Design of Nonuniformly Spaced Antenna Arrays using Orthogonal Coefficients Equating Method

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Abstract. *Orthogonal Coefficients Equating (OCE) method as an analytic method is proposed to synthesize nonuniformly spaced antenna arrays to have array factors nearly equal to that of a previously designed uniformly spaced antenna arrays. In this method, the orthogonal coefficients of array factors of nonuniformly spaced array are equated to those of uniformly spaced array. To this end, three orthogonal functions including Chebyshev polynomials, Legendre polynomials and exponential functions are discussed. Some examples are brought to verify the performance of the OCE method for all three orthogonal functions.*

Keywords

Nonuniformly spaced arrays, uniformly spaced arrays, orthogonal coefficients equating, Chebyshev polynomials, Legendre polynomials, exponential functions

1. Introduction

Nonuniformly Spaced Antenna Arrays (NSAAs) are desirable because they yield desired radiation patterns without using nonuniform amplitude excitation. Therefore, NSAAs do not need complex circuits providing nonuniform amplitude distribution like as for Uniformly Spaced Antenna Arrays (USAAs).

To find the optimum positions of the antennas of NSAAs is an essential aim for their synthesis. As yet, many methods have been introduced for the synthesis of NSAAs. These methods would be divided into two main groups. The first group includes various optimization procedures [1–6] and gives the results blindly without enough information about the features of the results. The second group includes various analytical procedures among them integral techniques, perturbation, iterations methods and probabilistic approaches [7–19]. In [17], Zeros Matching Method (ZMM) is introduced. In this method, the zeros of array factors of NSAAs are equated to the zeros of the exact-Chebyshev pattern to reach a near-Chebyshev pattern. In [18], ZMM method is generalized to design arbitrary array factors. In [19], Fourier's Coefficients Equating (FCE) method is proposed. In this method, the periodicity of array factors is used and the Fourier's coefficients of NSAA is equated to those of USAA. In [20], [21], NSAAs are synthesized with minimum number of elements at the cost of having nonuniform excitations.

In this article, another method is proposed for the second group of synthesis methods. It is tried to design NSAAs for having array factors as close as to those of a previously designed USAA [22], [23]. In this analytical method, called Orthogonal Coefficients Equating (OCE) method, the array factors are represented as a summation of orthogonal functions, first. Then the orthogonal coefficients of array factors of NSAAs are equated to those of array factor of USAA. The positions of the antennas are obtained by solving a system of linear equations, iteratively. Three orthogonal functions are used for presented OCE method consisting of Chebyshev polynomials, Legendre polynomials and exponential functions. Finally, some exhaustive examples are given to show the effectiveness of the proposed method to synthesize NSAAs.

The paper is organized as follows. In Sec. 2, the array factors of USAAs and NSAAs are reviewed. In Sec. 3, the OCE method using Chebyshev polynomials is introduced. In Sec. 4, the OCE method using Legendre polynomials is introduced. In Sec. 5, the OCE method using exponential functions is introduced. In Sec. 6, matrix equations are obtained for all presented OCE methods. In Sec. 7, some examples are provided to show the effectiveness of the OCE method.

2. USAA and NSAA Arrays

Figure 1 shows linear uniformly spaced antenna arrays (USAA) having $L = 2N + 1$ or $L = 2N$ elements of equal distances d_0. The array factors of uniformly spaced arrays are given by the following relations, for odd and even number of antennas, respectively:

$$F_{\mathrm{USAA}}(u) = \sum_{n=-N}^{N} A_n \exp(\mathrm{j}knd_0u), \tag{1}$$

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$$F_{\text{USAA}}(u) = \sum_{\substack{n=-N \\ n\neq 0}}^{N} A_n \exp\left(\mathrm{j}k(n \mp 0.5)d_0 u\right) \tag{2}$$

where $k = 2\pi/\lambda$ is the wavenumber and A_n is the excitation of the nth antenna. Also, $u = \cos\theta$ is the variable of array factors which varies within $u = [-1\ {+1}]$. The lower and upper signs in (2) stand for negative and positive n, respectively.

Figure 2 shows linear nonuniformly spaced antenna arrays (NSAA) with $L = 2N + 1$ or $L = 2N$ antennas. The position of the n-th antenna is deviated from that of uniformly spaced arrays by $e_n d_0$ as follows for even and odd number of antennas, respectively.

$$x_n = \begin{cases} (n + e_n) d_0; & n = -N, \ldots, -1, 0, 1, \ldots, N \\ ((n \mp 0.5) + e_n) d_0; & n = -N, \ldots, -1, 1, \ldots, N \end{cases} \tag{3}$$

where e_n is defined as the n-th deviation, assuming $e_0 = 0$ for the array of odd number of elements. Again, the lower and upper signs in the second case stand for negative and positive n, respectively.

