A Modified Vector Fitting Technique to Extract Coupling Matrix from S-Parameters

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Abstract. In this paper, a modified vector fitting technique to extract coupling matrix from S-parameters is introduced. This work allows designers to extract the coupling matrix of different or any pre-defined topologies from the simulated or measured S-parameter data. A study on vector fitting (VF) equations that can extract the rational polynomial of bandpass filter responses is carried out. VF is a robust numerical method which is applied widely in rational approximations due to its fast convergence and able to apply for high order polynomials. The rational polynomials are formed by applying the VF process to S-parameter responses without having to remove the phase offset and de-embedding the transmission lines. Focus fitting as the first enhancement can avoid VF overfitting spurious as poles; Poles forcing as the second enhancement is able to ensure the poles of all S-parameters are the same. Finally, the desired coupling matrix configuration is generated directly from the extracted polynomials using unconstrained and finitely bounded non-linear polynomials (NLP) optimization. Without the need for matrix transformation, the matrix elements are still able to show a one-to-one relationship in coupling values of resonators. Two bandpass filters are shown as examples to illustrate the performance of the new variation of VF.

Keywords
Coupling matrix, microwave filter, S-parameter extraction, vector fitting

1. Introduction

With the rise of the market value of the 5G communication system and beyond, there is no doubt that more and faster design processes are required to keep up with the growing pace in the communication field. Due to the high sensitivity of microwave filters, tuning is a necessary process in design because the effect of the mix-coupling of resonators [1] causes the filter frequency response to differ from the theoretical one. Therefore, the tuning process in filter design is labor-intensive and indirectly increases the design cost especially for high-order filters [2], [3].

The idea of computer-aided tuning (CAT) for microwave filters was introduced in [2]. Traditional CAT is based on matching the elements of a lumped element filter circuit model to a measured filter response in terms of the S-parameter by using a mathematical error minimization process [2].

Up until now, most of the research on rational function approximation has been devoted to the bandpass filter. It is well known that the extraction of the measured circuit model is the most challenging and critical part of the CAT procedure. Among rational function approximations, the Cauchy method is one of the techniques which generates polynomial interpolants from measurements of passive devices [4]. This method is also employed for the polynomial model synthesis of the S-parameters of microwave filters [5]. This approach is to generate reduced-order models from data samples obtained from measurement. The solution of the Cauchy method could be used as coefficients of characteristic polynomials needed for interpolating or extrapolating the complex S-parameter data [6] or the synthesis of the coupling matrix as mentioned in [7]. However, the Cauchy method consists of the ill-conditioning problem when dealing with high-order microwave filters or diplexer [8].

Among these methods, the VF is a robust numerical method [9] which is initially applied in power system transient modeling. Nowadays, VF is applied widely in the rational approximation of two-port filter response [10–12]. The working principle of VF is based on iteratively relocating the initial poles set to better locations. Compared to other rational approximation methods, VF can achieve fast convergence with fewer numbers of iterations. This method provides a fitting with guaranteed stable poles for the two-port network. Although VF is inefficient in the computation for a device with a high number of ports, this shortage does not affect the two-port microwave devices. In addition, a better approach is proposed to resolve the suffering of VF iteration convergence by replacing the high-frequency asymptotic condition with a more relaxed condi-
tion [12]. Later, [8] reported that original VF may also suffer from under-fitting and over-fitting problems. A modified approach, model-based vector fitting (MVF) is able to solve these problems by introducing a set of pole-located monomials to partially replace partial fractions as the basis function [8]. Recently, [13] applies quadrature-based vector fitting into S-parameter fitting which using a least squares objective that linked through quadrature rules by appropriately weighing the contribution of frequency sampling points that are chosen specifically.

The extraction of the coupling matrix also involves several machine learning algorithms such as artificial neural network (ANN) [14], [15] and deep neural network [16]. The downside of machine learning algorithms is they require complex training and a certain amount of data set to achieve a well-trained model.

In this work, two processes namely focus fitting and pole forcing are introduced on top of the conventional vector fitting method to achieve better fitting approximation on S-parameter responses. The simulated and measured fourth- and fifth-order Chebyshev bandpass filters responses are used in this study to prove the concept.

2. Vector Fitting Formulation

2.1 Focus Range Preprocessing

S-parameter responses are used instead of using Y-parameters as the rational function in this vector method. The original VF is less suitable for S-parameters due to the ability to approximate and fit the small magnitude values compared to Y-parameters. This results in the rational function not being able to fit all the poles within the passband and the spurious that is near to passband being mistaken as poles outside the passband as shown in Fig. 1. The poles of S-parameters shall always be within the passband. A spurious caused the wrong pole to be fitted and the fitted result to become one order less.

\[
\omega = \omega_0 \left( \frac{f_0}{\Delta f} \right) \left( \frac{f}{f_0} \right) \quad (1)
\]

where \( \Delta f \) and \( f_0 \) are the bandwidth and center frequency of the filter, respectively. Figure 2 and 3 show scaling of a fourth order filter from bandpass to lowpass domain.

