Design of \bar{h} -CPM-LFM Radar-Communication Integration Signal

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Abstract. In light of the increasing requirement for the electromagnetic spectrum, the integration of radar and communication is widely concerned because of its miniaturizing equipments and high efficiency of spectrum. To address the issue that the communication information in integration signal for radar and communication affects its detection performance. A novel integration signal is proposed in this paper. Inspired by the high communication efficiency of shaped octal phase-shift keying (S8PSK) and high spectral efficiency of the three-section integration waveform (k-CPM-LFM), we generate a new type of modulation \bar{h} -CPM by introduction of a precoding method with low complexity and a time-varying modulation index h, which is used to encode communication data into LFM radar waveform to form a novel integration waveform h-CPM-LFM. Numerical results show that the designed waveform is at least 10 dB less spectrum extension than other integration waveforms when carrying large amounts of communication information and has excellent BER performance under the condition of strong out-of-band interference. Ambiguity function analysis shows that the waveform has excellent detection performance comparable to LFM.

Keywords

LFM, joint communication-radar, adaptive waveform design, ambiguity function, bit error rate (BER)

1. Introduction

1.1 Related Work and Existing Problems

As the number of users and wireless devices increases, so does the demand for spectrum resources. Therefore, the integration and allocation of spectrum resources have become important hotspots in recent years. The combination of radar systems and communication systems has gained widely attention for its equipment miniaturizing and high efficiency of the spectrum [1]. In addition, the demand generated by 5G highspeed communication and high-performance radar [2] has promoted the rapid development of Dual-Functional Radar-Communication system (DFRC) [3]. Vehicles need to locate and transmit information to each other in the intelligent transportation system. The combat platform of the radar needs to detect targets and transmit information to the intelligence center [4]. Unmanned Aerial Vehicles (UAV) need to perceive the environment and transmit environmental data to the target users [5]. Free-Space Optical (FSO) communication system becomes more attractive due to the deployment of additional broadband channels [6], [7].

Radar and Communication Spectrum Sharing (RCSS) can be implemented in two main ways, one is a system combination and the other is co-design [8]. System combination solves the problem of interference between discrete radar and communication systems. Co-design refers to two functions implemented on one system [9]. The design of integration waveform is divided into two categories, which is the key element in the co-design of DFRC systems. On one hand, the communication signal is converted into pulse for detecting radar. For instance, Orthogonal Frequency Division Multiplexing (OFDM) has been widely researched and applied in recent years [10]. Communication symbols are embedded into OFDM waveforms which are used in waveforms for delayed Doppler radar [11]. In [12], the Direct Sequence Spread Spectrum (DSSS) coding method is used to form an integration waveform. A design scheme for OFDM using Multicarrier Complementary Phase-Coded (MCPC) was proposed in [13]. OFDM waveforms have problems of high autocorrelation sidelobe and Peak to Average Power Ratio (PARP) which will cause serious distortion and poor performance of radar detection and tracking when passing the non-linear region of the high-power amplifier.

On the other hand, the radar signal is modulated to carry communication information. The Linear Frequency Modulation (LFM) was first investigated in [14], which has been widely used for radar detection and tracking. The integration of radar and communication is realized by modulating communication symbols with positive and negative FM slope [15]. It can be seen that the scheme reduces radar transmit power significantly and affects detection performance. Thus, many researchers have studied the MFSK-LFM and MPSK-LFM for several years. In [16], the Frequency-Shift keying (FSK)-LFM waveforms were used to establish a continuous wave radar model for detecting stationary and moving objects in the air. In [17], the Minimum Shift Keying (MSK) spread spectrum signal is combined with the LFM waveforms to form MSK-LFM waveforms. The integration waveform of LFM and BPSK (LFM-BPSK) was put forward in [18]. These integration waveforms can realize the function of the DFRC system, but the discontinuous phase characteristics will lead to the decrease of spectrum efficiency and the low communication efficiency, which can not meet actual communication needs.

It is a favorable choice that the Continuous Phase Modulation (CPM) with constant envelope modulates the LFM [19]. In [20], [21], after time-frequency analysis by Short-time Fourier Transform (STFT), the mapped codebook of communication symbols is modified to obtain a threesection waveform (k-CPM-LFM). A cascade scheme of Low-Density Parity Check (LDPC) codes is carried out to adjust the communication symbols of CPM-LFM in [22]. Those results indicated that the spectrum of the integration waveform is limited to the original bandwidth for the radar system at the expense of the data transmission rate. In [23], a symbol with a shorter distance than the conventional is generated by the precoder, which is combined with CPM and Octal Phase-shift Keying (8PSK). Shaped Octal Phase-shift Keying (S8PSK) waveform was acquired. Then the LFM is used as the carrier to form the S8PSK-LFM waveform. The results show that the high spectral efficiency is obtained, nevertheless, it also increases the Bit Error Rate (BER) and the complexity of waveform generation.