The array factor of nonuniformly spaced arrays are given by the following relations for odd and even number of antennas, respectively:

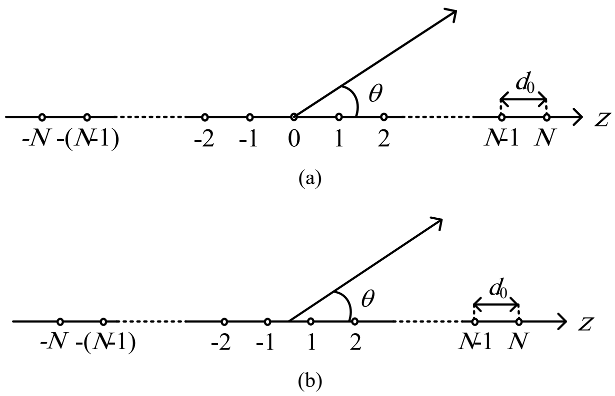

Fig. 1. Typical configuration of USAAs: a) Odd number of elements. b) Even number of elements.

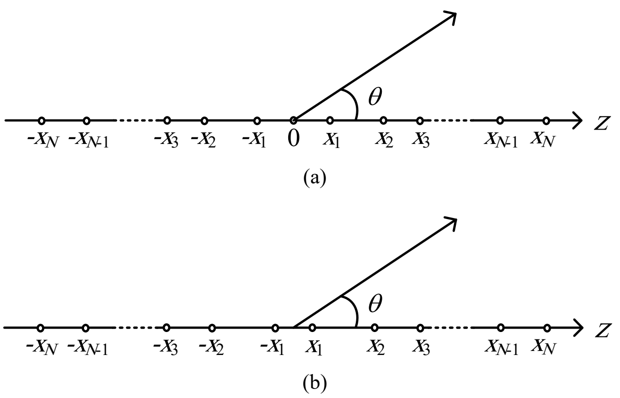

Fig. 2. Typical configuration of NSAAs: a) Odd number of elements. b) Even number of elements.

$$F_{\text{NSAA}}(u) = \sum_{n=-N}^{N} \exp(\mathrm{j}kx_n u), \tag{4}$$

$$F_{\text{NSAA}}(u) = \sum_{\substack{n=-N \\ n\neq 0}}^{N} \exp(\mathrm{j}kx_n u) \cdot \tag{5}$$

It is worth noting that all array factors of USAAs and NSAAs in (1), (2), (4) and (5) are functions of $u = \cos\theta$ and therefore can be represented by orthogonal functions defined within $u = [-1\ {+1}]$, which is the discussion of the following sections. Without loss of generality, here we discuss only for arrays containing odd number of elements.

It is an important note that the excitations of USAAs, A_n, must be scaled so that their sum equals L. This is done to equate $F_{\text{USAA}}(0) = F_{\text{NSAA}}(0)$.

3. Chebyshev Coefficients of Antenna Arrays

Both functions $F_{\text{USAA}}(u)$ and $F_{\text{NSAA}}(u)$ in (1) and (4) can be represented by Chebyshev series as follows:

$$F(u) = \sum_{m=0}^{\infty} F_m T_m(u) \tag{6}$$

where $T_m(u)$ is the m-th degree Chebyshev polynomial of the first kind which is orthogonal with other degrees of Chebyshev polynomials. Also, F_ms are the m-th coefficient given by

$$F_m = \frac{\varepsilon_m}{\pi} \int_{-1}^{1} F(u) \frac{T_m(u)}{\sqrt{1-u^2}} \mathrm{d}u \tag{7}$$

in which, ε_m is defined as one or two for $m = 0$ and $m \neq 0$, respectively.

Substituting functions $F_{\text{USAA}}(u)$ and $F_{\text{NSAA}}(u)$ in (7) and considering the following well-known identities

$$\int_{-1}^{1} \exp(\mathrm{j}xu) \frac{T_m(u)}{\sqrt{1-u^2}} \mathrm{d}u = \pi \mathrm{j}^m J_m(x), \tag{8}$$

$$J'_v(x) = \frac{v}{x} J_v(x) - J_{v+1}(x) \tag{9}$$

where $J_m(x)$ is the first kind Bessel function of order m and $J_m(x)$ is the derivative of the Bessel function, the orthogonal coefficients of USAA and NSAA are determined as follows, respectively.

$$F_m = \varepsilon_m \mathrm{j}^m \sum_{n=-N}^{N} A_n J_m(knd_0), \tag{10}$$

$$F_m = \varepsilon_m \mathrm{j}^m \sum_{n=-N}^{N} J_m(kx_n) \cong$$

$$\varepsilon_m \mathrm{j}^m \sum_{n=-N}^{N} J_m(knd_0) + \varepsilon_m \mathrm{j}^m \sum_{\substack{n=-N \\ n\neq 0}}^{N} kd_0 J'_m(knd_0) e_n =$$

$$
\begin{aligned}
&= \varepsilon_m \mathrm{j}^m \sum_{n=-N}^{N} J_m(knd_0) \\
&+ \varepsilon_m \mathrm{j}^m (kd_0) \sum_{\substack{n=-N \\ n\neq 0}}^{N} \left(\frac{mJ_m(knd_0)}{knd_0} - J_{m+1}(knd_0) \right) e_n .
\end{aligned} \tag{11}
$$

In (11), the approximation $ke_nd_0 << 1$ has been assumed.

