Improved Generalized Compound Distributed Clutter Simulation Method

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Abstract. *In modern radar systems, the use of generalized compound distributed models can more accurately describe the amplitude distribution characteristics of sea clutter, which is crucial for radar signal processing and sea target detection. However, traditional zero memory nonlinearity (ZMNL) method cannot simulate generalized compound distributed sea clutter with arbitrary shape parameters. To address this issue, an improved method for generating random variable was proposed, which combines the characteristics of the Gamma distribution and uses the additivity of its shape parameter. By increasing the branches for generating the Gamma distributed random variables, the Probability Density Function (PDF) of the Gamma function is transformed into a second-order nonlinear ordinary differential equation, and the Gamma distributed random variables with arbitrary shape parameters are solved. Finally, the Generalized Gamma (GΓ) distributed random variables under arbitrary shape parameter can be obtained through specific nonlinear transformations. This method extends the shape parameters of generalized compound distributed clutter to general real numbers. Through comparative experiments with measured data, the generalized compound distributed model has strong universality and can more accurately represent measured data. Finally, the results of clutter simulation experiments also indicate that the proposed method is not only suitable for clutter simulation with non-integer or non-semi- integer shape parameters, but also further improves the fitting degree.*

Keywords

Clutter simulation, generalized Gamma distribution, zero memory nonlinearity (ZMNL), Gamma distribution

1. Introduction

Clutter is one of the main factors restricting radar target detection and tracking performance. The research on sea clutter is of great significance for radar detection and system design [1], [2]. Early clutter models include Rayleigh distribution, Weibull distribution, Log-normal distribution, and K-distribution [3]. At high sea state and high radar resolution, the trailing effect of sea clutter becomes more obvious, when the Log-normal distribution can better fit the measured data, compared to the Rayleigh distribution [4]. The Weibull distribution provides a wider flexibility in the range of amplitude distributions, between the Log-normal and Rayleigh distributions. Therefore, it can more accurately describe the amplitude distribution of sea clutter under various sea conditions and radar resolutions [5]. The application of K-distribution in radar sea clutter simulation is affected by the radar bandwidth. As the radar bandwidth increases, the values of speckle and texture modulation components deviate, and it is necessary to consider other more suitable distribution models for modeling and simulating sea clutter [6], [7]. In order to fit actual data more accurately, Anastassopoulos V. et al. proposed the generalized compound model based on the GΓ distribution. This model has a large dynamic range, and if some parameters of the model take special values, the model can degenerate into classical models such as Weibull distribution and K-distribution. At the same time, the generalized compound model can also obtain clutter models with other distributions that are completely different from classical models, which has a wide range of applications [8]–[10].

In order to simulate the generalized compound distributed clutter accurately, many scholars at home and abroad have conducted research on coherent and incoherent clutter models. On this basis, document [11] proposed a simulation method for correlated generalized compound distributed radar clutter based on ZMNL method. This method is suitable for incoherent clutter model, the drawback is that the shape parameter of the clutter model must be integer or semi-integer, which will cause clutter simulation bias, and it cannot independently control the power spectrum and amplitude distribution. A new method for simulation of generalized compound distributed clutter is proposed in document [12], which allows the generation of clutter with an arbitrarily specified power spectrum and relatively simple operation compared to the traditional ZMNL method, but an approximate approach is made for the simulation when the shape parameter is non-integer or non-semi-integer. Documents

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[13] band [14] used the additivity of the Gamma distribution to simulate K-distributed clutter, dividing the shape parameter into the sum of the integer or semi-integer part named v_1 and the non-integer or non-semi-integer part named v_2, solving the problem of approximating the shape parameter v, but the specific Gamma distribution with v_2 is generated by the product of Beta distributed random variables and exponential distributed random variables. Although the approximation problem of shape parameters has been solved, the simulation of Beta distribution may deviate when the parameters are small, which will lead to a certain error between the final clutter simulation curve and the theoretical Probability Density Function (PDF). Document [15] proposed to use the product of inverse Gamma distributed random variables with integer or semi-integer shape parameters and inverse Beta distributed random variables to obtain Pareto distributed clutter with arbitrary values of shape parameters. This method is able to directly generate inverse Gamma distributed random variables with arbitrary values of the shape parameter, but the computation is more complicated.

