

# Robust and Fair Multi-Objective Power Allocation Problem Based on Efficient and Healthy Cognitive Radio

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**Abstract.** Cognitive radio networks (CRNs) is a technology that can alleviate the scarcity of radio resources, improve communication efficiency, and reduce electromagnetic radiation pollution. However, traditional research mostly concentrates on a single optimization function, which is too constrained to achieve global consideration. We suggest a multi-objective optimization problem (MOP) with the objectives of transmission rate and power efficiency. Then, we introduce a fairness factor with the minimum protection rate to ensure the quality of data transfer for each secondary user (SU). We use the ellipsoid set to characterize the uncertain parameters under the actual channel state information (CSI). In the worst case, the semi-infinite programming (SIP) problem is transformed into a second-order cone programming (SOCP) problem. The original problem is linearly combined using the weighted-sum method to construct a single objective problem (SOP), which is then turned into a solvable convex optimization problem and resolved using the Lagrange dual algorithm and sub-gradient method. The simulation results demonstrate the ability of our proposed algorithm to balance power and transmission rate optimization by adjusting the weighting values, while maintaining good robustness.

## Keywords

Cognitive Radio Networks (CRNs), multi-objective optimization, robust power allocation, fairness factor, rate constraint, weighting coefficient

## 1. Introduction

With the advancement of technology and the rapid development of communication technology, countries worldwide are already looking into the sixth generation of mobile communication technology (6G), which will be able to increase network speed and more effectively handle the demanding demands of daily life [1], [2]. However, spectrum resources are limited, restricting radio technology development [3]; cognitive radio technology can help to solve this problem. In the cognitive radio system, if the interference of the secondary user to the primary user (PU)

doesn't exceed the threshold that the primary users can tolerate, the secondary user (SU) can dynamically access the allowed spectrum [4]. Opportunistic access, perceptually enhanced access, and spectrum sharing access are the three different types of access methods. This technology can increase spectrum utilization to alleviate the present resource shortage issue [5], [6]. In addition, we require an effective resource allocation optimization technique to optimize system performance overall and protect the quality of service (QoS) for communication users. For cognitive radio systems, many studies have presented a variety of optimization techniques, and compared to other schemes, optimization with multiple objective constraints has many advantages [7], [8]. Particularly at the moment, multi-antenna technology is extensively applied in cognitive wireless networks, the system's performance is more easily influenced, and the optimization model of multiple objective functions may make radio resource allocation more balanced and stable [9], [10], [11].

### 1.1 Related Research

Xu proposed power and sub-channel allocation algorithm considering interference temperature thresholds and power constraints [12], which improves throughput and ensures proportional fairness. However, this algorithm is affected by uncertainty of channel information. Afterwards, they proposed to establish a robust optimization problem for maximizing energy efficiency under bounded uncertainty in channel gain [13]. Askari et al. studied robust beam forming in cognitive radio networks and obtains maximum achievable rate for secondary users [14]. Chen et al. optimized the system model to get robust transmit power, examined channel uncertainty error using normalized distribution, and enhanced system performance [15]. The above are all single optimizations, and the results have significant limitations.

Sun et al. developed a sticky bacteria algorithm that can balance interference minimization with capacity maximization in a multi-objective optimization problem [16]. Similarly, Ranjan et al. introduced interference index and used greedy algorithms to find a joint optimization method between capacity and interference [17]. He utilizes adap-

tive algorithms to convert multiple targets into a single target, improving the system's capacity [18]. Naseer et al. uses utility functions to reduce costs and achieve effective resource allocation to optimize global performance [19]. Baias et al. proposed a problem of minimizing total power and signal-to-noise ratio, and transformed it into a convex problem using linear fractional programming to obtain the optimal solution [20]. Although these papers consider multi-objective optimization, they do not consider the uncertainty of the system itself. Nguyen et al. used two convex function difference techniques and S-process theory under incomplete CSI, and optimized the rate and total harvesting energy [21]. We will improve on the above and use Lagrange algorithm and other algorithms to study multi-objective problems under channel uncertainty conditions.

### 1.2 Main Content and Contributions

In this paper, we study a multi-objective robust power allocation problem in CRNs, taking power efficiency and transmission rate as objective functions [22]. In addition, we consider the interference threshold of the PU and the limits of maximum power consumption and minimum work rate for each SU, in order to find a solution that can balance these two objectives to solve our proposed problem.

The main contributions of the optimization method in this paper are summarized as follows:

In multi-user cognitive radio networks, it is necessary to consider the overall optimization of system performance. This paper proposes a multi-objective power allocation model with combined optimization of power efficiency and transmission rate. There are two conflicting objectives in the model that are difficult to directly handle. We employ the weighted-sum method to solve this multi-objective problem and achieve a balance between the two optimization objectives.

Considering the quality of data transmission for secondary users we introduce fairness factors to ensure that each secondary user can work normally. Assuming that the uncertainty of channel gain is described by an ellipsoid set, the original multi-objective problem can be represented as a SIP problem. In the worst case, we convert the SIP problem into a finite constraint problem, establish a robust power allocation model, and finally transform the problem into a convex problem to solve.

The remaining parts of this paper are as follows: In Sec. 2, we introduce the system model and provide a description of the parameter uncertainty ellipsoid set and a transformation of the bounded uncertainty set. Section 3 describes the solutions for multi-objective optimization and the specific process of solving Lagrange functions and updating iterations. Section 4 includes experimental simulations, comparing the impact of robust uncertainties on the system. Section 5 analyzes the impact of changing the weighting coefficients on the objective function and system performance. Section 6 provides conclusions.