Now, the orthogonal coefficients of NSAA are equated with those of USAA. So, Equations (10) and (11) are equated to one another. This equating yields a system of equations as follows:

$$
\begin{aligned}
&k_0 d \sum_{\substack{n=-N \\ n\neq 0}}^{N} \left(\frac{mJ_m(knd_0)}{knd_0} - J_{m+1}(knd_0) \right) e_n \\
&\cong \sum_{n=-N}^{N} A_n J_m(knd_0) - \sum_{n=-N}^{N} J_m(knd_0).
\end{aligned} \tag{12}
$$

Equation (12) is identical to that obtained from FCE method [19]. Equation (12) holds for $m = 0$ to M which is the upper bound of m in truncated series in (6). The parameter M must be specified considering (10) and (11) so that the largest term in the M-th coefficient got less than the smallest term in the zero coefficient. It means that $|J_M(kNd_0)| < |J_0(kNd_0)|$. It would happen when $M > 1.3kNd_0$.

4. Legendre Coefficients of Antenna Arrays

Both functions $F_{\mathrm{USAA}}(u)$ and $F_{\mathrm{NSAA}}(u)$ in (1) and (4) can be represented by Legendre series as follows:

$$
F(u) = \sum_{m=0}^{\infty} F_m P_m(u) \tag{13}
$$

where $P_m(u)$ is the m-th degree Legendre polynomial which is orthogonal with other degrees of Legendre polynomials. Also, F_ms are the m-th coefficient given by

$$
F_m = \frac{2m+1}{2} \int_{-1}^{1} F(u) P_m(u) \mathrm{d}u \cdot \tag{14}
$$

Substituting functions $F_{\mathrm{USAA}}(u)$ and $F_{\mathrm{NSAA}}(u)$ in (14) and considering the following well-known identity

$$
\int_{-1}^{1} \exp(\mathrm{j}xu) P_m(u) \mathrm{d}u = \sqrt{2\pi}\, \mathrm{j}^{m-1} \frac{J_{m+0.5}(x)}{\sqrt{x}} \tag{15}
$$

the orthogonal coefficients of USAA and NSAA are determined as follows, respectively:

$$
F_m = k_m \sum_{n=-N}^{N} A_n \frac{J_{m+0.5}(knd_0)}{\sqrt{knd_0}}, \tag{16}
$$

$$
F_m = k_m \sum_{n=-N}^{N} \frac{J_{m+0.5}(kx_n)}{\sqrt{kx_n}} \cong
$$

$$
\begin{aligned}
&\cong k_m \sum_{n=-N}^{N} \frac{J_{m+0.5}(knd_0)}{\sqrt{knd_0}} \\
&+ k_m (kd_0) \sum_{\substack{n=-N \\ n\neq 0}}^{N} \left(\frac{mJ_{m+0.5}(knd_0)}{\sqrt{(knd_0)^3}} - \frac{J_{m+1.5}(knd_0)}{\sqrt{knd_0}} \right) e_n
\end{aligned} \tag{17}
$$

where $k_m = \sqrt{2\pi}(m+0.5)\mathrm{j}^{m-1}$. In (17), the approximation $ke_nd_0 << 1$ has been assumed.

Now, the orthogonal coefficients of NSAA are equated with those of USAA. So, Equations (16) and (17) are equated to one another. This equating yields a system of equations as follows.

$$
\begin{aligned}
&k_0 d \sum_{\substack{n=-N \\ n\neq 0}}^{N} \left(\frac{mJ_{m+0.5}(knd_0)}{\sqrt{(knd_0)^3}} - \frac{J_{m+1.5}(knd_0)}{\sqrt{knd_0}} \right) e_n \\
&\cong \sum_{n=-N}^{N} A_n \frac{J_{m+0.5}(knd_0)}{\sqrt{knd_0}} - \sum_{n=-N}^{N} \frac{J_{m+0.5}(knd_0)}{\sqrt{knd_0}}.
\end{aligned} \tag{18}
$$

In (16) and in the right hand side of (18), the existing term of $n = 0$, tends to zero for $m \neq 0$ and 0.798 for $m = 0$.

Equation (18) holds for $m = 0$ to M which is the upper bound of m in truncated series in (13). The parameter M must be specified in the light of (16) and (17) so that the largest term in the M-th coefficient got less than the smallest term in the zero coefficient. It means that $|J_{M+0.5}(kNd_0)| < |J_{0.5}(kNd_0)|$. It would happen when $M > 1.35kNd_0$.

