\} = E\{S_3\} + E\{S_4\}, \qquad (43)$$

in which:

$$S_{3} = \left| \sum_{m=1}^{M} \eta_{\rm mi}^{1/2} v_{\rm mk} v_{\rm mi}^{*} h_{\rm bt,m} h_{\rm ur,k} h_{\rm br,m}^{*} h_{\rm ut,i}^{*} \right|^{2}, \qquad (44)$$

$$S_4 = \left| \sum_{m=1}^{M} \eta_{\rm mi}^{1/2} v_{\rm mk} h_{\rm bt,m} h_{\rm ur,k} (\tilde{v}_{\rm mi}^{UL})^* \right|^2.$$
(45)

The term S_3 can be rearranged as:

$$S_{3} = \left(\sum_{m=1}^{M} \eta_{\text{mi}}^{1/2} v_{\text{mk}} v_{\text{mi}}^{*} h_{\text{bt,m}} h_{\text{ur,k}} h_{\text{br,m}}^{*} h_{\text{ut,i}}^{*}\right) \times \left(\sum_{n=1}^{M} \eta_{\text{ni}}^{1/2} v_{\text{nk}}^{*} v_{\text{ni}} h_{\text{bt,n}}^{*} h_{\text{ur,k}}^{*} h_{\text{br,n}} h_{\text{ut,i}}\right).$$
(46)

The development of S_3 leads to:

$$E\{S_3\} = E\left\{\sum_{m=1}^M S_5\right\} + E\left\{\sum_{n\neq m}^M S_6\right\},\tag{47}$$

in which:

$$S_5 = \eta_{\rm mi} |v_{\rm mk}|^2 |v_{\rm mi}|^2 |h_{\rm bt,m}|^2 |h_{\rm ur,k}|^2 |h_{\rm br,m}|^2 |h_{\rm ut,i}|^2, \quad (48)$$

$$S_{6} = \eta_{\rm mi}^{1/2} \eta_{\rm ni}^{1/2} v_{\rm mk} v_{\rm nk}^{*} v_{\rm mi}^{*} v_{\rm ni} h_{\rm bt,m} h_{\rm bt,n}^{*} \times |h_{\rm ur,k}|^{2} h_{\rm br,m}^{*} h_{\rm br,n} |h_{\rm ut,i}|^{2}.$$
(49)

Calculating the mean of S_3 , the term S_6 disappears, because it has independent r.v. with zero mean (v_{ni} and v_{mi}^* , for example). Therefore, it leads to:

$$E\{S_3\} = \sum_{m=1}^{M} \eta_{\rm mi} V_{\rm mk} V_{\rm mi} h_{\rm br,n} \xi_{\rm ur,k} \xi_{\rm br,m} \xi_{\rm ut,i}.$$
 (50)

Applying the same procedure to the second term of S_4 , its mean is given by:

$$E\{S_4\} = \sum_{m=1}^{M} \eta_{\rm mi} V_{\rm mk} h_{\rm br,n} \xi_{\rm ur,k} \zeta_{\rm mi}^f.$$
 (51)

Replacing (50), (51) in (43), it is obtained the expression for the interuser interference power.

The analytical SINR obtained in this work can be seen as an expansion of that presented in [21] and that obtained by De Mi et al and presented in [17]. An important difference for the latter is how the estimation error is introduced in the model. In that work, a variable coefficient between zero and one is used to represent the estimation error level. In this research the variances of the estimation errors are used in the analytical expression. Considering the absence of line-of-sight and large-scale fading ($\kappa_{mk} = 0$, $\beta_{mk} = 1$ and $\eta_{mk} = \eta \forall m, k$), no RF mismatch in user equipment and the same reciprocity error level at all access points, the equations presented here become similar to those presented in that work. So, taking as an example (35), one can obtain:

$$E\{P_{k,s}\} = \sqrt{\eta} [2M\xi_r\xi_t + (M-1)v_rv_r^*v_tv_t^*].$$
(52)