## 2. System Model and Problem Formula

This paper considers the Ad-Hoc distributed CRN which includes a secondary network composed of  $M$  cognitive users and a main network composed of one primary user. We apply the underlay model to share the spectrum, allowing each SU and PU access to the authorized spectrum together for communication.

Figure 1 shows the system model, where SU represents cognitive users, PU represents principal users, and the connecting lines, such as  $G_M$ ,  $g_{MM}$ ,  $h_m$  respectively represent interference connections between different users.

We use the Lagrange multiplier method and sub-gradient update iteration method to obtain an optimal power allocation scheme. To find the best allocation of power scheme, we apply the Lagrange multiplier method and sub-gradient update iteration method. According to the simulation study, the robust scheme improves the cognitive system's stability and achieves balanced optimization between two objectives, and it ensures the QoS of SUs and the system's robustness.

### 2.1 Non-Robust System Model

Secondary users need to meet the constraints of the PU interference threshold when accessing authorized frequency bands; that is, the total interference caused by SU on PU should not exceed the PU's interference threshold.

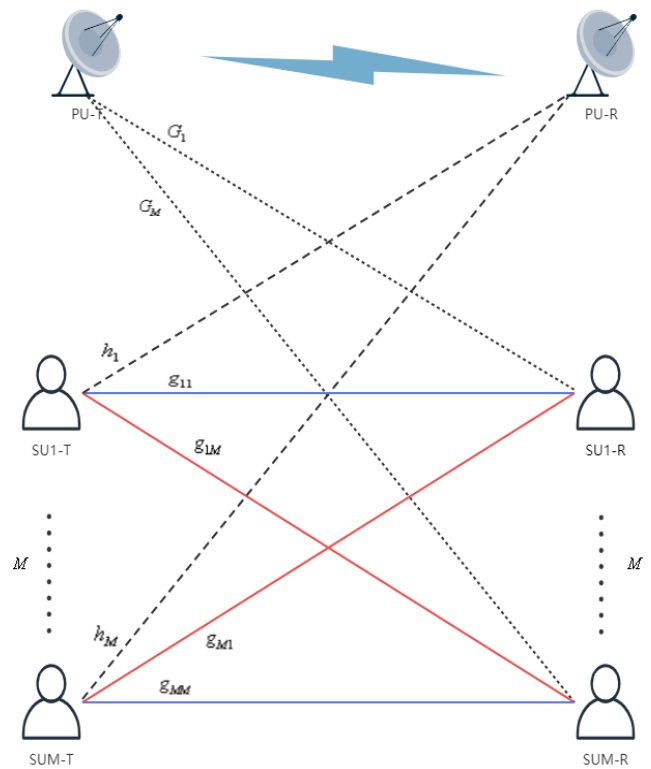


Fig. 1. Multi-user underlay cognitive radio system.

$$\sum_{i=1}^M h_i p_i \leq I^{\text{th}}, \forall i \in \{1, 2, \dots, M\} \quad (1)$$

where  $p_i$  is the transmit power of SU in the  $i$ th link, and  $h_i$  is the channel gain between the  $i$ th transmitter of SUs and the receiver of PU;  $I^{\text{th}}$  is the interference power threshold that the PU can tolerate. The constraint can ensure that the primary user can still communicate normally under certain interference conditions.

Considering the SU's own power consumption and interference mechanism, each SU's power cannot exceed its maximum transmission power limit to ensure that users can communicate normally

$$p_i \leq p_i^{\max}, \forall i \in \{1, 2, \dots, M\}. \quad (2)$$

In (2),  $p_i^{\max}$  is the maximum allowable transmission power of the SU in the  $i$ th link.

However, most research ignores the communication requirements of SUs in favor of ensuring the QoS of PUs. Considering the normal communication of PUs, SUs need to guarantee the QoS of PUs when they access the shared frequency band [23]. As a result, the transmission power of SUs and the interference they cause must be strictly limited. In this paper, we consider the minimum work rate required by cognitive users

$$\sum_{i=1}^M R_i \geq R^{\min}, \forall i \in \{1, 2, \dots, M\} \quad (3)$$

where  $R^{\min}$  denotes the minimum transmission rate at which SUs ensure the quality of their own communication.  $\sum_{i=1}^M R_i$  represents the actual total transmission rate of the SUs.

Using fairness factors, the original condition (3) can be relaxed to:

$$R_i \geq \xi_i R^{\min}, \quad (4)$$

$$R_i = \log_2 \left( 1 + \frac{p_i g_{ii}}{\sum_{j \neq i} p_j g_{ij} + p_0 G_0 + \sigma^2} \right). \quad (5)$$

In (5),  $g_{ii}$  is the channel gain between the transmitter of SU on link  $i$  and the receiver of SU on link  $i$ ;  $g_{ij}$  is the interference gain between the transmitter of SU on link  $j$  and the receiver of SU on link  $i$ ;  $p_j$  is the transmitting power of the SU transmitter on link  $j$ ;  $p_0$  is the transmitting power of the PU transmitter;  $G_0$  is the interference gain between the PU transmitter and the SU receiver on link  $i$ ;  $\sigma^2$  is background noise.

The multi-objective fair power allocation algorithm (MOFPA) we propose can be defined as follows:

$$\begin{aligned} W1: & \max \left( 1 - \frac{\sum_{i=1}^M p_i}{\sum_{i=1}^M p_i^{\max}} \right) \\ W2: & \max \sum_{i=1}^M R_i \end{aligned} \quad (6)$$

subject to (1), (2) and (4).