5. Exponential Coefficients of Antenna Arrays

Both functions $F_{\mathrm{USAA}}(u)$ and $F_{\mathrm{NSAA}}(u)$ in (1) and (4) can be represented by summation of orthogonal exponential functions as follows:

$$
F(u) = \sum_{m=-\infty}^{\infty} F_m \exp(\mathrm{j}m\pi u) \cdot \tag{19}
$$

Here, F_ms are the m-th coefficient given by

$$
F_m = \frac{1}{2} \int_{-1}^{1} F(u) \exp(-\mathrm{j}m\pi u) \mathrm{d}u \cdot \tag{20}
$$

Substituting functions $F_{\mathrm{USAA}}(u)$ and $F_{\mathrm{NSAA}}(u)$ in (20) and considering the following well-known identity

$$
\int_{-1}^{1} \exp(\mathrm{j}xu) \exp(\mathrm{j}m\pi u) \mathrm{d}u = 2\operatorname{sinc}(m + x/\pi) \tag{21}
$$

the orthogonal coefficients of USAA and NSAA are determined as follows, respectively:

$$
F_m = \sum_{n=-N}^{N} A_n \operatorname{sinc}\left(m + (knd_0)/\pi\right), \tag{22}
$$

$$F_m = \sum_{n=-N}^{N} \operatorname{sinc}\left(m+(kx_n)/\pi\right) \cong \sum_{n=-N}^{N} \operatorname{sinc}\left(m+(knd_0)/\pi\right)$$
$$+\frac{kd_0}{m\pi+knd_0}\sum_{\substack{n=-N\\n\neq 0}}^{N}\left(\cos(m\pi+knd_0)-\operatorname{sinc}(m+(knd_0)/\pi)\right)e_n. \tag{23}$$

In (23), the approximation $ke_nd_0 << 1$ has been assumed.

Now, the orthogonal coefficients of NSAA are equated with those of USAA. So, Equations (22) and (23) are equated to one another. This equating yields a system of equations as follows:

$$\frac{kd_0}{m\pi+knd_0}\sum_{\substack{n=-N\\n\neq 0}}^{N}\left(\cos(m\pi+knd_0)-\operatorname{sinc}(m+(knd_0)/\pi)\right)e_n \cong$$
$$\sum_{n=-N}^{N} A_n\operatorname{sinc}\left(m+(knd_0)/\pi\right)-\sum_{n=-N}^{N}\operatorname{sinc}\left(m+(knd_0)/\pi\right). \tag{24}$$

In the left hand side of (24), the existing term tends to zero when its argument $m\pi + knd_0$ became zero.

Equation (24) holds for $m = -M/2$ to $M/2$ which is the upper bound of m in truncated series in (19). The parameter $M/2$ must be specified in view of (22) and (23) so that the largest term in the $(M/2)$-th coefficient got less than the smallest term in the zero coefficient. It means that $|\operatorname{sinc}(M/2-(kNd_0)/\pi)|<|\operatorname{sinc}(kNd_0/\pi)|$ which results in $M > 4kNd_0/\pi$ which is near to $1.3kNd_0$, like Chebyshev and Legendre polynomials.

6. Finding the Unknown Deviations

All three Equations (12), (18) and (24) make a linear system of equations consisting of $M + 1$ linear equations and $2N$ unknown variables e_n, so they can be represented as a non-square matrix equation, as follows

$$\begin{aligned}&[\mathbf{P}]_{(M+1)\times 2N}[\mathbf{e}]_{2N\times 1} = \\ &[\mathbf{P}_0]_{(M+1)\times L}[\mathbf{A}]_{L\times 1}-[\mathbf{P}_0]_{(M+1)\times L}[\mathbf{1}]_{L\times 1}\end{aligned} \tag{25}$$

where $[\mathbf{1}]$ is a column matrix whose all elements are one. The unknown deviations are determined by solving the matrix equation (25) as follows

$$[\mathbf{e}]=[\mathbf{P}]^{-1}\left([\mathbf{P}_0][\mathbf{A}]-[\mathbf{P}_0][\mathbf{1}]\right) \tag{26}$$

in which $[\mathbf{P}]^{-1}$ is the pseudo inverse of matrix $[\mathbf{P}]$. To prevent probable divergence, one can limit deviations in each iteration, for instance in the range of $[-1.2\ \ +1.2]$.

In (26), the matrices $[\mathbf{P}_0]$ and $[\mathbf{P}]$ have purely real elements. On the other hand, the deviations e_ns for NSAAs must be determined purely real. Hence, the supposed excitations A_ns for USAAs must be real, as well. The other limitation is that the supposed excitations A_ns for USAAs must be positive because the excitations of the elements of NSAAs are positive as in (4) and (5). These are the two restrictions for the proposed OCE method to synthesize NSAAs.

Equation (26) gives us deviations which minimize the following errors, in fact.

$$\text{error1}=\sqrt{\frac{1}{M+1}\sum_{m=0}^{M}\left|F_m^{(\text{NSAA})}-F_m^{(\text{USAA})}\right|^2}\,, \tag{27}$$

$$\text{error2}=\sqrt{\frac{1}{\pi}\int_0^{\pi}\left|F^{(\text{NSAA})}(u)-F^{(\text{USAA})}(u)\right|^2\mathrm{d}\theta}\,. \tag{28}$$

The error2 evaluates the similarity of synthesized patterns to the desired one and its integral can be calculated by discretization.

To increase the accuracy of approximations existing in (11), (17) and (23), it is better to obtain deviations, e_n, in several iterations rather than in one iteration. This causes the needed deviations in each iteration become smaller than the final necessary deviations. To this end, one can use OCE methods several times to move from uniform excitation toward the desired excitation by using the following relation instead of A_n in those three equations.

$$A_n = 1+\left(A_n^{(\text{desired})}-1\right)\frac{it}{IT} \tag{29}$$

where $it = 1, 2, \ldots, IT$, in which IT is total number of iterations. Consequently, the matrix $[\mathbf{A}]$ in (25) and (26) must be changed in each iteration. By using (29) in each iteration, we start from uniform excitations which need zero deviations and move towards the desired excitations in IT iterations instead of in one iteration.