A new DFRC waveform \bar{h} -CPM-LFM is proposed in this paper. A precoder with low complexity and computation is designed to reduce the bandwidth. Meanwhile, a timevarying modulation index is introduced to replace the traditional one, and then \bar{h} -CPM-LFM waveform is obtained by using an LFM as carrier. Analysis show that the designed waveform has better performance than conventional signals in communication and radar.

1.2 Motivations and Contributions

The objective of this paper is to design a new DFRC waveform, which can maintain excellent radar detection and communication performance. A precoder and time-varying index \bar{h} are designed to improve spectrum efficiency and communication efficiency. A novel DFRC waveform is generated by combining this novel modulation with LFM, and it can be widely used in military and civil applications. In a word, this research's main contributions can be listed as follows:

(a) Inspired by *k*-CPM-LFM waveform efficient spectrum and excellent anti-interference performance [20], [21], and consider that guard symbol can be translated into guard bands at the edges of the base radar waveform

spectrum [24], a time-varying modulation index \bar{h} is introduced to form a new three-section waveforms and obtain better waveform performance.

- (b) The SOQPSK method [23] is adopted to encode the bi-polar odd symbol sequence into a bi-polar even symbol sequence with narrow distance and 0 symbols, the spectral efficiency is much better than CPM. But it is complicated that the XOR operation is performed from the second place in sequence for this pre-coding approach. Therefore, a symbol-corresponding precoding method is proposed in this paper, which has the advantages of low complexity and computation.
- (c) A new modulation method \bar{h} -CPM is generated by combining a precoding method with a novel three-section waveform formed by time-varying modulation index \bar{h} , which is used to encode communication data into LFM radar waveform to construct a novel integration waveform \bar{h} -CPM-LFM. The simulation analysis show that it has high spectrum and communication efficiency when transmitting the same information, and has the same excellent radar detection performance as LFM.

1.3 Organization

The remainder of this paper is organized as follows. In Sec. 2, a new modulation method is produced by combining the precoding method and a time-varying modulation index \bar{h} with CPM, which uses LFM as the carrier. Analysis of the communication performance and radar performance are presented in Sec. 3. The designed waveform is compared with other integration waveforms in communication and radar performance in Sec. 4. Finally, some conclusions are drawn in Sec. 5. For illustrative purposes, the main symbols and parameters used in this paper are listed in Tab. 1.

Symbol	Notation				
$f_{\rm c}$	Carrier frequency				
h	Modulation index				
\bar{h}	Time-varying modulation index				
e_i	Original binary symbol				
a_i	Transmitted symbol by CPM				
b_i	Transmitted symbol by \bar{h} -CPM				
М	Modulation order				
Ts	Symbol duration				
Tp	Pulse duration				
L	Correlation length				
Ν	Number of symbols				
В	Bandwidth of LFM				

Tab. 1. List of major symbols and notations.

2. Signal Model

2.1 Generation of \bar{h} -CPM

CPM is an efficient modulation echnique that has been widely used in high spectral efficiency communication because of its constant envelope and high spectrum utilization [25]. The modulated CPM sequence is

$$S(t;a) = A \exp\left[2\pi f_{c}t + \varphi(t;a)\right], \qquad (1)$$

$$\varphi(t;a) = 2\pi h \sum_{i=-\infty}^{N} a_i q \left(t - iT_{\rm s}\right).$$
⁽²⁾

To facilitate the following discussion, given A = 1, the modulation index $h \in [0, 1]$, the continuous phase-encoded sequence $\varphi(t; a)$ controlled by $a_i \in \{\pm 1, \pm 3, \pm 5, \dots, \pm (M - 1)\}$, and time index $i \in Z$, the type of modulation \bar{h} -CPM is

$$S(t;b) = A \exp\left[2\pi f_c t + \varphi(t,b)\right], \qquad (3)$$

$$\varphi(t;b) = 2\pi \bar{h} \sum_{i=-\infty}^{N} b_i q \left(t - iT_{\rm s}\right). \tag{4}$$

