Optical Ray Transfer Matrix Model of the Turbulent Cells Cascade

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Abstract. *The paper offers a new approach to modeling atmospheric turbulence consisting of turbulent cells whose size is larger than the optical beam width. Particular turbulent cells are approximated by an optical element matrix. The ray transfer matrix method is presented, through which the optical elements can be described in the matrix form. A deflection simulation was performed that indicated the behavior of the optical beam by passing through the optical element. Furthermore, the calculation of the deflection vector is described together with a cascade model of turbulent cells. The matrix calculation for the cascade of optical elements is also expressed.*

Keywords

Atmospheric turbulence, matrix optics, cascade of turbulent cells, ABCD matrix, reciprocity

1. Introduction

Optical Wireless Communication (OWC) refers to transmission in unguided propagation media in the visible, infrared (IR) and ultraviolet (UV) bands [1]. When implementing the OWC link in an optical channel, the serious effects of atmospheric phenomena should be considered and eliminated. Significant dependence of the transmission properties of the Free Space Optical (FSO) system on the state of the atmosphere, causing deterioration of the FSO system's availability [2], [3], is one of the system drawbacks. Despite the stochastic nature of the atmosphere, the FSO system has advantages such as unlicensed bandwidth, more flexibility in designing optical network architecture, easy installation, insensitivity to electromagnetic interference and data security [4].

Constituent particles of the atmosphere, mainly water molecules, carbon dioxide, and ozone, cause absorption, scattering, and atmospheric attenuation [5]. Furthermore, there is a beam divergence loss in FSO communication, which depends on the free space loss (wavelength dependant) [3]. Another atmospheric effect that has a significant impact on the performance of the FSO system is atmospheric turbulence. Atmospheric turbulence leads to irradiance fluctuations, beam spreading, beam wandering and loss of spatial coherence of a laser beam [2], [6]. There are several works focused on modeling and analysis of FSO systems. Some of them focus on describing the various limitations of FSO systems and the various ways to improve the performance of such an atmospheric channel [5]. Further works consists of analyzes of modulation techniques in the presence of strong or weak turbulence in the communication channel or modeling of optical wave diffraction [7–11].

Several mathematical methods [12] are used to quantify atmospheric turbulence and its effect on the optical system. Each of these methods works with turbulence through the parameters of the transmission medium, whether it is air temperature, air pressure, or time and space changes in refractive index [12–14].

This paper offers a new perspective on turbulence as a cascade of the turbulent cells. The turbulent cells of the cascade are approximated by optical elements defined by ray transfer matrices. Due to the model are observed the deflection parameters, which together form the deflection vector.

The turbulent matrix model can be linked to instantaneous quantification methods working with deflection parameters, especially in a dynamically changing communication channels [15]. By extending the model to the influence of other physical phenomena (e.g. absorption, diffraction, scattering, etc.), the model can be applied for prediction of the optical beam propagation in turbulent medium. In such a case, it would be possible to determine the matrix defining the turbulence and, based on its parameters, determine the deflection vector of the input beam.

2. Introduction to Ray Transfer Matrix

Turbulence consists of turbulent cells of various scale sizes [12]. The ray transfer matrix method, which is based on the principles of geometric optics, is used for our work. It deals with large-scale turbulent cells that cause the optical

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beam deflection in the optical wireless links. This method offers a way to perceive a turbulent cell as an optical element. The following technique works with the beam description of optical radiation, which is part of geometric optics. The presence of paraxial rays is a requirement for using this technique [16], for which the following applies

$$\tan(\theta) \approx \theta \tag{1}$$

where θ represents the angle subtended by the optical beam with the auxiliary axis parallel to the optical system axis.

The propagation of paraxial rays through various optical elements is described by ray transfer matrices [17] also known as ABCD matrices. Optical elements can be optical prisms, thick or thin lenses, mirrors or interfaces where the refraction of an optical beam occurs [17].