Also, in the iteration number it, the parameter n existing in the first and third terms of (12), (18) and (24) must be replaced with $\left(n+\sum_{s=1}^{it-1}e_n^{(s)}\right)$. This means that the elements of the matrix $[\mathbf{P}]$ and the second matrix $[\mathbf{P}_0]$ in (25) and (26) must be changed in each iteration. Here, $e_n^{(s)}$ is the small deviations obtained in the iteration number s and $\sum_{s=1}^{it-1}e_n^{(s)}$ is the summation of all small deviations obtained before the iteration number it.

At last, the final deviations are determined by summing all small deviations obtained in all IT iterations, as follows

$$e_n=\sum_{s=1}^{IT}e_n^{(s)}\,. \tag{30}$$

7. Verifying OCE Methods

To verify the proposed OCE methods, some examples are presented and discussed. They are Chebyshev, Taylor and Raised Linear excitations. The spaces between the antennas of all USAAs are assumed to be $d_0 = \lambda/2$.

7.1 Chebyshev Excitation

A NSAA with $L = 2N + 1 = 21$ elements is designed to have a desired array factor identical to that of a USAA having equi-ripple (Chebyshev) excitation with Side Lobe Level (SLL) equal to –17, –20, or –25 dB. The parameter M is chosen equal to 40 for all three OCE methods, i.e., Chebyshev, Legendre and Exponential. Figure 3 illustrates the defined error2 versus the total number of iterations, IT, for three OCE methods, i.e., Chebyshev, Legendre and Exponential. It is seen that there is an optimum IT for which the error2 is minimum. Usually, the optimum IT is as large as possible. Also, the type of variation of defined error1 is similar to that of defined error2. Therefore, one may find the same optimum IT from both defined error1 and error2.

Figures 4–6 show the magnitude of array factors of designed NSAAs for SLL = –17, –20 and –25 dB, respectively, obtained by three OCE methods. It is seen that the designed array factors have near-Chebyshev patterns [17] for all three OCE methods. As the SLL decreases, the difference between resultant and desired SLLs becomes larger. This difference is about 1.5 dB in the case of SLL = –25 dB.

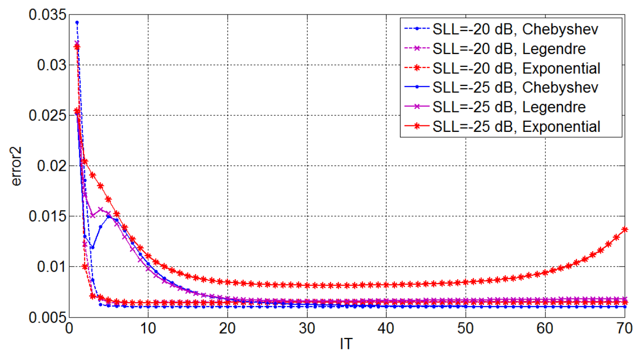

Fig. 3. Defined error2 versus total number of iterations, IT for two Chebyshev excitations.

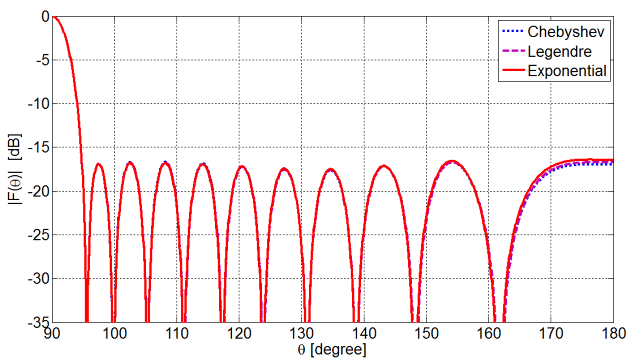

Fig. 4. Array factors of NSAA designed by three OCE methods for Chebyshev excitation of SLL = –17 dB.

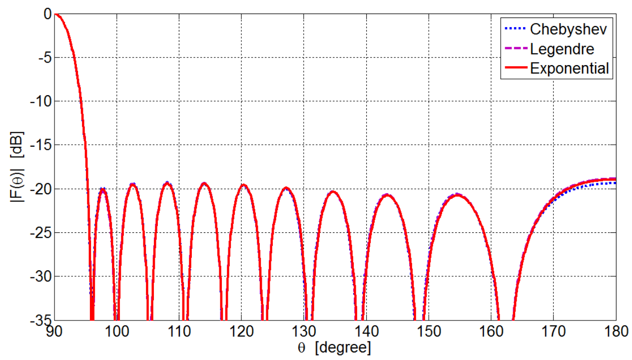

Fig. 5. Array factors of NSAA designed by three OCE methods for Chebyshev excitation of SLL = –20 dB.