On the other hand, assuming that RF mismatch occurs only in the transceivers of access points in a cell-free system, the development of (34) provides:

$$E\{P_{k,s}\} = \sum_{m=1}^{M} \eta_{mk} E\{|v_{mk}|^{4}\} h_{br,n} \xi_{br,m} + \sum_{\substack{n \neq m}}^{M} \eta_{mk}^{1/2} \eta_{nk}^{1/2} V_{mk} V_{hk} v_{bt,m} v_{bt,n}^{*} v_{br,m}^{*} v_{br,n}.$$
 (53)

It is worth to mention that the SINR expression obtained in this section can also be manipulated in order to test other scenarios, with diverse parameter arrangements.

5. SINR for Slow RF Mismatch

In a scenario in which there is slow RF mismatch, the SINR of the k-th user can be obtained in a similar way used for the rate with fast variation. The difference is that the terms referring to mismatch can be taken as constant [19]. Therefore, considering the RF mismatch in both ends of the link, one can obtain:

$$E\{P_{k,s}\} = \sum_{m=1}^{M} \eta_{mk} E\{|v_{mk}|^{4}\} |h_{bt,m}|^{2} |h_{br,m}|^{2} |h_{ut,k}| |h_{ur,k}|^{2} + \sum_{n \neq m}^{M} \eta_{mk}^{1/2} \eta_{nk}^{1/2} V_{mk} V_{nk} h_{bt,m} h_{bt,n}^{*} h_{br,m}^{*} h_{br,n} |h_{ut,k}| |h_{ur,k}|^{2},$$
(54)

$$E\{P_{k,e}\} = \sum_{m=1}^{M} \eta_{mk} V_{mk} |h_{bt,m}|^2 |h_{ur,k}|^2 \zeta_{mk}^s, \qquad (55)$$

$$E\{S_3\} = \sum_{m=1}^{M} \eta_{\rm mi} V_{\rm mk} V_{\rm mi} |h_{\rm bt,m}|^2 |h_{\rm br,m}|^2 |h_{\rm ut,i}|^2 |h_{\rm ur,k}|^2, \quad (56)$$

$$E\{S_4\} = \sum_{m=1}^{M} \eta_{\rm mi} V_{\rm mk} |h_{\rm bt,m}|^2 |h_{\rm ur,k}|^2 \zeta_{\rm mi}^s.$$
 (57)

In the case in which the mismatch is slow, it has been observed that the analytical expression becomes a function of the devices response coefficients, instead of the distribution parameters that characterize the reciprocity errors.

6. Results

This section presents simulations results in order to assess the impact of RF mismatch on the performance of cell-free systems considering the model with multiplicative errors. All simulations are based on Matlab ®codes. The analyzes are based on the Empirical Cumulative Density Function (ECDF) curves for the downlink achievable rates of a cell-free system affected by reciprocity error due RF mismatch. The objective is not only to evaluate the influence of multiplicative reciprocity errors due the RF mismatch on achievable rates in cell-free systems, but also the quality of the estimate obtained with the usage of the extended analytical expressions proposed in this paper. To plot each ECDF curve, 10,000 large-scale fading realizations were used. This includes varying the M AP and K UE coordinates, as well as shadowing. At each realization, K achievable rate samples were produced. Hien et al [11] used 200 channel realizations, despite considering larger values of K. With K = 40, it would be 8,000 achievable rate samples. Therefore, the number of achievements used here is sufficient to obtain almost the same number of samples that would be obtained with the values adopted in that work. Two homogeneous scenarios were considered for visibility conditions: NLoS for all links and LoS for all links.