In the above problem,  $\sum_{i=1}^M p_i$  and  $\sum_{i=1}^M R_i$  represent the actual total power and transmission rate of the SUs. W1 represents maximized power consumption efficiency, W2 represents the maximized sum rate. The essence of maximizing power efficiency is actually to minimize transmission power. However, the transmission rate must be increased at the cost of increasing power consumption, which could result in a decrease in power efficiency. As a result, the model in this study is an optimization problem with two conflicting objectives, and there isn't an optimal solution that meets both objectives simultaneously. To achieve a balancing between power efficiency and transmission rate, we have to choose the proper weighting parameters based on demand.

## 2.2 Robust System Model

Most of the research on the allocation of power in CRNs proceeds under ideal CSI. On considering the fact that there are parameter disturbances in actual channels, we provide a robust algorithm that takes channel information error into account. At the same time, we conducted uncertainty parameter planning for channel gain [24], [25]. This robust algorithm enhances system stability, obtains a reliable power allocation scheme and guarantees that the system can achieve smooth communication under worst-case conditions.

The channel gain between SUs can be expressed as:

$$\phi_{ij} = \frac{g_{ij}}{g_{ii}}, \forall i \neq j, i \in \{1, 2, \dots, M\}. \quad (7)$$

The actual standardized channel gain can be expressed as two parts

$$\phi_{ij} = \bar{\phi}_{ij} + \Delta\phi_{ij}. \quad (8)$$

In (8),  $\bar{\phi}_{ij}$  represents the channel gain's nominal value and  $\Delta\phi_{ij}$  represents the corresponding deviation.

The channel gain between PU and SUs is expressed as:

$$h_i = \bar{h}_i + \Delta h_i. \quad (9)$$

Similarly,  $\bar{h}_i$ ,  $\Delta h_i$  denotes the nominal value of channel gain and the disturbance part of the channel.

For describing the uncertainty set of channel parameters and the disturbance of channel gain at link  $i$  [25], [26], we use the ellipsoid approximation. Using ellipsoidal sets, we construct a robust multi-objective optimization system model.

Using ellipsoidal sets to describe the channel gain uncertainty between SUs:

$$\varphi_i = \left\{ \phi_i \mid \bar{\phi}_{ij} + \Delta\phi_{ij} : \|\Delta\phi_{ij}\|_2 \leq \varepsilon_\alpha^2, \forall j \neq i, \forall i \in \{1, 2, \dots, M\} \right\}. \quad (10)$$

The channel gain uncertainty set  $h_i$  under ellipsoidal description is represented as:

$$H = \left\{ h_i \mid \bar{h}_i + \Delta h_i : \|\Delta h_i\|_2 \leq \varepsilon_\beta^2, \forall i \in \{1, 2, \dots, M\} \right\} \quad (11)$$

where  $\varphi_i, H$  are ellipsoidal representations of the uncertainty set of channel gains;  $\|x\|_2$  refers to the Euclidean norm.

The size of the uncertainty region in the ellipsoid sets is represented by  $\varepsilon_\alpha, \varepsilon_\beta$ , which are the maximum accepted deviation of the channel gain [27], [28]. The channel's uncertain perturbation increases with increasing  $\varepsilon_\alpha, \varepsilon_\beta$ .

Based on ellipsoidal sets (10) and (11), the multi-objective fair robust power allocation algorithm (MOFRPA) can be expressed as follows:

$$\begin{aligned} W1: & \max \left( 1 - \frac{\sum_{i=1}^M p_i}{\sum_{i=1}^M p_i^{\max}} \right) \\ W2: & \max \sum_{i=1}^M R_i \\ C_1'': & \sum_{j \neq i} (\bar{h}_i + \Delta h_i) p_j \leq I^{\text{th}} \\ C_2'': & p_i \leq p_i^{\max} \\ C_3'': & R_i \geq \xi_i R_i^{\min}. \end{aligned} \quad (12)$$

In the problem (12),  $R_i$  can be written as follows:

$$R_i = \log_2 \left( 1 + \frac{p_i}{\sum_{j \neq i} (\bar{\phi}_{ij} + \Delta\phi_{ij}) p_j + \frac{Q_i}{g_{ii}}} \right). \quad (13)$$

In (13),  $Q_i$  can be written as:

$$Q_i = p_0 (\bar{G}_0 + \Delta G_0) + \sigma^2 \quad (14)$$

where  $Q_i$  denotes the sum of interference and background noise from primary user;  $\sigma^2$  represents background noise;  $\bar{G}_0$  represents the nominal value of the channel gain from the PU-T to the SU-R,  $\Delta G_0$  represents the perturbation part.