In the lowpass domain, initial poles are easier to be identified because the poles exist only within the filter in-band and the range of each reflection zero/pole is close to each other. The imaginary parts of the initial poles selected are linearly distributed from \(-1\) until \(1\) and the real parts are \(1\%\) of the imaginary part which is always negative as shown in (2) and (3) [9]

\[
a_n = -p + jq, \quad a_{n+1} = -p - jq \quad (2)
\]

where

\[
q = 100p. \quad (3)
\]

\( a_n \) is the initial pole, \( p \) and \( q \) are real and imaginary part of the poles.
To avoid VF fitting the spurious as the poles, the focus range is applied to the lowpass response excluding the outer band that might contain irregular spurious or harmonics. The range is recommended to be within ±1.2 rad/s to ±2 rad/s to ensure the fitting is matched better in the focus fitting process. By applying the focus range, the fitting accuracy of the poles improves even in the case of the high in-band return loss response. However, the drawback is the out-band has a magnitude drop due to irregular poles for S-parameters. This drawback can be reduced using S-parameters pole forcing in the next section.

2.2 S-Parameters Pole Forcing

There are two processes in conventional VF which are pole relocation and residues identification [9]. Initially, the S-parameters, $S_{11}$, $S_{21}$, and $S_{22}$ from the S2P file must go through both processes with a few iterations to obtain the converged results. Polynomials of S-parameters can be constructed using the resultant poles and residues [9] as

$$S_{ij}(s) = \sum_{n=1}^{N} \frac{r_n^j}{s - a_n^i} + d_{ij} \quad (4)$$

where $r_n$ and $a_n$ are residues and poles respectively which are complex values, $d$ is the leading coefficient of S-parameters, $N$ is filter order, and $s$ is the frequency variable. However, by using conventional VF, the poles of the S-parameters polynomials are different and do not satisfy the unitary condition even though the focus range is applied.

Therefore, the second enhancement in this VF process is forcing the poles of all S-parameters to be identical. To do that, firstly, the $S_{21}$ parameter requires run-through pole relocation iterations to obtain a set of resultant poles, $a_n$ as mentioned [8]. Since the resultant poles are used for all S-parameters, the remaining steps are to calculate the residues of each S-parameter solving the overdetermined linear matrix as (5) and (6)

$$F \begin{bmatrix} 0 & 0 & r_{11} \\ 0 & 0 & r_{21} \\ 0 & 0 & r_{22} \end{bmatrix} = \begin{bmatrix} s_{11} \\ s_{21} \\ s_{22} \end{bmatrix} \quad (5)$$

where $k^{th}$ row of $F = \begin{bmatrix} 1/(s_k - a_1) & 1/(s_k - a_2) & \cdots & 1/(s_k - a_N) \end{bmatrix}$, $F \in [m \times (N+1)]$.

$$\begin{bmatrix} r_1^j \\ r_2^j \\ \vdots \\ r_N^j \end{bmatrix}^T \in [(N+1) \times 1], \quad (6a)$$

$$S_y = \begin{bmatrix} s_{y_1} \ s_{y_2} \ \cdots \ s_{y_n} \end{bmatrix} \in [m \times 1]. \quad (6c)$$

In (6a), $a_n$ is the resultant pole from the pole relocation process and $N$ is the filter order. In (6c), $m$ is the number of sampling points. Without repeating the residue identification, multiple iterations, the resultant residues are obtained directly by solving (4) using the least square method. Indirectly, the VF process is shortened by reusing the poles.

After obtaining the resultant poles, $a_n$, residues, $r_n$, and leading coefficient, $d$ can reconstruct the S-parameters polynomials based on (4). The full process of new VF variation is summarized in the flow chart in Fig. 4.

3. Coupling Matrix Formation

Instead of using characteristic polynomial extraction to obtain the coupling matrix, non-linear polynomial (NLP) optimization is used to determine the coupling matrix with its desired topology. This way allows filter to achieve any topology without performing matrix rotations. The NLP optimization method can be any unconstrained or finitely bounded method. To skip the matrix rotation, the formation of the objective function is based on comparing the reconstructed S-parameter polynomials and the general template coupling matrix with the desired topology.