The beam transmission technique shown in Fig. 1 operates with two reference planes, input and output planes. Both planes are perpendicular to the optical axis of the system. At each point in the optical system, an optical axis corresponding to the central beam is defined. The optical beam enters the inhomogeneous area through the input plane at a distance x_1 from the optical axis and propagates in the direction that makes an angle θ_1 with an auxiliary optical axis. The inhomogeneous area is defined by the refractive index n_2, where the refractive index n_1 represents the surrounding medium. After propagation to the output plane, this beam is at a distance x_2 from the optical axis and propagates further at an angle θ_2 [17].

The beam deflection on the output plane is evaluated by the equation

$$x_2 = A \cdot x_1 + B \cdot \theta_1 \tag{2}$$

and the angle of deflection of the beam from the optical axis is evaluated from the following relation

$$\theta_2 = C \cdot x_1 + D \cdot \theta_1. \tag{3}$$

These expressions can be written in the matrix form [17]

$$\begin{pmatrix} x_2 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} x_1 \\ \theta_1 \end{pmatrix} \tag{4}$$

where the ABCD matrix represents the area between input and output planes where the optical element is situated. This matrix combines the deflection of the outgoing beam with the deflection of the incident beam.

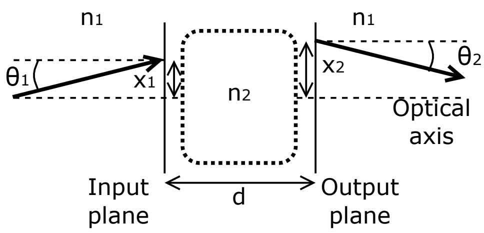

Fig. 1. The optical beam transmission technique, where $x_{1,2}$ represent the deflections of the beam from the optical axis, $\theta_{1,2}$ indicate the angles of deflection and d is the distance between the input and output planes.

2.1 Approximation of Turbulent Cells and Inter-Turbulent Spaces

A turbulent atmosphere is considered to be a nonstationary medium where the refractive index n changes. Thick lens is a proper optical element for approximating a turbulent cell. Defined matrix of thick lens assumes a change in the refractive index as the optical beam passes through the optical structure [18].

The ABCD matrix defined for a thick lens (see Fig. 2) is given by multiplying the three matrices containing the descriptive parameters of the lens [16]

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{n_2-n_1}{R_2 n_1} & \frac{n_2}{n_1} \end{pmatrix} \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{n_1-n_2}{R_1 n_2} & \frac{n_1}{n_2} \end{pmatrix} \tag{5}$$

where n_1 is refractive index outside of lens, n_2 is refractive index of the lens itself, R_1 is radius of curvature of the first surface, R_2 is radius of curvature of the second surface and t represents the center thickness of lens.

The ABCD matrix defined for medium of constant refractive index (see Fig. 3), which represents the inter-turbulent space in the turbulent cell cascade model, is given by [16]

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix} \tag{6}$$

where l denotes the width of the area with constant refractive index. The matrix is derived using a trigonometric function and paraxial approximation. A more detailed description of the derivation and calculation can be found in Sec. 3.

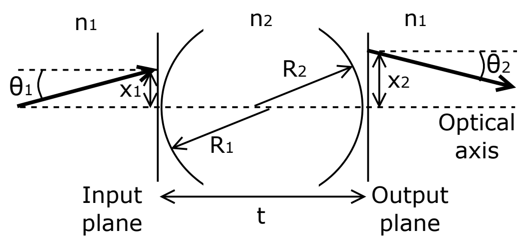

Fig. 2. Beam propagation model through a thick lens, where t denotes the center thickness of the lens, which also represents the distance between the input and output planes. $R_{1,2}$ are the radii of curvature of the lens surfaces [18].

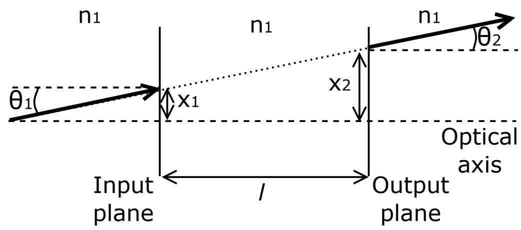

Fig. 3. Beam propagation through a medium with a constant refractive index, where l denotes the distance between the input and output planes and the equality of angles applies $\theta_1 = \theta_2$.