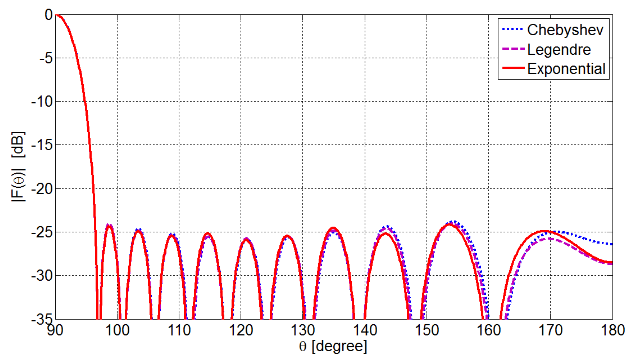

Fig. 6. Array factors of NSAA designed by three OCE methods for Chebyshev excitation of SLL = –25 dB.

Figures 7–9 illustrate the magnitude of orthogonal coefficients of array factors of USAAs and NSAAs, for three OCE methods, individually, for SLL = –20 dB. There is a good equality between two groups of orthogonal coefficients. The odd numbered coefficients are zero for Chebyshev and Legendre polynomials. High level coefficient in Fig. 9 is related to the frequency of ripples of equal amplitudes existing in array factor. Also, it is seen from Fig. 9 that the number of exponential functions can be nearly halved.

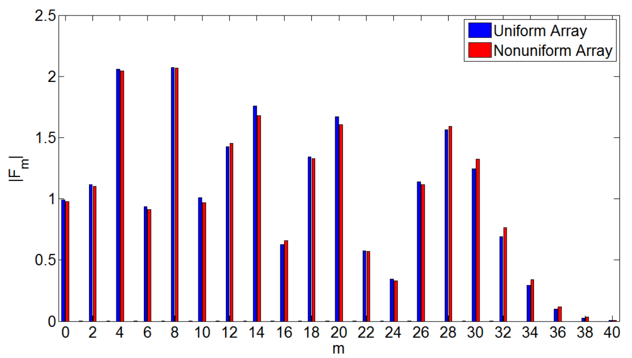

Fig. 7. Magnitude of orthogonal coefficients of Chebyshev polynomials for Chebyshev excitation of SLL = –20 dB.

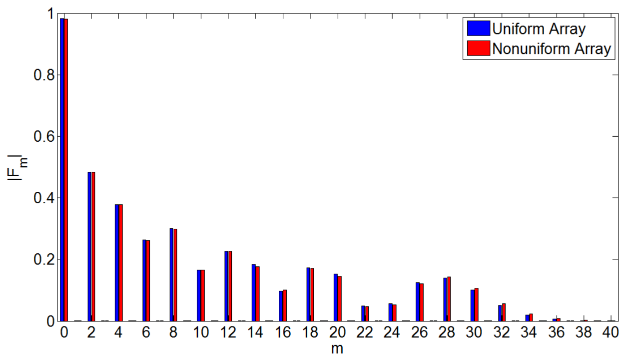

Fig. 8. Magnitude of orthogonal coefficients of Legendre polynomials for Chebyshev excitation of SLL = –20 dB.

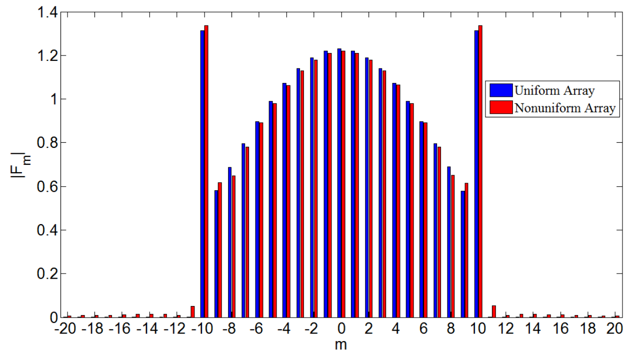

Fig. 9. Magnitude of orthogonal coefficients of Exponential functions for Chebyshev excitation of SLL = –20 dB.

7.2 Taylor Excitation

A NSAA with $L = 2N + 1 = 21$ elements is designed to have a desired array factor identical to that of a USAA of Taylor excitation with SLL < –20 dB and *nbar* = 3. The parameter M is chosen equal to 40. Figure 10 illustrates the defined error2 versus the total number of iterations, *IT*, for three OCE methods, i.e., Chebyshev, Legendre and Exponential.

Figure 11 shows the magnitude of array factors of designed NSAAs obtained by three OCE methods. It is seen that the designed array factors are near to the desired one.

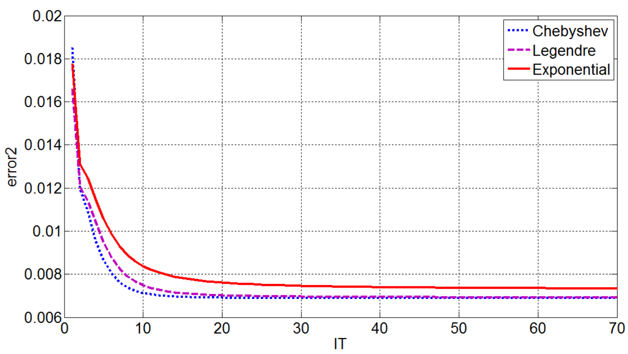

Fig. 10. Defined error2 versus total number of iterations, *IT* for Taylor excitation.