Problem (12) is a model for the infinite number constraint problem of set  $i$ , which is essentially a SIP problem. The kind of problem is difficult to solve. In the worst case,

according to Cauchy Schwartz inequality, the SIP problem can be converted into a problem with finite constraints.

$$\begin{aligned} \max \left\{ \sum_{j \neq i} (\bar{\phi}_{ij} + \Delta\phi_{ij}) p_j \right\} &= \max \left( \sum_{j \neq i} |\bar{\phi}_{ij} p_j| + \sum_{j \neq i} |\Delta\phi_{ij} p_j| \right) \\ &= \sum_{j \neq i} \bar{\phi}_{ij} p_j + \varepsilon_\alpha \sqrt{\sum_{j \neq i} p_j^2}, \end{aligned} \quad (15)$$

$$\begin{aligned} \max \left\{ \sum_{j \neq i} (\bar{h}_i + \Delta h_i) p_j \right\} &= \sum_{j \neq i} \bar{h}_i p_j + \varepsilon_\beta \sqrt{\sum_{j \neq i} p_j^2} \\ &= \sum_{i=1}^M (\bar{h}_i + \varepsilon_\beta) p_i. \end{aligned} \quad (16)$$

In the same way, the interference of PU with SU can be expressed as follows:

$$I_G = (\bar{G}_0 + \varepsilon_\gamma) p_0. \quad (17)$$

In the worst case, the original SIP problem can be transformed into the following multi-objective fair robust power allocation (MOFRPA) problem:

$$\begin{aligned} W1: & \max \left( 1 - \frac{\sum_{i=1}^M p_i}{\sum_{i=1}^M p_i^{\max}} \right) \\ W2: & \max \sum_{i=1}^M \log_2 \left( 1 + \frac{p_i}{\sum_{j \neq i} \bar{\phi}_{ij} p_j + \varepsilon_\alpha \sqrt{\sum_{j \neq i} p_j^2} + \frac{Q_i}{g_{ii}}} \right) \\ C_1'': & \sum_{j \neq i} (\bar{h}_i + \varepsilon_\beta) p_j \leq I^{\text{th}} \\ C_2'': & p_i \leq p_i^{\max} \\ C_3'': & R_i \geq \xi_i R_i^{\min}. \end{aligned} \quad (18)$$

In the above equation,  $R_i, Q_i$  can be rewritten as follows:

$$R_i = \log_2 \left( 1 + \frac{p_i}{\sum_{j \neq i} \bar{\phi}_{ij} p_j + \varepsilon_\alpha \sqrt{\sum_{j \neq i} p_j^2} + \frac{Q_i}{g_{ii}}} \right), \quad (19)$$

$$Q_i = p_0 (\bar{G}_0 + \varepsilon_\gamma) + \sigma^2. \quad (20)$$

### 3. Multi-Objective Joint Optimization Scheme

This paper handles the multi-objective optimization problem defined in (18) by using the weighted-sum method to get the optimal power for the aforementioned joint optimization problem. The transmission rate and power efficiency can be linearly combined into a single objective problem by applying a weighting coefficient and the size of the weighting coefficients represents the degree of preference in the optimization process [28], [29]

$$\max \alpha_i^t \left( 1 - \frac{\sum_{i=1}^M p_i}{\sum_{i=1}^M p_i^{\max}} \right) + (1 - \alpha_i^t) \left( \sum_{i=1}^M R_i \right). \quad (21)$$

In (21),  $\alpha_i^t$  ( $0 \leq \alpha_i^t \leq 1$ ) is the weighting coefficient. Large values assigned to  $\alpha_i^t$  tend to optimize power consumption efficiency; on the other hand, smaller values promote increasing transmission rate. By adjusting  $\alpha_i^t$ , we can control the proportion to different objective functions during the optimization process and choose the appropriate  $\alpha_i^t$  values for different requirements. In addition, when working practically, we also need to consider the different impacts of two objectives on the system.

We rewrite (21) as follows to make the calculation that follows more convenient

$$- \min \left[ \alpha_i^t \left( 1 - \frac{\sum_{i=1}^M p_i}{\sum_{i=1}^M p_i^{\max}} \right) + (1 - \alpha_i^t) \left( \sum_{i=1}^M R_i \right) \right]. \quad (22)$$

In this way, using the Lagrange multiplier algorithm, a new Lagrange function can be defined:

$$\begin{aligned} L(\{p_i\}, \{\mu_i\}, \{v_i\}, \{\chi_i\}) = & -\alpha_i^t \left( 1 - \frac{\sum_{i=1}^M p_i}{\sum_{i=1}^M p_i^{\max}} \right) - (1 - \alpha_i^t) \left( \sum_{i=1}^M R_i \right) \\ & + \mu_i \left( \sum_{i=1}^M (\bar{h}_i + \varepsilon_\beta) p_i - I^{\text{th}} \right) \\ & + v_i (p_i - p_i^{\max}) + \chi_i (\xi_i R_i^{\min} - R_i) \end{aligned} \quad (23)$$

where  $\mu_i \geq 0, v_i \geq 0, \chi_i \geq 0$  is the Lagrange multiplier for the three constraints in the (23) problem.

The Lagrange multipliers updating function can be expressed as follows according to sub-gradient algorithm

$$\begin{aligned} \mu_i(t+1) &= \max(\mu_i(t) + \theta L_{-} \mu_i(t), 0), \forall i \in \{1, 2, \dots, M\}, \\ v_i(t+1) &= \max(v_i(t) + \beta L_{-} v_i(t), 0), \forall i \in \{1, 2, \dots, M\}, \\ \chi_i(t+1) &= \max(\chi_i(t) + o L_{-} \chi_i(t), 0), \forall i \in \{1, 2, \dots, M\}. \end{aligned} \quad (24)$$

In (24),  $\theta, \beta, o$  are the non-negative step sizes, and  $t$  is the number of iterations.