2.2 ABCD Turbulence Simulation

In the following text, $\mathbf{L}$ will denote the ray transfer matrix (ABCD matrix) for medium of constant refractive index (inter-turbulent space) and $\mathbf{T}$ will denote the ray transfer matrix for the thick lens (turbulent cell). Figure 4 shows two variations of the optical beam transition. One of them represents a reference model consisting of a medium with a constant refractive index. The refractive index of the optical element has the same value as the refractive index of the surrounding medium. The second variation is a model of turbulent cell approximated by a thick lens. The simulation based on Fig. 4 points to five cases of beam propagation:

- The first case is the reference model $\mathbf{L}_0$ with a medium with a constant refractive index
- The remaining four cases $\mathbf{T}_1$–$\mathbf{T}_4$ are formed by a turbulent cell model with different sizes and refractive indices.

Geometric parameters (radii of curvature, central thickness of the lens) and refractive indices for specific simulation models are given in the Tab. 1. According to relationship (10), the refractive index $n(\lambda)$ depends on the wavelength of the used laser (λ = 0.6328 µm), the atmospheric pressure (P = 102850 Pa), and the thermodynamic temperature T varying according to Tab. 1.

Model	***R*** [cm]	***t*** [cm]	***T*** [K]	***n*** [-]
L_0	1	2	295.65	1.027502
T_1	1.5	3	296.15	1.027456
T_2	2	4	296.65	1.027410
T_3	2.5	5	295.15	1.027549
T_4	3	6	294.65	1.027596

Tab. 1. Parameters for each case of the model. R stands for radius of curvature ($R = R_1 = R_2$), t is central thickness of lenses, T presents thermodynamic temperature, and n is the refractive index calculated according to (10).

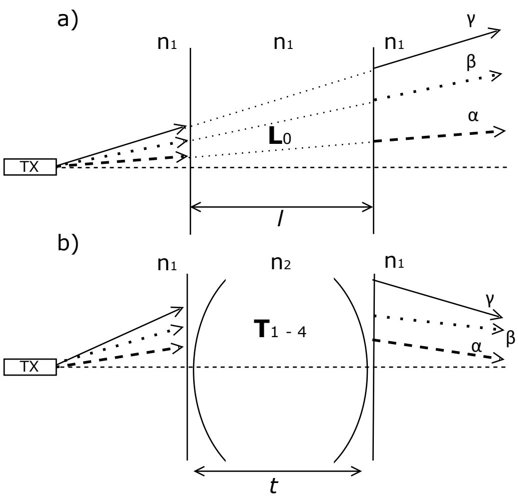

Fig. 4. Three different deflected optical beams propagate a) in a medium with a constant refractive index $\mathbf{L}_0$ and b) a medium with a thick lens $\mathbf{T}_{1-4}$. α, β, and γ indicate specific optical beam deflections in each model.

These turbulent models show how the direction and the size of the optical beam deflection can change after passing through a turbulent cell compared to passing through a stationary medium. Final results of the simulation are evaluated for three different beams (α, β, γ) passing through each model. Each of the rays has a different input deflection x_1 according to Tab. 2, and its subsequent transition is realized for different large lenses with different values of refractive indices. With increasing matrix numbering ($\mathbf{T}_1$–$\mathbf{T}_4$), cell size and refractive index increase.