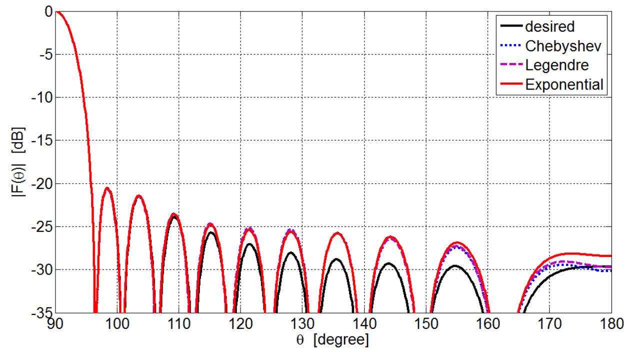

Fig. 11. Array factors of NSAA designed by three OCE methods for Taylor excitation of SLL < –20 dB.

7.3 Raised Linear Excitation

The excitation of a USAA with $L = 2N + 1 = 21$ elements is chosen as a raised linear one with max/min = 2. This excitation is real but asymmetric so the resulted pattern would be conjugate symmetric. The parameter M is chosen equal to 40. Total number of iterations, *IT*, has a little impression on defined errors. Hence, $IT = 1$ is chosen for this example. Figures 12 and 13 show the amplitude and phase of designed array factors, respectively, obtained by three OCE methods, i.e., Chebyshev, Legendre and Exponential. The agreement between the phases is not as good as the agreement between the amplitudes. Of course, the amplitude of array factors is important for us, usually.

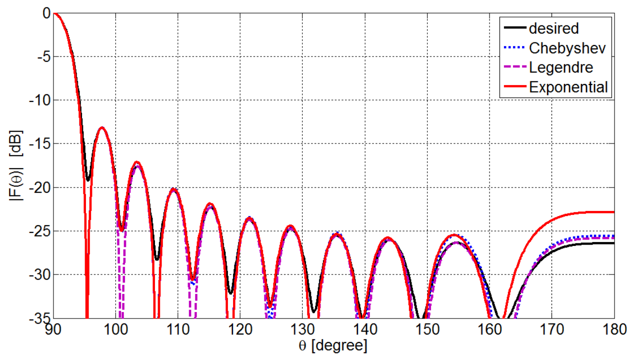

Fig. 12. Magnitude of the array factors of NSAA designed by three OCE methods for raised linear excitation.

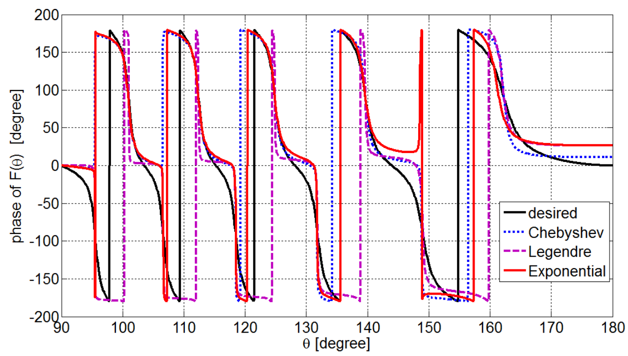

Fig. 13. Phase of the array factors of NSAA designed by three OCE methods for raised linear excitation.

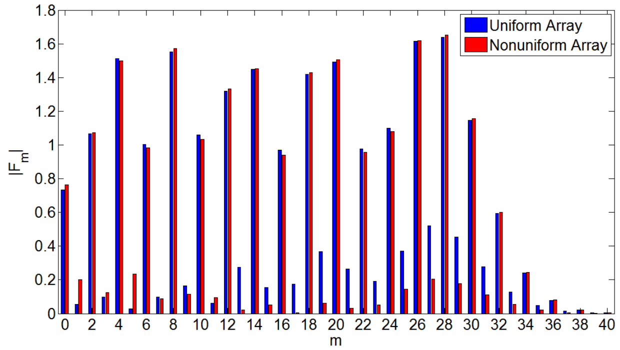

Fig. 14. Magnitude of orthogonal coefficients of Chebyshev polynomials for raised linear excitation.

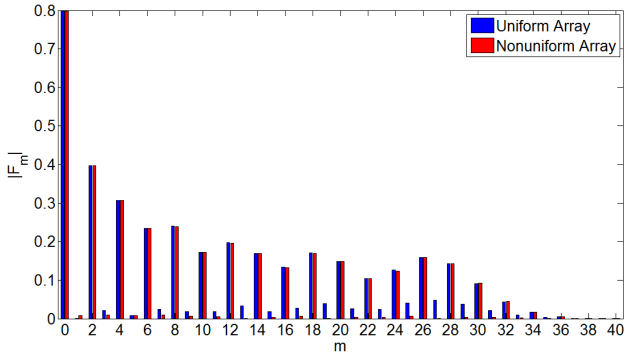

Fig. 15. Magnitude of orthogonal coefficients of Legendre polynomials for raised linear excitation.