The corresponding sub-gradient can be expressed as follows according to sub-gradient algorithm:

$$\begin{aligned} L_{-} \mu_i &= \sum_{i=1}^M (\bar{h}_i + \varepsilon_\beta) p_i - I^{\text{th}}, \\ L_{-} v_i &= p_i - p_i^{\max}, \\ L_{-} \chi_i &= \xi_i R_i^{\min} - R_i. \end{aligned} \quad (25)$$

The optimal power of each SU can be obtained according to the Karush Kuhn Tucker (KKT) conditions [30], [31] through the following equation:

$$\frac{\partial L(\{p_i\}, \{\mu_i\}, \{v_i\}, \{\chi_i\})}{\partial p_i} = 0. \quad (26)$$

The multi-objective robust power allocation scheme's optimal solution for a given Lagrange multiplier is

$$\begin{aligned} p_i^* &= \frac{(1 + \chi_i - \alpha_i^t)}{\ln 2 \left[ \left( \frac{\alpha_i^t}{\sum_{i=1}^M p_i^{\max}} \right) + \mu_i (\bar{h}_i + \varepsilon_\beta) + v_i \right]} \\ & - \left( \sum_{j \neq i} \bar{\phi}_{ij} p_j + \varepsilon_\alpha \sqrt{\sum_{j \neq i} p_j^2} + \frac{Q_i}{g_{ii}} \right), \quad i \in \{1, 2, \dots, M\} \end{aligned} \quad (27)$$

We apply the Lagrange multipliers updating function and sub-gradients algorithm to handle the proposed problem and solve the power expression (27). The power gradually converges and stabilizes as the number of iterations  $t$  increases, which is the power we need. The aforementioned multi-objective power allocation problem can be solved using robust algorithms and ellipsoidal set programming.

The robust power allocation algorithm is summed up as follows:

1. Initialize variables:  $0 \leq p_i(0) \leq p_i^{\max}, \mu_i(0) \geq 0, v_i(0) \geq 0, \chi_i(0) \geq 0, \theta \geq 0, \beta \geq 0, o \geq 0$ .

2. Variable calculation: Calculate the sum of interference generated by all secondary users to the primary user according to (16), calculate the transmission rate value of the secondary user using (19), and similarly use (17) to obtain the interference generated by the primary user to the secondary user, while background noise is generated by a random function. Bring the obtained variable into the update sub-gradient expression (25), calculate the update function, and finally bring in the power expression (27) to obtain  $p_i^*$ .

3. Update the function:  $\mu_i(t+1), v_i(t+1), \chi_i(t+1)$ .

4. Iteration is performed according to step size and number of iterations, with a step size of 0.00001 and 30 iterations. If the power can converge to a certain value, the iteration is ended, and if not, the iteration is continued by returning to step 2.

## 4. Simulation Analysis

This section introduces the simulation results in MATLAB. In order to evaluate the performance of the proposed MOFRPA, we first compare the MOFPA with the multi-objective power allocation algorithm (MOPA) and then analyze the MOFPA and MOFRPA under fair rate constraints. Finally, the focus was on analyzing how the MOFRPA's power, transmission rate, and interference changed

Parameter	Initial value
$p_0$ (mW)	1.88
$p_i$ (mW)	[0.30, 0.31, 0.32]
$\sigma^2$	0.1 * rand()
$f^{th}$ (mW)	0.95
$\xi_i$	[0.25, 0.35, 0.40]
$R^{min}$ (bit/s/Hz)	6
Step size	0.00001
Iterations	30

Tab. 1. Parameter setting.

under different conditions in order to verify the flexibility and efficiency of the proposed method.

This paper analyzes CRNs in the underlay mode, where a PU and three SUs share an authorized frequency band. In Tab. 1, more simulation parameters are listed.

### 4.1 Analysis of Traditional Algorithms and Fair Algorithms

As seen in Fig. 4, the transmission rate of MOPA is slightly higher than that of MOFPA. However, it can be shown from Fig. 2 and Fig. 3 that MOPA's power and interference greatly exceed the threshold that the secondary and primary users were able to accept. Although MOPA sacrificed power efficiency in exchange for an increase in transmission rate, both power and interference exceeded the threshold, which affects the QoS of primary users and is not advisable. In this simulation, CR1, CR2, and CR3 stand for cognitive users 1, 2, and 3, respectively.

By setting the weighting coefficient  $\alpha_i' = 0$ , the multi-objective optimization has become the traditional maximizing rate power allocation model (MRPA) that only optimizes the rate. As shown in Fig. 5, when only considering the maximum transmission rate, it is easy to ignore the power constraints of secondary users, resulting in excessive transmission power and interference with primary user communication.

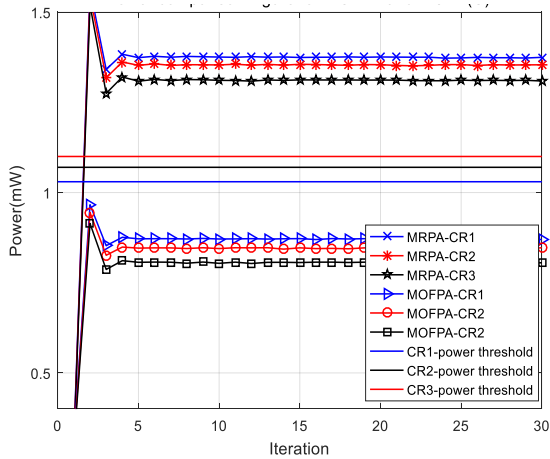


Fig. 2. Power for MOFPA and MOPA.