The results of simulation based on these five models are shown in the Fig. 5. The absolute difference of the output deflections from the reference model Δ_{x_2} is plotted on the y-axis and the values of the input deflection x_1 of the optical beam is plotted on the x-axis. The x-axis on Fig. 5 is formed by three points, which indicate the deflections of the beam on the input plane x_1 (α, β, γ). For each model ($\mathbf{L}_0$–$\mathbf{T}_4$), the beam deflections on the output plane $x_{2\mathbf{L}_0}$, $x_{2\mathbf{T}_{1-4}}$ were determined according to the model type. Since the relatively small dimensions of the turbulent cells and the small changes in the refractive index are taken into account, the output deflections x_2 are slightly different. Therefore, it is more efficient to monitor the absolute difference of output deflections Δ_{x_2} from the reference model given by relation

$$\Delta_{x_2} = |x_{2\mathbf{L}_0} - x_{2\mathbf{T}_{1-4}}| \tag{7}$$

where values of $x_{2\mathbf{L}_0}$, $x_{2\mathbf{T}_{1-4}}$ for individual models are obtained from the matrix (4). Their dependence on the input deflection x_1 is evident. It is clear that the value of Δ_{x_2} will be zero at all points of the reference model.

Beams	$\boldsymbol{x_1}$ [cm]	$\boldsymbol{\theta_1}$ [rad]
α	0.5	1.75 e-04
β	1	3.49 e-04
γ	1.5	5.24 e-04

Tab. 2. Values of input deflections x_1 and angles of deflection θ_1 for three different deflected optical beams denoted as α, β, γ. The input deflections x_1 are determined based on the transmitter distance, turbulent cell size, and paraxial condition.

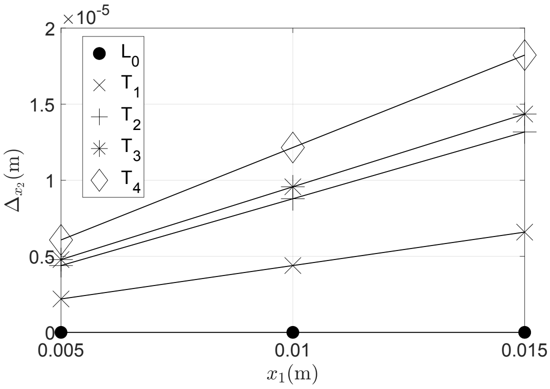

Fig. 5. Dependence of the absolute difference of the output deflections from the reference model on the values of the input deflection of the optical beam. The dependence is realized for five models $\mathbf{L}_0$–$\mathbf{T}_4$.

As turbulent cell sizes and refractive indices increase, so does the overall difference in output deflections from the reference model. By passing through a larger turbulent cell, a more significant deflection will occur.

3. Cascade System

The idea of a cascade system is to imagine atmospheric turbulence as several consecutive turbulent cells. This cascade represents part of the optical transmission path. Each turbulent cell has its own defined geometric parameters that best reflect the size of the actual turbulent cell. By modeling the turbulent cascade, it is possible to monitor the deflection of the beam not only on the detector of the receiver after passing through the entire cascade but also within it.

The following section offers a mathematical procedure for calculating the optical beam deflection with a matrix solution of a cascade of turbulent cells.

3.1 Calculation of Beam Deflection by Passing Through a Cascade of Two Turbulent Cells

To better illustrate the procedure for determining the individual deflections of the paraxial optical beam, the following calculation procedure will be based on the turbulent cascade model from the Fig. 6.

In the first place, it is necessary to determine the distance from the laser radiation source to the turbulent area l_{TX} and the deflection of the beam x_1. Using the trigonometric tangent function, it is possible to calculate the parameters of the beam deflection on the input plane of the lens

$$x_1 = l_{\mathrm{TX}} \cdot \tan(\theta_1) \tag{8}$$

where x_1 is the deflection on the input plane, l_{TX} indicates the distance from the laser source to the turbulent cell. The value of the initial deflection angle θ_1 of the laser beam must be greater than zero degrees ($\tan(0) = 0$), which is a condition for the proper use of ray transfer matrix calculation. The beam deflection parameters at the input plane of the first turbulent cell of the cascade form the input vector denoted as

$$\mathbf{X}_{\mathrm{IN}} = \begin{pmatrix} x_1 \\ \theta_1 \end{pmatrix}. \tag{9}$$