The transmitted symbol is b_i with bipolar even-length data alphabets $\{\pm 1, \pm 2, \pm 3, \dots, \pm (M/2)\}$. A time-varying modulation index \bar{h} is introduced as

$$\bar{h} = \begin{cases} h, & k_0 \le i \le N - k_0, \\ 0, & \text{otherwise,} \end{cases}$$
(5)

 k_0 is the segmentation points of which is the latest the integration waveform [21]

$$k_0 = \frac{h}{2\mu T_8^2} (M/2). \tag{6}$$

The phase response function q(t) is decided by the integral of the shaping pulse g(t),

$$g(t) = \begin{cases} \frac{1}{2LT_{\rm s}}, & 0 \le t \le LT_{\rm s}, \\ 0, & \text{otherwise,} \end{cases}$$
(7)

$$q(t) = \begin{cases} 0, & t \le 0, \\ \int_0^t g(\tau) d\tau, & 0 < t < LT_s, \\ 0.5, & t \ge LT_s \end{cases}$$
(8)

where g(t) is the rectangular pulse with low pulse compression sidelobe and the duration is LT_s . Now, when $(N-1)T_s \le t \le NT_s$, the phase coding of \bar{h} -CPM is

$$\phi_N(t,b) = 2\pi \bar{h} \left(\sum_{i=N-L+1}^N b_i \frac{t}{2LT_s} + \sum_{i=0}^{N-L} \frac{b_i}{2} \right).$$
(9)

The modulation index h of the CPM is a fixed value for the duration. Meanwhile, the modulation index \bar{h} of the \bar{h} -CPM is time-dependent and controlled by k_0 . Figure 1(a) is the communication symbol mapping of k-CPM-LFM waveforms in [21]. The communication symbol mapping \bar{h} -CPM is shown in Fig. 1(b). When the communication symbol is located in $[0, k_0 - 1]$ and $[N - k_0 + 1, N]$, the transmitted symbol is called 0 and $\bar{h} = 0$. The transmitted binary bits e_i are transformed into a_i with odd-length data alphabets by modulation order M in CPM

$$e_i(0,1) \to a_{i(\text{CPM})}(\pm 1, \pm 3, \dots, \pm (M-1)).$$
 (10)

In *h*-CPM, the transmitted binary bits are converted to decimal firstly, and then the symbol is converted. The steps are as follows:

Step 1. Let $e_i = (e_i^1, e_i^2, e_i^3)$ stand for the set of transmitted binary bits by the *i*th phase.

Step 2. The transmitted binary bits $e_i = (e_i^1, e_i^2, e_i^3)$ are first converted to decimal,

,

$$e_i = \left(e_i^1, e_i^2, e_i^3\right) \to a_{i(\bar{h}\text{-}CPM)}(0, 1, 2, 3, \dots, M-1).$$
 (11)

Step 3. Then convert a_i to $b_i \in (\pm 1, \pm, 2, ..., \pm M/2)$ by the following formula

$$b_{i} = \begin{cases} a_{i}, & 1 \le a_{i} \le \frac{M}{2} - 1, \\ \frac{M}{2}, & a_{i} = 0, \\ a_{i} - M, & \frac{M}{2} \le a_{i} \le M - 1. \end{cases}$$
(12)

In this paper, the modulation order *M* is 8 and effect of the phase states is shown in Fig. 2. The CPM symbol $a_{i(CPM)}$ valuing $\pm 1, \pm 3, \pm 5$ and ± 7 respectively represented as $\pm \pi/8$, $\pm 3\pi/8, \pm 5\pi/8$ and $\pm 7\pi/8$. They change between previous and current phase. As for the \bar{h} -CPM, symbol $a_{i(\bar{h}-CPM)}$ valuing $\pm 1, \pm 2, \pm 3$ and ± 4 stand for $\pm \pi/8, \pm 2\pi/8, \pm 3\pi/8$ and $\pm 4\pi/8$. Furthermore, as given in (9), the phase of \bar{h} -CPM is generated by the integral of the shaping pulse g(t) during the symbol duration T_s multiplied by b_i and \bar{h} . The obtained phase changes $\pi/8$ for each symbol duration T_s . Moreover, the spectral energy dispersed more uniformly due to the reduction of the linear phase mutation.



Fig. 1. The communication symbol mapping.

	←→							
e_i	0 0 0	0 0 1	0 1 0	0 1 1	1 0 0	1 0 1	1 1 0	1 1 1
						-		
$a_{i(\text{CPM})}$	-7	-5	-3	-1	1	3	5	7
$a_{i(\bar{h} - CPM)}$	0	1	2	3	4	5	6	7
b_i	4	1	2	3	-4	-3	-2	-1

Fig. 2. \bar{h} -CPM precoder of M = 8.