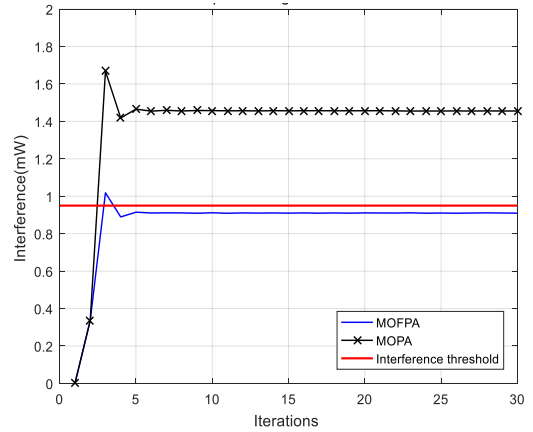


Fig. 3. Interference for MOFPA and MOPA.

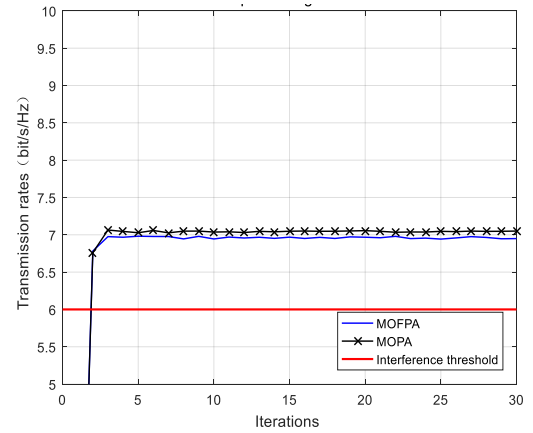


Fig. 4. Transmission rate for MOFPA and MOPA.

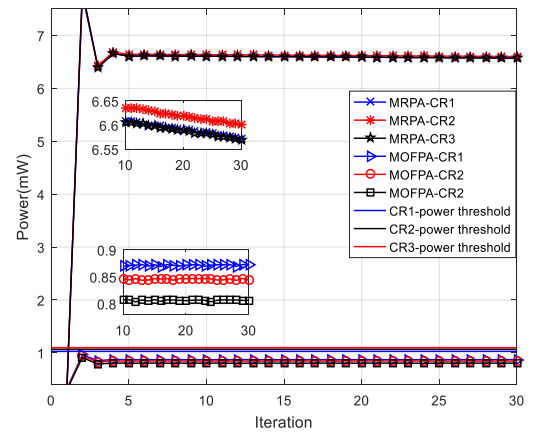


Fig. 5. Power for MOFPA and MRPA.

### 4.2 Analysis of Robust and Non-Robust Algorithms

In this section, we analyze the performance of MOFPA and MOFRPA with the influence of robust parameter size on the system, comparing different parameters under ideal channel information and worst-case conditions, taking into account the uncertainty of the channel information. In this simulation, CR1, CR2, and CR3 stand for cognitive users 1, 2, and 3, respectively.

As shown in Fig. 6, both MOFPA and MOFRPA can eventually reach stable and convergent values without going above the transmission power threshold. MOFRPA's optimal power is lower than MOFPA's, which is more in line with the demand for minimizing power. In the same channel, primary and secondary users must share spectrum resources, the power value during data transmission can be appropriately reduced by our proposed MOFRPA to overcome the influence of channel parameter disturbances on communication. A larger robust jitter indicates that an increase in uncertainty means a worsening state of the system, and the power in Fig. 6 also decreases accordingly. This better satisfies the power constraints of cognitive users and guarantees the system's stability in case of uncertain conditions.

In Fig. 7, the interference of MOFRPA is lower than the interference of MOFPA and does not exceed the threshold. In order to prevent the PU's communication from being interrupted by excessive interference, SUs must not only achieve the minimum working rate requirements but also reduce interference in their own communication process. The MOFRPA proposed in this paper takes channel parameter uncertainty into account, so it is necessary to reduce interference and overcome channel jitter. When the jitter increases from 3% to 5%, the channel uncertainty increases and the interference generated by SU decreases.

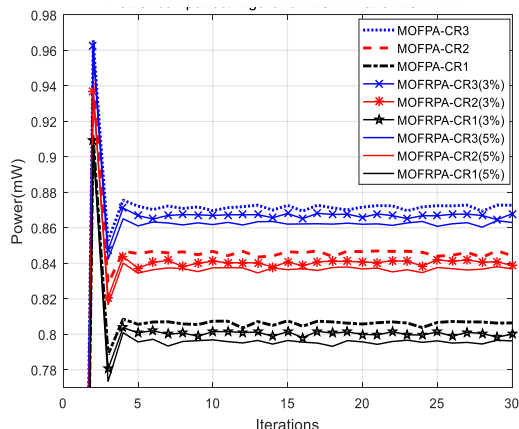


Fig. 6. Power comparison for MOFPA and MOFRPA.

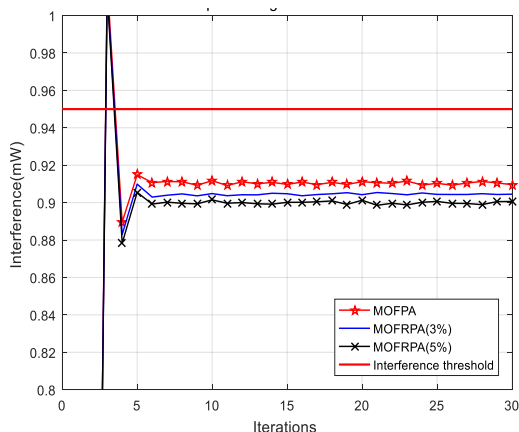


Fig. 7. Interference comparison for MOFPA and MOFRPA.