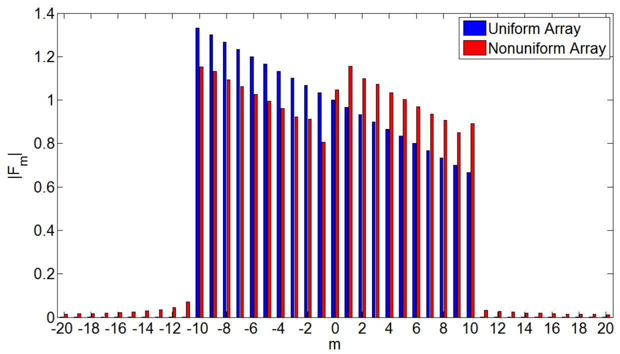

Fig. 16. Magnitude of orthogonal coefficients of Exponential functions for raised linear excitation.

Figures 14–16 illustrate the magnitude of orthogonal coefficients of array factors of USAAs and NSAAs, for three OCE methods, individually. The array factors are complex symmetric (even absolute and odd phase), because the excitations are asymmetric. Also, both even and odd numbered coefficients are non-zero for Chebyshev and Legendre polynomials.

Figure 17 shows the excitation of all cases in the previous examples. All excitations have been scaled so that their sum equals $L = 21$. Also, Figure 18 shows the unknown deviations, e_n, for all cases in the previous examples obtained by Chebyshev polynomials. Based on these deviations and according to (3), Figure 19 illustrates the position of NSAAs against those of USSAs for all cases in the previous examples. It is seen from Figs. 17 and 19 that how designed NSAAs use nonuniform spaces to avoid from intensely nonuniform excitations USSAs need. In fact, nonuniform spaces substitute for nonuniform excitations.

The deviations obtained by Legendre polynomials and Exponential functions are very close to those obtained by Chebyshev polynomials. These deviations have obtained after optimum iterations IT mentioned for each example. They are symmetric for symmetric excitations and asymmetric for asymmetric excitations.

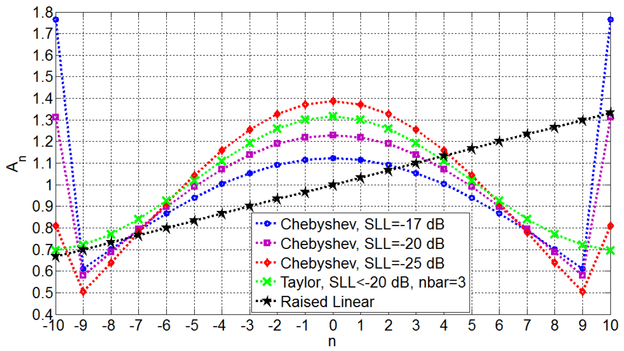

Fig. 17. Excitations of USSA in the presented examples.

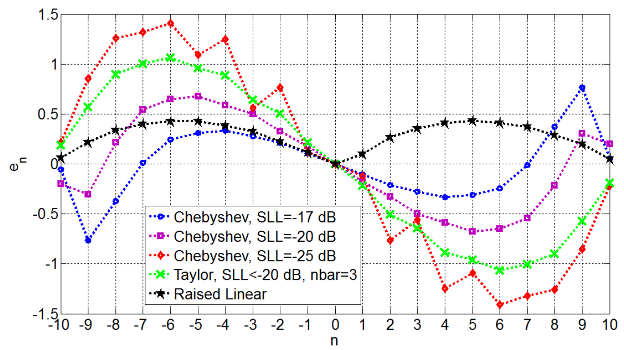

Fig. 18. Deviations obtained by Chebyshev polynomials for NSAAs in the presented examples.

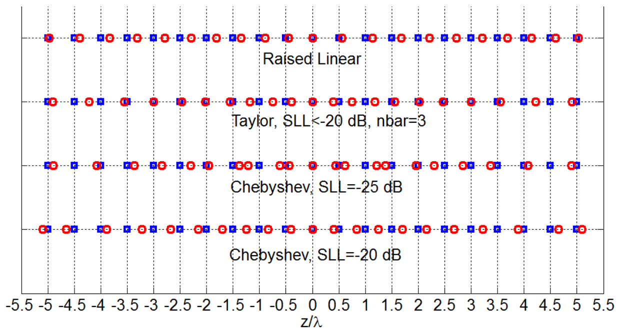

Fig. 19. Positions of USSAs and NSSAs obtained by Chebyshev polynomials. ☐: USAA. O: NSAA.

8. Conclusion

An analytic method, called Orthogonal Coefficients Equating (OCE) method was proposed to synthesize Nonuniformly Spaced Antenna Arrays (NSAAs). In this method, the orthogonal coefficients of array factors of NSAAs are equated to those of USAAs. Three orthogonal functions are used for OCE method consist of Chebyshev polynomials, Legendre polynomials and exponential functions. The limitation of the OCE method is that the excitations of the supposed USAAs must be real and positive. The introduced OCE method is merged with an iteration approach to increase its accuracy. Some exhaustive examples were given to show the effectiveness of the OCE method for designing NSAAs. As a whole, the array factors of synthesized NSAAs have an acceptable agreement with those of USAAs.

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