As in [4], the downlink symbol transmission power is $P_d^{cf} = 200 \text{ mW}$ and the pilot symbol transmission power is $P_p^{cf} = 100 \text{ mW}$. Besides that, there are M = 100 access points and d = 500 m. In order to observe the impact of multiplicative errors and the validity of the equation, the values used to model phase and amplitude errors were the double of those used by De mi et al [17]. Such values are in the range of those presented in [24]. The high level reciprocity errors was simulated using (0; 2; [-8, +8]) dB and $(0; 1; [-50, +50])^\circ$ as parameters of the Gaussian r.v. associated with the truncated Gaussian r.v. that describe magnitude and phase of a device response, respectively. For the normal level of reciprocity, these parameters are (0; 1; [-2, +2]) dB and $(0; 0.5; [-20, +20])^\circ$.

The first evaluation was performed for a homogeneous scenario with and without line-of-sight, considering $\tau = 15$ and considering the occurrence of high level reciprocity errors in the access point and user equipment transceivers. Were considered two different quantities of user equipment: 15 and 25. When K = 15, there is no reuse of pilot sequences, while in the second case, ten pilot sequences are reused, which results in the occurrence of pilot contamination. According to Buzzi and D'andrea [32], the first quantity would correspond to a sparsely populated scenario, while the second would correspond to an intermediate scenario. The impact of the number of users on the values obtained with the use of analytical expression will be evaluated later.



Fig. 2. ECDF curves of the achievable rates obtained with Monte Carlo simulation in (27) and by using the analytical expression in a cell-free system affected by high level and fast reciprocity errors in the AP and UE.





In this first test, fast varying RF mismatch and two channel gain estimation methods (LMMSE and LS) were considered. In Fig. 2, for K = 15, it is shown that both curves overlap, indicating that the performance of the two estimation methods is similar in the absence of pilot interference, and this is an indication that they were correctly derived. Besides that, there is a gap between the curves obtained by using Monte Carlo method in (27) and by using the analytical expression. When K = 25, this gap also exist. However, the gaps observed in the LoS scenario were smaller.

When the terms of the numerator (power of the signal of interest) and the denominator (interference and noise) of (27) are obtained by using the Monte Carlo method in (30), both curves coincide, as observed in Fig. 3 (for NLoS condition) and Fig. 4 (for LoS). This reveals that the gap stems from the approximations made in (29) and (30), which were necessary to obtain the SINR expression. This error occurs because the coefficients representing the reciprocity error increase the dependency among the signal of interest and the interference. Also in Figs. 3 and 4, the curves obtained with the occurrence of RF mismatch are shown only in the access point transceivers (RFx1). Comparing these with those obtained with the mismatch at both ends (RFx2), it is observed that the system performance worsens even more. On the other hand, the values obtained with the analytical expression almost did not vary with the use of mismatch at one or two ends of the link. This is confirmed by the usage of the Monte Carlo method in (30), for RFx1 and RFx2.

Even though there is a dependency between terms in the numerator, it is important to observe whether the increase in the number of users affects the gap between the data obtained with the Monte Carlo simulation in (27) and the theoretical values. For that, four *K* values were considered, without the occurrence of pilot interference. In Fig. 5, the ECDF curves of the downlink achievable rates obtained with K = 30, 15, 10 and 8 are illustrated. This simulation considered a set of 30 pilot sequences with length 30 samples.



Fig. 4. ECDF curves of the achievable rates obtained with Monte Carlo simulation in (27), analytical expressions and an Monte Carlo simulation in (30), considering only high level reciprocity errors in all transceivers and only in access points, for LoS condition.

From them, it appears that, in the absence of pilot contamination, the variation in the number of users does not affect the quality of the data obtained by using the analytical expression, because the gap relatively to the curve obtained with data generated by using Monte Carlo method does not increase considerably. Without and with line-of-sight, it is ≈ 0.4 and 0.2 bits/s/Hz, for the lowest rate observed among 60% of users.

The next test considered different levels of reciprocity errors. In Fig. 6, the ECDF curves of achievable rates obtained with three conditions of the reciprocity error are illustrated, considering fast varying mismatch: only normal level, only high level and a mixed scenario, where each transceiver can have its own reciprocity errors level. It is noticed that the increase in the level of mismatch impair the adherence of the data provided by the analytical expression. The relative gap among the curves obtained with the analytical expression is less than that observed in the curves obtained with the application of the Monte Carlo method in (27). This is also explained by the dependence between the terms of the SINR.