2.2 Generation of \bar{h} -CPM-LFM

LFM waveform is widely used in radar for its large bandwidth-delay product [26]. It can be expressed as

$$S(t)_{\rm LFM} = A \exp\left(j\left(2\pi f_{\rm c}t + \pi\mu t^2\right)\right). \tag{13}$$

Similarly, A = 1 in this part for the following discussion. The linear frequency modulation rate is $\mu = B/T$, and the spectrum is concentrated at $[f_c, f_c + B]$. The integration waveform \bar{h} -CPM-LFM can be formed by \bar{h} -CPM which can encode the communication data into the LFM carrier. It can be expressed as

$$S(t) = \sum_{i=0}^{N-1} \operatorname{rect}\left(\frac{t-iT_{s}}{T_{s}}\right) \exp\left(j2\pi\left(f_{c}t + \frac{\mu}{2}t^{2}\right)\right)$$

$$\times \exp\left(j2\pi\bar{h}\left(\sum_{i=N-L+1}^{N}b_{i}\frac{t}{2LT_{s}} + \sum_{i=0}^{N-L}\frac{b_{i}}{2}\right)\right),$$

$$\operatorname{rect}(x) = \begin{cases} 1, & 0 < x < 1, \\ 0, & \text{otherwise.} \end{cases}$$
(15)

The block diagram of waveform generation is shown in Fig. 3. The original binary sequence e_i is converted into b_i with bipolar even-length data alphabets by the designed \bar{h} -CPM precoder. The b_i is limited to three segments by the time-varying modulation index \bar{h} , and the phase function $\phi(t)$ which contains communication information is yielded after the forming filter and integration. After orthogonal modulation, $\phi(t)$ is embedded into the I and Q signals, and then multiply with the LFM carrier signal cos $(2\pi f_c t + \pi \mu t^2)$ and sin $(2\pi f_c t + \pi \mu t^2)$. Finally, the DFRC waveform \bar{h} -CPM-LFM is yielded by combining the signals from the I, Q channels.

3. Performance Analysis

3.1 Communication Performance

The STFT is widely used in the time-frequency analysis of non-stationary signals [27]. The STFT of S(t) can be regarded as

$$STFT(t,f) = \int_{-\infty}^{+\infty} S(\tau)g_{\rm w}^*(\tau-t)\exp(-j2\pi f\tau)d\tau \quad (16)$$

where $g_w(t)$ is the real symmetric window of STFT. Different windows yield different time and frequency resolutions. The Gaussian function is utilized in this analysis, and expressed as

$$g_{\rm w}(t) = \exp\left(-\frac{t^2}{2\sigma^2}\right). \tag{17}$$

The STFT results of \bar{h} -CPM-LFM are as follows

$$STFT(t, f) = A_N \Phi_N(t, f)$$
$$\times \sum_{i=0}^{N-1} \operatorname{rect}\left(\frac{t - iT_s}{T_s}\right) A_i(t, f) \Phi_i(t, f). \quad (18)$$



Fig. 3. The block diagram of \bar{h} -CPM-LFM implementation.

The power spectrum of \bar{h} -CPM-LFM is

$$S_{\rm P}(t,f) = |STFT(t,f)|^2 = |A_N|^2 \left| \sum_{i=0}^{N-1} \operatorname{rect}\left(\frac{t-iT_{\rm s}}{T_{\rm s}}\right) A_i(t,f) \right|^2.$$
(19)

As for $(N-1)T_s \le t \le NT_s$, the power spectrum can be simplified as

$$S_{\rm P}(t,f) = |A_N|^2 \exp\left(-\frac{(2\pi\sigma)^2 (f - f_0(t))^2}{(2\pi\mu\sigma^2)^2 + 1}\right),\qquad(20)$$

$$f_0(t) = f_c + \mu t + \frac{\bar{h}}{2LT_s} \sum_{i=0}^{N-1} \operatorname{rect}\left(\frac{t - iT_s}{T_s}\right) \sum_{i=N-L+1}^{N} b_i.$$
(21)

According to (20), the power spectrum is accompanied by a negative exponential distribution. The energy of the most \bar{h} -CPM-LFM power spectrum is concentrated near $f_0(t)$ when $(N-1)T_s \le t \le NT_s$. As the main component of the power spectrum, $f_0(t)$ is analyzed mainly in this part.

Usually to facilitate the analysis, let f_c is 0 in (21), the instantaneous frequency of the \bar{h} -CPM-LFM within one pulse width can be expressed as

$$f_{\bar{h}\text{-CPM-LFM}} = \mu i T_{\rm s} + \frac{\bar{h}}{2LT_{\rm s}} \sum_{i=N-L+1}^{N} b_i.$$
 (22)

From (22), we can see that the actual bandwidth of the integration waveform loaded with communication symbols will be extended and is larger than *B*, and the extension of the spectrum is proportional to the distance between the transmitted communication symbols. Firstly, the integration waveform spectrum extension is limited by a designed precoder, which converts the bipolar odd-symbol $a_i(0, \pm 1, \pm 3, \pm 5, \pm 7)$ sequence of the CPM into a narrow bipolar even-symbol sequence $b_i(0, \pm 1, \pm 2, \pm 3, \pm 4)$ (in octal). Then \bar{h} -CPM-LFM can achieve better spectral efficiency.