To overcome the shortcomings of existing methods, an improved method is proposed, which is to change the generation method of GΓ distributed random variables. Firstly, the Gaussian distributed random variables are used to generate Gamma distributed random variables, and then the GΓ distributed random variables are obtained through specific nonlinear transformation. By adding the Gamma distribution generating branches and transforming the PDF of the Gamma function into a second-order nonlinear ordinary differential equation, Gamma distributed random variables with scale parameter of 1 and shape parameter of any value can be generated before performing nonlinear transformation. Then specific nonlinear transformations are performed on them to obtain the GΓ distributed random variables under arbitrary parameters. This method not only extends the shape parameters of generalized compound distributed clutter to general real numbers, but also fits better with theoretical distributions.

The organizational structure of the remaining sections of this paper is delineated as follows. In Sec. 2, the mathematical model of generalized compound distributed clutter is introduced, including the impact of changes in various parameters on the model. Section 3 introduces the traditional generalized compound clutter simulation method based on ZMNL and how to estimate the parameters of the generalized compound distributed model. Section 4 presents the improved generalized compound distributed clutter simulation method and improved Gamma distributed random variables generation method. Section 5 first fitted the generalized compound distribution and other typical distributions with IPIX measurement data [16] demonstrating the usefulness of the generalized compound distributed model. Then, the experimental results of the improved generalized compound model and the traditional generalized compound model were compared and analyzed, verifying the superiority of the proposed method. Finally, Section 6 provides an overview of our research findings and potential future research directions.

2. Generalized Compound Distributed Clutter Simulation Data Model

2.1 Mathematical Model of Generalized Compound Distributed Clutter

From [17], it can be seen that the generalized compound distributed model can accurately describe the scattering statistical properties and modulation component information. The generalized compound distributed model, i.e., is described by the product of two independent GΓ distributed variables, namely:

$$Z = XY \tag{1}$$

where X is the scattering component with short correlation time (fast-varying component), Y is the modulation component with long correlation time (slow-varying component).

The PDF of X and Y are shown in (2) [18], [19]:

$$\begin{aligned} f(X) &= \frac{b_1}{\Gamma(v_1)} X^{b_1 v_1 - 1} \exp\left(-X^{b_1}\right), \\ g(Y) &= \frac{b_2}{a \cdot \Gamma(v_2)} \left(\frac{Y}{a}\right)^{b_2 v_2 - 1} \exp\left(-\frac{Y^{b_2}}{a}\right) \end{aligned} \tag{2}$$

based on (1) to (2), the expression for the PDF of generalized compound distributed clutter can be written as:

$$\begin{aligned} & f_{\text{GC}}\left(Z; a, b_1, b_2, v_1, v_2\right) \\ & = \int_0^{+\infty} \frac{1}{s} f\left(Z/s\right) g\left(s\right) \mathrm{d}s \\ = & \frac{b_1 b_2}{\Gamma(v_1)\Gamma(v_2)} \cdot \frac{Z^{b_1 v_1 - 1}}{a^{b_2 v_2}} \cdot \int_0^{+\infty} s^{b_2 v_2 - b_1 v_1 - 1} \exp\left(-\left(\frac{s}{a}\right)^{b_2} - \left(\frac{Z}{s}\right)^{b_1}\right) \mathrm{d}s \end{aligned} \tag{3}$$

where v_1 and v_2 are the shape parameter; b_1 and b_2 are the power parameter; a is the scale parameter; $\Gamma(\cdot)$ is the Gamma function. Among them, v_1, v_2, b_1, b_2 and a are all greater than zero.

The k-th moment function of the generalized compound model is [20]:

$$E = \left(Z^k\right) = a^k \frac{\Gamma\left(\frac{k}{b_1} + v_1\right)\Gamma\left(\frac{k}{b_2} + v_2\right)}{\Gamma(v_1)\Gamma(v_2)}. \tag{4}$$

When the generalized compound distribution takes a special value, it degenerates into other common distributions [21]: 1) When $b_1 = b_2$, it corresponds to the Generalized K-distribution; 2) When $b_1 = b_2$, $v_1 = 1$ and $v_2 = 0.5$ corresponding to the Weibull distribution; 3) When $b_1 = b_2 = 2$ and $v_1 = 1$, corresponding to the K-distribution. According to the different values of different parameters, the generalized compound distribution can also evolve into other distributions, as shown in Fig. 1.