Considering the slow mismatch and high level reciprocity errors in a scenario with no line-of-sight, the impact on transmission rates is different than that observed in a scenario in which the mismatch is fast and reciprocity errors level is high, as can be observed in Fig. 7.



Fig. 5. ECDF curves of the achievable rates obtained with Monte Carlo simulation in (27) and by using the analytical expression in a cell-free system affected by high level reciprocity errors in the AP and UE, with K = 8, 10 and 15.



Fig. 6. ECDF curves of the achievable rates of cell-free systems with K = 15 and different levels of RF mismatch.



Fig. 7. ECDF curves of the achievable rates of cell-free systems with K = 15, without mismatch and with high level reciprocity errors due slow and fast varying RF mismatch.

This can be seen in the ECDF curves in which slow mismatch impacted the variance of the achievable rates more than the fast mismatch. In terms of the 10%-outage rate, that is, the smallest rate among 90% of the users, systems with slow mismatch, fast mismatch and without mismatch provide \approx 1.5, 2.0 and 2.5 bits/s/Hz. It makes sense that slow and fast mismatches provide different results. In the former, the product of the reciprocity error coefficients through the complex Gaussian channel g_{mk} , results in Gaussian r.v. with different statistics. In the second, the product of the error coefficients by the Gaussian channel results in r.v. with another distribution and another variance. For this scenario, assuming that the mismatch coefficients are also known, because it varies slowly, (30) can be used to estimate the achievable rates and obtain the ECDF curves. In this case, the SINR expression terms are those obtained in Sec. 5.

In Fig. 8, it is shown the ECDF curves of the achievable rates obtained considering K = 15 and 25 for slow mismatch with homogeneous error level scenarios, with high level RF mismatch, when applying the LS and LMMSE channel estimators. In this case, the values provided by the analytical expression, when compared to that obtained with fast mismatch, provided better adherence to the data provided by Monte Carlo method applied in (27). In the case where the mismatch is slow, the terms related to the signal of interest and interference are reduced to Gaussian products, as in systems without mismatch.



Fig. 8. ECDF curves of the achievable rates obtained with Monte Carlo simulation in (27) and by using the analytical expression in a cell-free system affected by high level slow reciprocity error in the AP and UE.

Therefore, the amount of r.v. common to such terms is reduced. This explains the reduction of the gap between the curves obtained with the theoretical expression and with the Monte Carlo method.

7. Conclusions

This paper proposed the use of multiplicative models to represent RF mismatch in the context of cell-free systems. In addition, the mean and variances of the estimation error of the estimated channel were obtained for the LS and LMMSE channel estimators under multiplicative reciprocity errors linked to RF mismatch. Novel analytical expressions of the downlink rates achievable for cell-free systems subject to fast and slow varying mismatch were obtained and applied for homogeneous LoS and NLoS scenarios.

It has been demonstrated that, aside from a small approximation error, the expression provides an useful estimate of the cell-free system achievable rates, when there are reciprocity errors. It has been observed that such approximation error is sensitive to the level of mismatch and the presence of a line-of-sight, although insensitive to the number of users in the system.

From simulation results, it is observed that the fast varying RF mismatch is more prone to deteriorate the performance of cell-free systems, compared to the slow one. Finally, a modelling of cell-free systems affected by RF mismatch is presented, with different levels of reciprocity errors at each access point or user equipment, since different devices may react differently to the environmental conditions commonly associated to the occurrence of mismatch. In this case, the analytical expression also proved to be useful.

As future work, one can seek to improve the approximation of the SINR, in order to reduce the gap between it and the data obtained in the Monte Carlo simulation, in addition to the inclusion of other effects in the cell-free models. It would also be interesting to derive an analytical expression when using zero-forcing processing.

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