In order to calculate the deflection vectors using matrix (5), the refractive indices in the given areas are determined based on the values of surrounding pressure, thermodynamic temperature, and laser wavelength. The refractive index $n(\lambda)$ is calculated from the relation [13]

$$n(\lambda) = 1 + 77.6 \cdot 10^{-6} \left(1 + 7.52 \cdot 10^{-3} \cdot \lambda^{-2}\right) \frac{P}{T} \tag{10}$$

where λ is optical wavelength in μm, P is air pressure in Pa and T is thermodynamic temperature at the location in K. Spectral dependence of the refractive index of the atmosphere must be taken into account, so that its value can be evaluated for specific optical wavelengths. Therefore, the used equation depends not only on the state quantities, but also on the wavelength of the laser.

The turbulent cell size defined by the central thickness t and radii of curvature $R_{1,2}$, and the refractive index n are used to calculate the matrix $\mathbf{T}_1$ (Fig. 6). The $\mathbf{T}_1$ label represents the ABCD matrix for the first turbulent cell. By the parameters of deflection on the input plane (x_1, θ_1) and the matrix $\mathbf{T}_1$ the deflection on the output plane are obtained

$$\begin{pmatrix} x_2 \\ \theta_2 \end{pmatrix} = \mathbf{T}_1 \begin{pmatrix} x_1 \\ \theta_1 \end{pmatrix}. \tag{11}$$

After finding the value of the parameters x_2 and θ_2, the value of the deflection parameters on the input plane of the second lens (x_3, θ_3) are calculated.

The trigonometric tangent function determines the deflection in the input plane of the second lens due to the known angle of deflection and inter-turbulent distance l. In this simplified model, the optical beam always propagates toward the optical axis of the system. The deflection of the laser beam in the inter-turbulent area is defined for four potential cases, which are shown in Fig. 7:

(a) The beam deflects from the area above the optical axis to above it with a positive deflection value, but a negative value of deflection angle (Fig. 6 - $\mathbf{L}_1$).

(b) The beam deflects from the area above the optical axis to below it. The deflection angle is negative, and deflection on the output plane acquires a negative value (Fig. 6 - $\mathbf{L}_1$).

(c) The beam deflects from the area below the optical axis to below it. The deflection angle is positive, but deflection acquires a negative value (Fig. 6 - $\mathbf{L}_2$).

(d) The beam deflects above the optical axis with a positive value of both deflection parameters (Fig. 6 - $\mathbf{L}_2$).

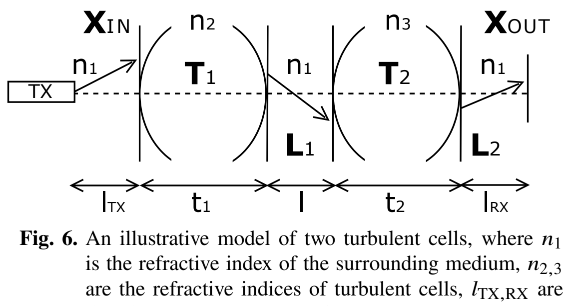

Fig. 6. An illustrative model of two turbulent cells, where n_1 is the refractive index of the surrounding medium, $n_{2,3}$ are the refractive indices of turbulent cells, $l_{\mathrm{TX,RX}}$ are the distances TX and RX from the turbulent cascade, l is the inter-turbulent distance, $t_{1,2}$ are the central cell thickness, $\mathbf{T}_{1,2}$ are ABCD matrices of the turbulent cells, $\mathbf{L}_{1,2}$ are ABCD matrices of the inter-turbulent spaces and $\mathbf{X}_{\mathrm{IN,OUT}}$ are the deflection vectors at the input and output planes of the turbulent cascade.