Secondly, to limit the spectrum within $[0, \mu NT_s]$ (assume $\mu > 0$, if $\mu < 0$ the symmetrical conclusion can be obtained), bandwidth must be satisfied

$$0 \le f_{\bar{h}-\text{CPM-LFM}} = \mu i T_{\text{s}} + \frac{\bar{h}}{2LT_{\text{s}}} \sum_{i=N-L+1}^{N} b_i \le \mu NT. \quad (23)$$

When the transmitting symbol is -(M/2), then the left inequality of (23) can be regarded as

$$0 \le \mu kT_{\rm s} - \frac{\bar{h}}{2LT_{\rm s}} \sum_{i=k_0-L+1}^{k_0} (M/2) \tag{24}$$

where k is the position of the symbol. Considering a full response in this paper, namely L = 1,

$$k \ge \frac{\bar{h}}{2\mu T_{\rm s}^2} (M/2),\tag{25}$$

 k_0 is the minimum value of k. Or vice versa, when the transmission symbol is M/2

$$\mu kT_{\rm s} + \frac{\bar{h}}{2LT_{\rm s}} \sum_{i=k-L+1}^{k} (M/2) \le \mu NT_{\rm s},$$
 (26)

$$k \le N - \frac{\bar{h}}{2\mu T_{\rm s}^2} (M/2),$$
 (27)

so, the symbol is now at $k \le N - k_0$. Therefore, the timevarying modulation index \bar{h} is introduced in this part, which can be regarded as

$$\bar{h} = \begin{cases} h, & k_0 \le i \le N - k_0, \\ 0, & \text{otherwise,} \end{cases} \quad h \in [0, 1].$$
(28)

Then, the DFRC waveform \bar{h} -CPM-LFM with better spectral efficiency is obtained. Under the limitation of timevarying index, the waveform is divided into three segments. The symbol 0 is introduced at the beginning and end of the pulse. Although the communication efficiency is reduced, it can not only limit the spectral broadening of \bar{h} -CPM but also be used as a guard symbol to protect the edges of the base radar waveform spectrum [24].

In the AWGN channel [28], the BER of CPM waveform was demodulated by Maximum Likelihood Sequence Estimation (MLSE) or Viterbi algorithm

$$P_{\rm e} = K_{\rm min} Q \left(\sqrt{\frac{E_{\rm b}}{N_0} d_{\rm min}^2} \right) \tag{29}$$

where K_{\min} is the number of paths can reach the minimum distance in the observation interval, d_{\min}^2 is the minimum Euclidean metric of different symbols. As is given in (29), the Euclidean metric between adjacent symbols is the key to control BER performance [29]. The BER of the CPM-LFM waveform is the same as the CPM waveform, because loading CPM on LFM has little effect on the Euclidean Metric between the adjacent symbols. As for h-CPM-LFM waveform, although the precoding method obtains better spectral performance, it reduces the Euclidean Metric between adjacent symbols. Therefore, the BER of *h*-CPM-LFM is worse than CPM-LFM under the condition of matched filtering and no strong and random interference outside the original bandwidth. The following simulation analysis shows that the BER is sacrificed for better spectrum performance and the BER performs well in such conditions.

3.2 Radar Performance

The ambiguity function of signal can be expressed as

$$\chi(\tau, f_{\rm d}) = \int_{-\infty}^{+\infty} S(t)S^{*}(t-\tau) \exp(j2\pi f_{\rm d}t) dt$$

= $\int_{-\infty}^{+\infty} e^{j2\pi f_{\rm d}t} \sum_{k=1}^{N} \operatorname{rect}\left(\frac{t-(k-1)T_{\rm s}}{T_{\rm s}}\right) e^{j\pi u^{2}+j\phi(t,b_{i})}$
 $\times \sum_{l=1}^{N} \operatorname{rect}\left(\frac{t-(l-1)T_{\rm s}}{T_{\rm s}}\right) e^{j\pi u(t-\tau)^{2}+j\phi(t-\tau,b_{i})} dt,$
(30)

$$\begin{aligned} \text{et } b &= \left(\sum_{m=k-L+1}^{\sum} b_m - \sum_{n=l-L+1}^{\sum} b_n \right), \text{ then,} \\ \chi &(\tau, f_d) = \begin{cases} ((k-l+1)T_s - \tau) D_{kl} \\ \times \operatorname{sinc} \left\{ \left[\mu \tau + f_d + \frac{\bar{h}\bar{b}}{2LT_s} \right] ((k-l+1)T_s - \tau) \right\}, \\ (k-l)T_s &\leq \tau \leq (k-l+1)T_s. \\ ((l-k+1)T_s - \tau) D_{kl} \\ \times \operatorname{sinc} \left\{ \left[\mu \tau + f_d + \frac{\bar{h}\bar{b}}{2LT_s} \right] ((l-k+1)T_s - \tau) \right\}, \\ (k-l-1)T_s &\leq \tau \leq (k-l)T_s \end{cases}$$