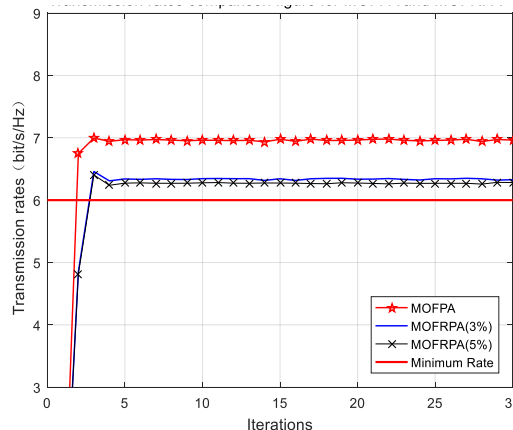


Fig. 8. Transmission rates comparison for MOFPA and MOFRPA.

Figure 8 depicts the influence of uncertain parameters on transmission rate. Although MOFRPA's transmission rate is a little bit lower than MOFPA's when channel disturbance is taken into account, both can still reach the minimum required transmission rate, showing that both can communicate normally. In the case of channel parameter perturbations, the robust algorithm sacrifices some transmission rates to guarantee that all users can share the spectrum. From Fig. 8, it can be seen that the transmission rate decreases as the size of uncertain parameters increases, yet it can still reach the minimum working rate requirements. This indicates that our proposed MOFRPA improves the robustness of the system without affecting user communication.

In our proposed algorithm, the larger the robust parameter, the greater the channel's uncertainty. Even if MOFRPA's transmission rate is a little bit lower than MOFPA, it is able to satisfy the need of the minimum working rate while lowering transmission power and interference, and achieves a good balance between interference, power, and transmission rate. When channel information is uncertain, conservative allocation of power can protect the interests of each user and improve system stability.

### 5. Analysis of Weighting Coefficients

The size of the weighting coefficient directly affects the multi-objective optimization's focus direction. How to choose the right  $\alpha_i$  to achieve a reasonably balanced power allocation between the two objectives without causing the transmission rate of secondary users to be quite low or excessive transmission power disrupt the communication of primary users? We will discuss the effects of adjusting the weighting coefficient on the two objectives in this section, CR1, CR2, and CR3 stand for cognitive users 1, 2, and 3, respectively.

Figure 9(a) and 9(b) show the changes in power values for each user and total power efficiency. When the robust parameter of MOFRPA is set to 5% and the weighted parameter  $\alpha_i$  is set to 0.75, 0.78, and 0.81, re-



spectively, by comparing Fig. 9(a) and Fig. 9(b), we can see that an increase in the weighting coefficient  $\alpha_i^t$  results in an increase in the proportion of the optimization process that focuses on maximizing power efficiency, resulting in an increase in power efficiency and a decrease in transmission power, demonstrating the importance of the previously mentioned weighting coefficient  $\alpha_i^t$ . It will arrive at a steady value for convergence quite rapidly as the number of iterations increase. Multi-objective optimization can be flexible carried out by adjusting the size of  $\alpha_i^t$ .

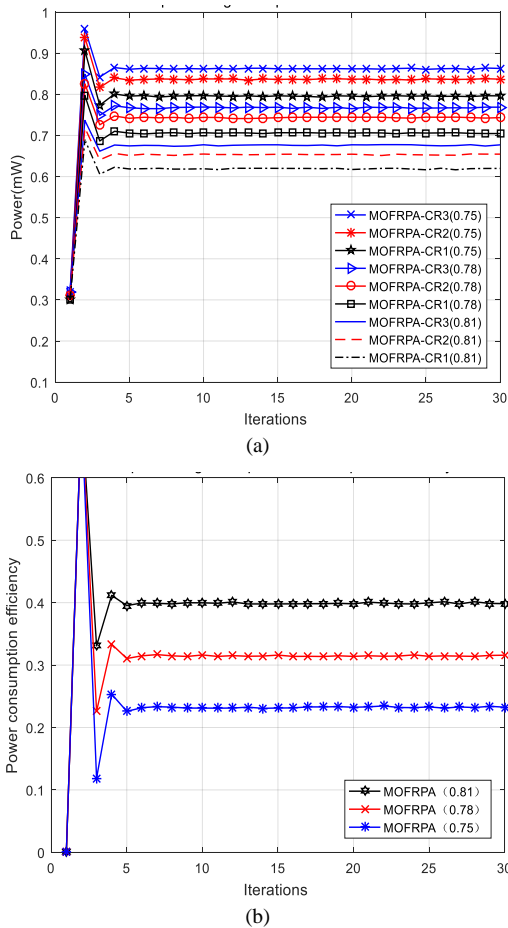


Fig. 9. (a) Power comparison figure. (b) Power consumption efficiency comparison figure.

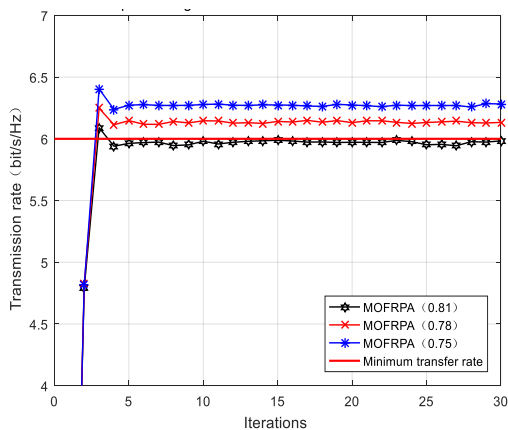


Fig. 10. Transmission rate comparison figure.