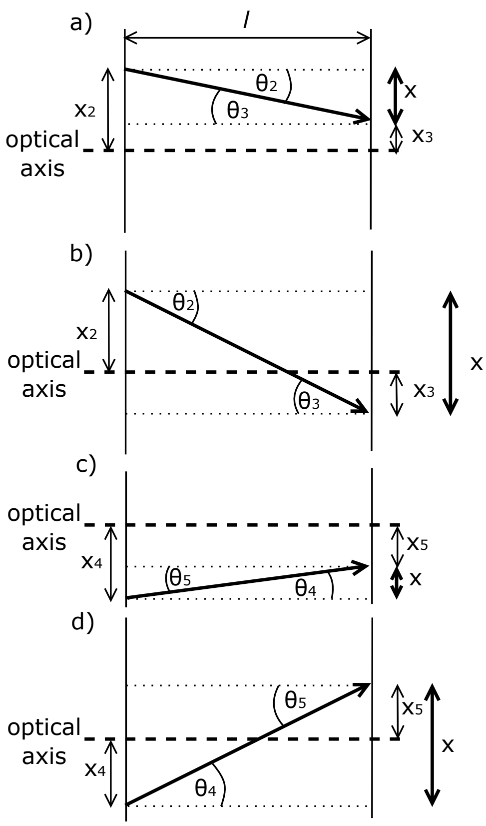

Fig. 7. Four cases of the laser beam transition from the space below and above the optical axis of the system. a) and b) represent the area $\mathbf{L}_1$ in Fig. 6, and c) and d) represent the $\mathbf{L}_2$ region in the same figure.

The calculation of the deflection vector at the input plane of the second turbulent cell has the form

$$\begin{pmatrix} x_3 \\ \theta_3 \end{pmatrix} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_2 \\ \theta_2 \end{pmatrix} \tag{12}$$

where the negative sign in the deflection parameter x_3 indicates the position of the beam deflection under or above the optical axis. Subsequently, $\mathbf{L}_1$ represents the ABCD matrix for the inter-turbulent space

$$\begin{pmatrix} x_3 \\ \theta_3 \end{pmatrix} = \mathbf{L}_1 \begin{pmatrix} x_2 \\ \theta_2 \end{pmatrix}. \tag{13}$$

The calculation of the deflection at the output plane of the second lens is similar to the calculation of the first lens, where $\mathbf{T}_2$ represents the ABCD matrix for the second turbulent cell

$$\begin{pmatrix} x_4 \\ \theta_4 \end{pmatrix} = \mathbf{T}_2 \begin{pmatrix} x_3 \\ \theta_3 \end{pmatrix}. \tag{14}$$

After determining the position of the laser beam on the output plane of the second lens (Fig. 6), the position of the received beam on the detector's plane (x_5, θ_5) can be calculated based on the matrix definition for a medium with a constant refractive index

$$\begin{pmatrix} x_5 \\ \theta_5 \end{pmatrix} = \begin{pmatrix} 1 & l_{\mathrm{RX}} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_4 \\ \theta_4 \end{pmatrix} \tag{15}$$

where the matrix in this formula represents the matrix $\mathbf{L}_2$ on the Fig. 6

$$\mathbf{X}_{\mathrm{OUT}} = \begin{pmatrix} x_5 \\ \theta_5 \end{pmatrix} = \mathbf{L}_2 \begin{pmatrix} x_4 \\ \theta_4 \end{pmatrix}. \tag{16}$$

For all cases of transmission of an optical beam in the inter-turbulent space, depending on the inter-turbulent distance and the size of the parameters of the input deflection vector, a positive or negative beam deflection can be detected.

3.2 Matrix Cascade Calculation

Imagine a cascade of N turbulent cells according to Fig. 8. Each turbulent cell is defined by its ray transfer matrix $\mathbf{T}_1$–$\mathbf{T}_n$ and each inter-turbulent space is defined by matrix $\mathbf{L}_1$–$\mathbf{L}_n$. Vector $\mathbf{X}_{\mathrm{IN}}$ points to a primary deflection of the optical beam at the input of the turbulent cascade and $\mathbf{X}_{\mathrm{OUT}}$ points to the vector of the resulting beam deflection by passing through the whole cascade.