where

$$D_{kl} = \exp\left\{ j\pi \left[-\mu\tau^{2} + \bar{h} \sum_{i=0}^{k-L} b_{a} - \bar{h} \sum_{i=0}^{l-L} b_{i} + \frac{h}{L} \right] \times \left(\sum_{m=k-L+1}^{k} b_{m}(k-m) - \sum_{n=l-L+1}^{l} b_{n}(l-n) - (f_{d} + \mu\tau) \right) \times T_{p} + \left(\mu\tau + f_{d} + \frac{\bar{h}\bar{b}}{2LT_{s}} \right) \left((k+l+1)T_{s} + \tau - T_{p} \right) \right\}.$$
(32)

The peak value of the \bar{h} -CPM-LFM ambiguity function is

$$\tau = -\left\{\frac{f_{\rm d}}{\mu} + \frac{\bar{h}N}{2LB}\left(\sum_{m=k-L+1}^{k} b_m - \sum_{n=l-L+1}^{l} b_n\right)\right\}.$$
 (33)

In (33), the peak value of the ambiguity function appears near $\tau = f_d/\mu$ randomly, it is affected by modulation parameters and communication symbols, resulting in the high pulse-compression sidelobe of the integration waveform. Compared to the unprecedented integration waveform, \bar{h} -CPM-LFM is pre-coded to reduce the distance between the symbols, and lead to lower partials and Peak Sidelobes Ratio (PSLR).

In (31), $\chi(\tau, f_d) \neq 0$ only when k = l with the precondition of $\tau = 0$. The Doppler resolution is

$$|\chi(0, f_{\rm d})| = |T_{\rm p}\operatorname{sinc}(T_{\rm p}f_{\rm d})|.$$
 (34)

In (34), the Doppler resolution of \bar{h} -CPM-LFM is similar to LFM, which is inversely proportional to the pulse width T_p and independently with the number of communication symbols and the way in which the phase is modulated.

(31)

If the Doppler shifts are the same between different targets, namely, $f_d = 0$. when one pulse contains fewer communication symbols and the symbol duration T_s is longer than the delay τ , i.e. $0 \le \tau \ll T_s$,

$$|\chi(\tau,0)| \begin{cases} |(T_{s}-\tau) \operatorname{sinc} (\mu\tau (T_{s}-\tau))|, \ k = l, \\ (k-l)T_{s} \leq \tau \leq (k-l+1)T_{s}. \\ |\tau \operatorname{sinc} \left\{ \left[\mu\tau + \frac{h}{2LT_{s}} (b_{k}-b_{l-L+1}) \right] \tau \right\} |, \\ k = l+1, \ (k-l-1)T_{s} \leq \tau \leq (k-l)T_{s}, \end{cases}$$
(35)

if $0 \le \tau \ll T_s$, the delay resolution is

$$|\chi(\tau,0)| = |T_p \operatorname{sinc}(B\tau)|.$$
(36)

According to (36), the delay resolution, as the same as LFM, is inversely proportional to the bandwidth *B*. A sufficient number of communication symbols are transmitted within a single pulse to effectively guarantee communication efficiency. So, if the condition cannot meet $0 \le \tau \ll T_s$, the bandwidth is expanded inevitably according to spectrum analysis. Therefore, integration waveform can obtain a better delay resolution at the expense of spectral resources, but it leads to a higher range sidelobe, and this will be further illustrated in the simulation analysis.

4. Simulations and Discussion

In this section, we provide numerical analysis results based on MATLAB to assess the performance of the proposed novel integration waveform \bar{h} -CPM-LFM. The parameters of the integration waveform are shown in Tab. 2. In Fig. 4, the waveform and phase diagram of \bar{h} -CPM-LFM are plotted. It is clear that the phase of \bar{h} -CPM-LFM is three consecutive parts, as described in Sec. 2.1.

4.1 Communication Performance

The partial enlargement spectrum of LFM and four integration waveforms with varying symbol numbers are displayed in Fig. 9. Simulation parameters are shown in Tab. 2, we can observe that spectrum of communication symbols are embedded into integration waveform compared with LFM's spectrum. With the increase of the number of communication symbols, the spectrum gap of the different waveforms is larger. However, under the condition of all communication symbols, the spectrum expansion of \bar{h} -CPM-LFM is obviously smaller than that of other integration waveforms and the specific numerical results are shown in Tab. 3. Figure 5 shows the spectrum expansion trend of integration waveforms as the number of symbols increases. The spectrum of S8PSK-LFM signal approximates that of k-CPM-LFM, outperforms CPM-LFM about 6 dB, whereas spectrum of \bar{h} -CPM-LFM differs from that of S8PSK-LFM and k-CPM-LFM by less than 10 dB when N = 200, 500, 1000. It can be seen that the number of communication symbols is inversely proportional to the width of the symbols within a pulse, and the spectrum spread of the integration waveform is more serious.