To match both equations, the frequency points of maximum and minimum S-parameters values within the $[-1, 1]$ range are selected as shown in Fig. 5. Matching the maximum and minimum S-parameters values improve the accuracy and speed of the optimization. These frequency points are used to evaluate the resulting VF and the desired-topology of S-parameter polynomials. Having compared and solved the polynomials, the template matrix elements can be found.
The matrix in (7) shows the general coupling matrix template of inline topology without cross-coupling

$$\mathbf{M}_1 = \begin{bmatrix}
0 & x_1 & 0 & 0 \\
x_1 & y_1 & x_2 & 0 \\
0 & x_2 & \ddots & \ddots \\
0 & 0 & \ddots & y_{N-1} & x_N \\
0 & 0 & 0 & x_N & 0
\end{bmatrix}$$

(7)

where $x_n$ and $y_n$ are main-coupling and self-coupling values respectively. The asymmetric filter topology is used to make the optimization better for the fact that the practical filter is always asymmetric. $y_n$ values shall be very close to zero. It is worth mentioning that the coupling matrix template is not limited to inline topology and (7) is just one of the examples to illustrate the coupling matrix extraction approach.

S-parameter can be generated based on the optimized coupling matrix [7]. To validate the result, the S-parameter of coupling matrix is compared to the frequency response of the original file.

4. Result and Discussion

To illustrate the proposed method, two Chebyshev bandpass filters are design and simulated. The fourth order filter’s responses are obtained from simulation and the fifth-order filter’s responses are obtained from the measurement. The conventional and proposed VF methods are applied to both filters’ responses. The details of the processes are discussed in the next sections.

4.1 Simulated Fourth-Order Chebyshev Waveguide Bandpass Filter

The Chebyshev bandpass filter having the FBW and center frequency of 6.75%, and 4.65 GHz has been designed and simulated. This filter is realized as a waveguide filter based on inline topology using EM simulation from ANSYS 3D High Frequency Simulation Software (HFSS). The conventional VF is not able to fit the in-band poles with high return loss as shown together in Fig. 6.

During the pre-processing, the de-embedded simulation response is scaled down to the lowpass domain and the sampling points outside of ±1.5 rad/s are removed. The four initial poles are selected as shown in (8)

$$a_1 = -0.01 - j,  
a_2 = -0.01 + j,  
a_3 = -0.0033 - 0.33j,  
a_4 = -0.0033 + 0.33j.$$  

(8)

By applying the focus range only, the in-band poles fit better but with the drawback of $S_{11}$ magnitude drop in the out-band as shown in Fig. 7. The magnitude drop is caused by the poles’ differences in $S_{11}$ compared to $S_{21}$.

To remove the magnitude drop, the overdetermined linear matrix in (5) is formed based on the initial poles, frequency points, and their corresponding S-parameters and solved by the least square approximation. By forcing the

Fig. 5. Maximum and minimum of S-parameter within [-1, 1] range of a fourth-order Chebyshev filter response.

Fig. 6. Comparison between simulation and conventional VF response of a simulated fourth-order Chebyshev waveguide bandpass filter.

Fig. 7. Comparison between simulation and new VF response of a simulated fourth-order Chebyshev waveguide bandpass filter with focus fitting only.
Fig. 8. Comparison between simulation and full VF response of a simulated fourth-order Chebyshev waveguide bandpass filter with focus fitting and poles forcing.

Poles of S-parameters, all S-parameter polynomials can be formed using the resultant poles and residues as shown in (9a) and (9b), and the final fitting response is shown in Fig. 8.

\[
S_{21} = \frac{-0.326 - 0.157j}{s + 0.301 + 1.238j} + \frac{-0.165 + 0.506j}{s + 0.434 - 1.247j} + \frac{0.538 + 0.800j}{s + 0.763 + 0.646j} + \frac{-0.029 - 1.133j}{s + 0.878 - 0.361j}, \tag{9a}
\]

\[
S_{11} = S_{22} = \frac{-0.300 - 0.140j}{s + 0.301 + 1.238j} + \frac{0.154 - 0.444j}{s + 0.434 - 1.247j} + \frac{-0.447 - 0.632j}{s + 0.763 + 0.646j} + \frac{-0.027 - 0.892j}{s + 0.878 - 0.361j}. \tag{9b}
\]

After reconstructing the S-parameter polynomials, the inline coupling matrix can be obtained by using preconditioned conjugate gradient (PCG) as the NLP optimization method as

\[
M = \begin{bmatrix}
0 & 1.1394 & 0 & 0 & 0 & 0 \\
1.1394 & 0.0422 & 0.9996 & 0 & 0 & 0 \\
0 & 0.9996 & 0.0249 & 0.7179 & 0 & 0 \\
0 & 0 & 0.7179 & 0.0279 & 0.9400 & 0 \\
0 & 0 & 0 & 0.9400 & 0.0403 & 1.0372 \\
0 & 0 & 0 & 0 & 1.0372 & 0
\end{bmatrix}. \tag{10}
\]

4.2 Fifth-Order Chebyshev Waveguide Bandpass Filter

The fifth order Chebyshev waveguide filter is designed and fabricated. The measurement is performed using a Rhode & Schwarz ZVL vector network analyzer (VNA). By having 5.11% of FBW and center frequency 2.445 GHz, in-band poles with high return loss are also not able to fit nicely using the conventional vector fitting as shown in Fig. 9.