Through the paraxial approximation [16] according to the equation (1), it is possible to write the ABCD matrix of this form for the inter-turbulent space

$$\mathbf{L}_n = \begin{pmatrix} 1 & l_n \\ 0 & 1 \end{pmatrix}, \tag{17}$$

as was shown in the previous subsection. As a result of this simplification, neither the matrices defined for turbulent cells nor the matrices for inter-turbulent space contain evolving deflection parameters. Therefore, it is possible to perform matrix multiplication of individual optical elements of the turbulent cascade.

Figure 9 shows a diagram pointing to a matrix cascade calculation of the output deflection vector $\mathbf{X}_{\mathrm{OUT}}$. The technique of calculating the output deflection of the beam incident on the detector is evident from the given diagram. Multiplication of matrices is performed by notation

$$\mathbf{X}_{\mathrm{OUT}} = \mathbf{L}_n \cdot \mathbf{T}_n \ldots \mathbf{L}_2 \cdot \mathbf{T}_2 \cdot \mathbf{L}_1 \cdot \mathbf{T}_1 \cdot \mathbf{X}_{\mathrm{IN}} \tag{18}$$

where $\mathbf{X}_{\mathrm{IN}}$ denotes the vector of cascade deflection input parameters and $\mathbf{X}_{\mathrm{OUT}}$ denotes the vector of cascade deflection output parameters. By reversing the order of the matrices, it is possible to calculate for the given channel the deflection of the optical beam propagating in the opposite direction.

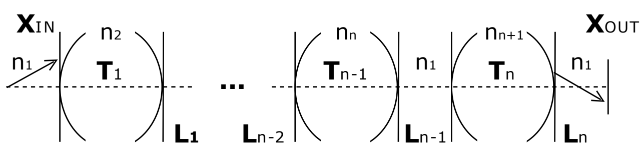

Fig. 8. Illustration model of the cascade of N turbulent cells.

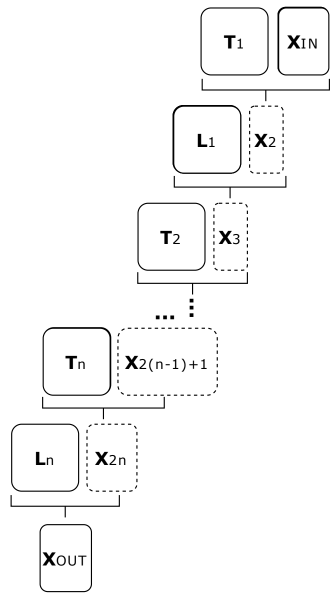

Fig. 9. Diagram showing the procedure of cascade matrix calculation. The bold blocks represent the input deflection vector, the ABCD matrices, and the output deflection vector. Dashed blocks indicate deflection vectors by passing through the individual elements of cascade.

4. Conclusion

In summary, this paper presents an introduction to the ray transfer matrix method used for a mathematical description of atmospheric turbulence as an optical element. The designed turbulent model consists of a cascade of optical elements approximating turbulent cells. From the cascade of turbulent cells and in the presence of certain simplifications and requirements, there is the potential to work with mentioned mathematical technique. In this way, the model currently serves to observe the optical beam deflection parameters that create the deflection vector.

The paper also includes a simulation aimed at comparing the deflection of the optical beam through different large turbulent cells via a matrix model. In addition, it presents a mathematical procedure for calculating the deflection vectors inside or outside the cascade, at the receiver.

Turbulence in FSO systems can also be analyzed using vector notation and matrix definition of the transmission medium. Via the matrix calculation technique, it would be possible to determine the matrix of a turbulent medium based on initial and detected optical beams. The backward matrix calculation will be able to specify the initial beam deflection so that the optical link works as efficiently as possible.

Implementation of this method would be possible in FSO systems in the presence of local temperature inhomogeneity, which is larger than the width of the optical beam - turbulent cells in the desert or planetary boundary layer. Also, the results are applicable in other transmission media - underwater communications in the analysis of the propagation of optical beams through layers with different temperatures, i.e. different refractive indices. However, the work is in the initial phase of the modeling for further processing in the time and space domain.

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