Paremeter	Value				
f_{c}	0				
$f_{\rm s}$	5 MHz				
h	1/4				
М	8				
L	1				
Ν	50, 200, 500, 1000				
В	1 MHz				
$T_{\rm p}$	1000 µs				
g(t)	REC				

Tab. 2. The parameters of the integration waveform in simulation.



Fig. 4. Waveform and phase diagram of \bar{h} -CPM-LFM.



Fig. 5. The spectrum spread amplitude of LFM and integration waveforms at different numbers of symbols.

Number of symbols	LFM	CPM-LFM	S8PSK	k-CPM-LFM	h-CPM-LFM
50	-62.5	-58.7	-56.8	-59.2	-61.3
200	-62.5	-30.9	-39.5	-38.7	-50.1
500	-62.5	-26.3	-33.8	-34.5	-42.5
1000	-62.5	-24.9	-29.5	-30.1	-32

Tab. 3. The spectrum spread amplitude of lfm and integration waveforms (unit: dB).

According to (22), the bandwidth range of LFM is $[f_c, f_c + B]$ and we can get the \bar{h} -CPM-LFM bandwidth $B_0 = f_c + \mu i T_s + \frac{\bar{h}}{2LT_s} \sum_{i=N-L+1}^N b_i$. Since b_i is drawn from $[\pm 1, \pm 2, \pm 3, \pm 4]$ for \bar{h} -CPM-LFM, b_i is drawn from $[0, \pm 1, \pm 2, \pm 3, \pm 4]$ for S8PSK-LFM and b_i is drawn from $[\pm 1, \pm 3, \dots, \pm (M-1)]$, M = 8 for k-CPM-LFM and conventional CPM-LFM, then we can conclude that the bandwidth range for \bar{h} -CPM-LFM and S8PSK-LFM are $[f_c + B - \frac{4\bar{h}}{2T_s}, f_c + B + \frac{4\bar{h}}{2T_s}]$ and $[f_c + B - \frac{4\bar{h}}{2T_s}, f_c + B + \frac{4\bar{h}}{2T_s}]$ respectively, and *k*-CPM-LFM and CPM-LFM have the same bandwidth range $[f_c + B - \frac{7\bar{h}}{2T_s}, f_c + B + \frac{7\bar{h}}{2T_s}]$ when the number of communication symbols N is large. Thus, the bandwidth of all integration waveforms expands than that of LFM, but the bandwidth of h-CPM-LFM is closest to LFM due to the introduction of the time-varying index \bar{h} . Overall, for fixed amount of transmitting communication symbols, h-CPM-LFM achieve higher spectrum efficiency compared with other signals.

The number of bits transmitted by CPM-LFM, k-CPM-LFM and \bar{h} -CPM-LFM respectively in one pulse is:

$$N_{\rm b} = N \log_2 M, \tag{37}$$

$$N_{\rm b0} = (N - 2k_0) \log_2 M, \tag{38}$$

$$N_{\rm b1} = (N - 2k_1)\log_2 M,\tag{39}$$

the k_0 and k_1 are obtained from (25) and (27)

$$k_0 = \frac{h}{2\mu T_{\rm s}^2} (M - 1), \tag{40}$$

$$k_1 = \frac{h}{2\mu T_{\rm s}^2} (M/2), \tag{41}$$

therefore, the ratio of the sacrificed efficiency of the communication is

$$r_{\rm b0} = \frac{N_{\rm b} - N_{\rm b0}}{N_{\rm b}},\tag{42}$$

$$r_{\rm b1} = \frac{N_{\rm b} - N_{\rm b1}}{N_{\rm b}}.$$
 (43)

The sacrificial ratio of \bar{h} -CPM-LFM communication efficiency in which communication symbols are pre-coded is smaller than k-CPM-LFM to narrow distance resulting in $k_1 < k_0$. Thus \bar{h} -CPM-LFM is more efficient in communication.



Fig. 6. BER of integrated waveform with or without strong interference outside the original bandwidth.

BER of integration waveform subject to AWGN with and without strong interference outside the original bandwidth is evaluated by the Viterbi algorithm shown in Fig. 6. In the absence of interference outside the original bandwidth, the BER of \bar{h} -CPM-LFM is worse than CPM-LFM due to the Euclidean metric between the communication symbols is reduced after precoding. Meanwhile, in the three-segment waveform, the transmitted effective symbols are located in the middle part of the waveform, which provides better antiinterference and noise immunity in practical communication [23]. Hence, \bar{h} -CPM-LFM has better BER performance than S8PSK-LFM. After adding strong interference outside the original bandwidth, the BER of \bar{h} -CPM-LFM is much smaller than that of other integration waveforms because the three-section waveform method can reduce the sensitivity to interference.