Following the same process as shown in Sec. 4.1, five initial poles in (11) are required to perform the VF. Based on (4), the value of one of the initial poles shall be zero. However, having a zero initial pole might cause the iteration to fail to converge. Therefore, a small value is used instead.

\[
a_1 = -0.01 - 1\text{j}, \quad a_2 = -0.01 + 1\text{j}, \tag{11}
a_3 = -0.005 - 0.5\text{j}, \quad a_4 = -0.005 + 0.5\text{j}, \quad a_5 = -0.0001 - 0.01\text{j}.
\]

After applying the focus fitting pre-processing by selecting 2 rad/s and poles forcing, the S-parameter polynomials are shown in (12a), (12b), and (12c).

\[
S_{21} = \frac{-0.164 - 0.096j}{s + 0.188 - 1.054j} + \frac{-0.011 + 0.204j}{s + 0.199 + 1.083j} + \frac{0.553 + 0.003j}{s + 0.296 + 0.517j} + \frac{-0.675 + 0.410j}{s + 0.468 - 0.678j} + \frac{-0.500 + 0.640j}{s + 0.599 - 0.036j}, \tag{12a}
\]

\[
S_{11} = S_{22} = \frac{-0.165 + 0.093j}{s + 0.188 - 1.054j} + \frac{-0.058 + 0.152j}{s + 0.199 + 1.083j} + \frac{-0.468 + 0.050j}{s + 0.175 + 0.270j} + \frac{-0.417 + 0.392j}{s + 0.468 - 0.678j} + \frac{-0.500 + 0.640j}{s + 0.599 - 0.036j}, \tag{12b}
\]

\[
S_{22} = \frac{-0.163 + 0.060j}{s + 0.188 - 1.054j} + \frac{0.003 + 0.218j}{s + 0.199 + 1.083j} + \frac{-0.372 + 0.042j}{s + 0.468 - 0.678j} + \frac{-0.381 + 0.431j}{s + 0.500 + 0.640j} + \frac{-0.453 + 0.153j}{s + 0.599 - 0.036j}. \tag{12c}
\]

Based on the comparison result of simulation, partial new VF without pole forcing and full new VF response, the sampling within the in-band responses match very well. It is observed that there is a very small discrepancy, which can be negligible, in the out-of-band frequency using the pole forcing as observed around 4 GHz and 5.5 GHz.
<table>
<thead>
<tr>
<th></th>
<th>4th-Order Filter</th>
<th>5th-Order Filter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_{11}/S_{22}$</td>
<td>$S_{11}$</td>
</tr>
<tr>
<td>Conventional VF</td>
<td>25.88%</td>
<td>11.39%</td>
</tr>
<tr>
<td>Partial new VF</td>
<td>16.53%</td>
<td>5.29%</td>
</tr>
<tr>
<td>Full new VF</td>
<td>6.34%</td>
<td>4.74%</td>
</tr>
</tbody>
</table>

Tab. 1. Mean absolute error (MAE) percentages of S-parameters on different VF results.

$$\begin{align*}
M &= \begin{bmatrix}
0 & 0.953 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.953 & -0.035 & 0.796 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.796 & -0.007 & 0.602 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.602 & 0.006 & 0.610 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.610 & 0.008 & 0.805 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.805 & -0.020 & 0.934 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.934 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\end{align*}$$

(13)

Mean absolute error (MAE) is used to calculate the difference of the value between each fitted filter response. It is obvious that the in-band response of $S_{11}$ and $S_{22}$ parameters greatly improved by using this variation of VF. There is some slight trade-off on the $S_{21}$ parameter which is more obvious in starting and ending of measured fifth-order Chebyshev filters response. This shows that this variation of VF is less suitable for measurement response due to some losses inherit in the prototype. Overall errors of every S-parameter are still below 7%. Table 1 summarizes the mean absolute error percentages of both filters’ S-parameters on different VF results.

5. Conclusion

A modified vector fitting technique to extract coupling matrix from S-parameters is developed and demonstrated. With the pre-defined template matrix topology, NLP optimization can help to extract the coupling matrix from the simulated or measured S-parameter data. In addition, the synthesis and reconfiguration of the filter coupling matrix using NLP optimization provides more information on the circuit modal extraction without matrix rotation limits. By implementing preprocessing of focus range selection and pole forcing methods into the procedure, the results show this method does improve the accuracy of rational system function from the S2P files of simulated and fabricated filter responses with a very small discrepancy at out-of-band frequencies.

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