Similar to Fig. 9, Figure 10 shows the impact of  $\alpha_i^t = 0.75$ ,  $\alpha_i^t = 0.78$ , and  $\alpha_i^t = 0.81$  on the system's overall transmission rate. The transmission rate is at its highest at  $\alpha_i^t = 0.75$ , and it gradually drops as  $\alpha_i^t$  rises. When  $\alpha_i^t = 0.81$ , it can be seen, the transmission rate is unable to meet the minimum working rate limitation at the moment. In order to ensure normal communication between secondary users, even if increasing the weight coefficient has a small impact on the transmission rate, it cannot always increase. To guarantee the system's performance and achieve a balance between two objectives, it is important to appropriately distribute the weight coefficient's size.

As shown in Fig. 11, we use transmission power as the main research parameter and analyze the relationship between different weighting coefficients and corresponding user power in the same cognitive link. In order to observe the changes in user power more significantly, we chose MOFPA and introduced MOFPA2 for comparison. We changed the weighting coefficient for each user in MOFPA2, raising the  $\alpha_i^t$  for user 2 and lowering it for user 3, while maintaining the weighting coefficient for user 1. In MOFPA2, the power of user 2 is lower than that of MOFPA because increasing  $\alpha_i^t$  means increasing the proportion of optimized power efficiency, i.e. reducing transmission power. Similarly, the user 3's power in MOFPA2 is greater than that in MOFPA, while user 1's power remains unchanged.

This system allows different users in the same cognitive link to flexibly adjust their weighting coefficients, thereby changing the power allocation scheme according to their individual needs. This emphasized the significant impact of weighting coefficients on the distribution of power in the cognitive system.

Figure 12 shows the effect of changing the weighting coefficient  $\alpha_i^t$  on the power after the 10th iteration. Considering that there are many users using the communication system, and that users occasionally have new requirements during transmission, we can adapt the original scheme quickly and flexibly using the weighting coefficient. The power was found to have significantly decreased after the

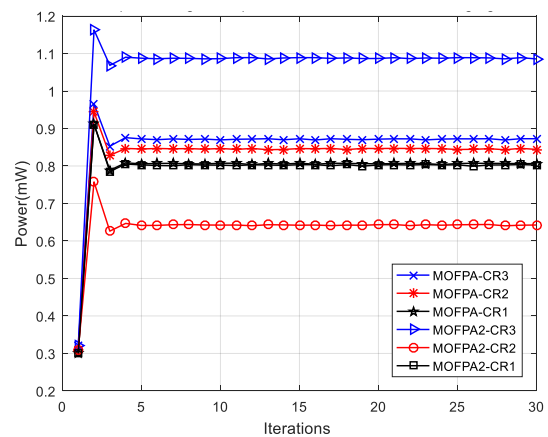


Fig. 11. Comparison figure of power of each user with different  $\alpha_i^t$ .



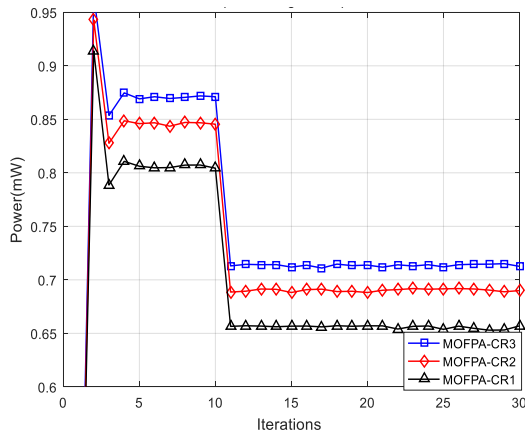


Fig. 12. Comparison figure of power.

tenth iteration. This is because, during the iteration process, we increased the size of the weighting coefficient  $\alpha_i^t$ , which increases the proportion of power efficiency during optimization and caused us to tend to lower transmission power, which resulted in a decrease in power.

The impact of  $\alpha_i^t$  on the system has been verified in this experiment, demonstrating the necessity of choosing appropriate weighting coefficients to enable a reasonable allocation of system resources.

## 6. Conclusion

A relatively efficient and environmentally friendly method of multi-objective fair allocation of power has been proposed. This method can jointly optimize the two objective functions of power efficiency and transmission rate, converting the multi-objective linear combination into a single objective problem using the weighted sum method. We add a fairness factor and a minimum transmission rate constraint. A robust power allocation scheme is proposed to enhance the system's anti-interference ability. Under the condition of uncertain CSI, the original uncertain problem is transformed into a finite constraint problem applying ellipsoid sets and the Cauchy-Schwartz inequality. Then relevant mathematical methods to solve this multi-objective problem are applied. Simulation experiments show that the robust scheme's transmission power is relatively low meaning that power efficiency will improve and interference will reduce. Due to the simultaneous reduction of interference to primary users, when certain channel interference occurs, each user in the system can communicate normally. Finally, we further discuss the impact of changing the weights of the two objectives on system performance, with a focus on analyzing changes in power and verifying the role of weight coefficients. Combining the results of the robust scheme, it is found that although the weight of the transmission rate accounts for a small proportion, it still has a significant impact on the system.

Simulation results verify the effectiveness of the proposed algorithm, and also demonstrate its flexibility in controlling the trend and degree of multi-objective optimi-

zation, achieving balanced resource allocation. And this algorithm improves spectrum utilization and can alleviate the problem of spectrum resource shortage.

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