4.2 Radar Performance

The ambiguity functions of LFM and \bar{h} -CPM-LFM are presented in Fig. 7, the parameters are also shown in Tab. 1. It can be seen that the peak values of \bar{h} -CPM-LFM and LFM ambiguity function are both distributed near the slope $\mu = B_0/T_p$. As shown in Fig. 8, communication symbols are modulated in \bar{h} -CPM-LFM, which leads to the spreading of the contour around the slope. The results indicate that \bar{h} -CPM-LFM has worse Doppler resistance and weaker sensitivity to high-speed moving targets compared with LFM, which is inevitable when the signal is modulated communication symbol [30].



Fig. 7. The ambiguity function of LFM and \bar{h} -CPM-LFM.

In the cases of the different numbers of symbols, the same trend between LFM and integration waveform is shown in Fig. 10. Because the Doppler resolution is inversely proportional to the pulse width and independent of the number of communication symbols and the way of phase modulation, which is consistent with the analysis above.

To evaluate the delay resolution of the LFM and integration waveforms, zero-Doppler cuts with different numbers of symbols are shown in Fig. 11. Firstly, as shown in Fig. 9(a), the difference of spectrum is small between LFM and the integration signal when N is 20. It can be seen that the delay resolution of LFM and integration waveform have the same trend from Fig. 11(a) since the delay resolution is in inverse proportion to the bandwidth B. The integration waveform has high sidelobes due to the randomness of modulation communication symbols. Secondly, with the increase of the number of symbols, the bandwidth of the integration waveform is extended beyond the LFM from Fig. 11(b), which results in the main lobe width of the integration waveform become narrower and enhancing the detection of adjacent weak signals. When the main lobe width of CPM-LFM and \bar{h} -CPM-LFM are one-fourth and one-half of LFM respectively, statistics indicate that \bar{h} -CPM-LFM spectrum extension is limited. Therefore, the presented waveform has excellent performance for radar tracking and detection.



Fig. 8. The contour of LFM and \bar{h} -CPM-LFM.

5. Conclusion

To improve the integration waveform performance of radar and communication, a novel integration waveform \bar{h} -CPM-LFM is proposed in this paper. The precoding method and a time-varying index \bar{h} are introduced to enhance the spectrum efficiency and BER. The simulation results show that the designed waveform can provide radar resolution equivalent to LFM. Moreover, the analysis also indicate that the \bar{h} -CPM-LFM behaves better communication performance and is better suitable for target detection compared with conventional signals. In the future, we will focus on signal processing and demodulation at the receiving end to enable the waveform to be widely used in civil and military applications.

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Fig. 9. Partial enlargement of LFM, CPM-LFM, S8PSK-LFM, k-CPM-LFM and \bar{h} -CPM-LFM spectrum.



Fig. 10. Partial enlargement of LFM, CPM-LFM, S8PSK-LFM, k-CPM-LFM and h-CPM-LFM integration waveforms Zero-Doppler cut.



Fig. 11. Partial enlargement of LFM, CPM-LFM, S8PSK-LFM, k-CPM-LFM and h-CPM-LFM integration waveforms zero-Doppler cut.

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Appendix A: Proof of Lemma

The abbreviations in (18) are shown below

$$A_N = \frac{2\sigma^2 \sqrt{\pi}}{\sqrt{\left(2\pi\sigma^2 \mu\right)^2 + 1}},\tag{A1}$$

$$\Phi_N(t, f) = \exp\left(j\pi \left(2\left(f_c - f\right)t + \mu t^2\right)\right),\tag{A2}$$

$$A_i(t, f) = \exp\left(B_i(t, f)\right),\tag{A3}$$

$$B_{i}(t,f) = -\frac{2\pi^{2}\sigma^{2}}{\left(2\pi\mu\sigma^{2}\right)^{2} + 1} \times \left(f - f_{c} - \mu t - \frac{\bar{h}}{2L_{s}T}\sum_{i=N-L+1}^{N} b_{i}\right)^{2}, \quad (A4)$$

$$\Phi_{i}(t,f) = \exp\left(j\pi \frac{\bar{h}}{LT_{s}} \sum_{i=N-L+1}^{N} b_{i} \frac{t-iT_{s}}{T_{s}}\right)$$

$$\times \exp\left(2\pi \sum_{i=0}^{N-L} b_{i} - \mu B_{i}\right).$$
(A5)