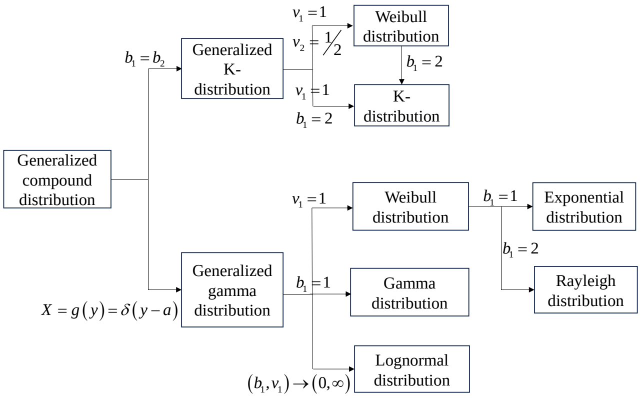


Fig. 1. Relationship between generalized compound distribution and other distributions.

2.2 Effect of Various Parameters on the PDF of Generalized Compound Distribution

According to (3), the PDF of the generalized compound distribution contains five parameters a, v_1, v_2, b_1, b_2. Figures 2–4 show the comparison of the PDF of the generalized compound distribution for different values of a, v_1, and b_1. The effects of v_1 and v_2, as well as b_1 and b_2, on the generalized compound distribution are the same. The values of each group of data are shown in the annotations in the figures. From Fig. 2, it can be seen that parameter a does not affect the shape of the distribution. From Fig. 3, it can be seen that v_1 is the skewness parameter that affects the asymmetry of the distribution. The smaller its value, the more significant the asymmetry of the distribution, and the greater the skewness. From Fig. 4, it can be seen that the power parameter b_1 affects the tail of the distribution, and the smaller its value, the longer the tail of the distribution.

3. Generalized Compound Distributed Clutter Simulation Method

3.1 Traditional Generalized Compound Distributed Clutter Modeling

The flow diagram of generating generalized compound distributed clutter based the ZMNL method is shown in Fig. 5. The generalized compound distributed clutter simulation of ZMNL method consists of two branches, one of which is to generate X_i, which is generated by the sum of squares of M independent Gaussian random variables ($w_{1,i}$, $w_{2,i}$, …, $w_{M,i}$) to the power of $1/b_1$. The other branch is used to generate Y_i, which generates the Y_i through the sum of squares of N independent Gaussian random ($w_{M+1,i}$, $w_{M+2,i}$,…, $w_{M+N,i}$) variables to the power of $1/b_2$. Among them, $M = 2v_1$, $N = 2v_2$, M and N are integers determined by the shape parameters v_1 and v_2 It can be seen that this simulation method can only simulate generalized compound distributed sea clutter with shape parameters v_1 and v_2 being integers or semi-integers.

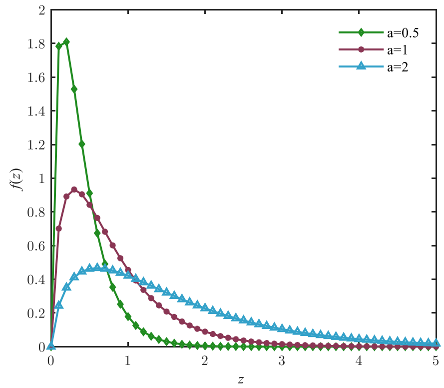


Fig. 2. The influence of parameter a on the generalized compound distribution.

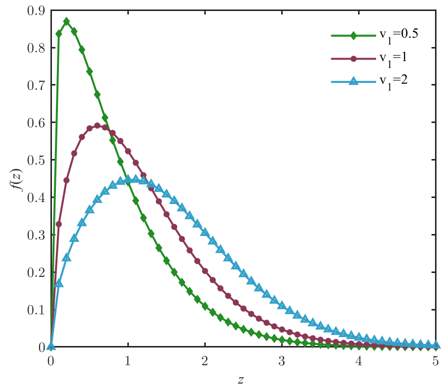


Fig. 3. The influence of parameter v_1 on the generalized compound distribution.

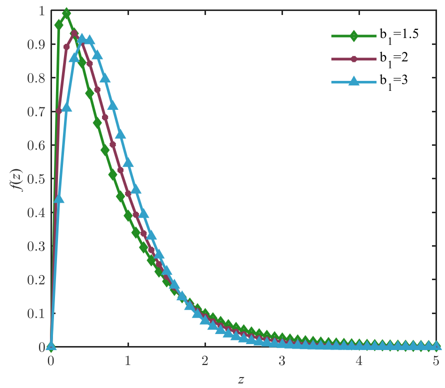


Fig. 4. The influence of parameter b_1 on the generalized compound distribution.

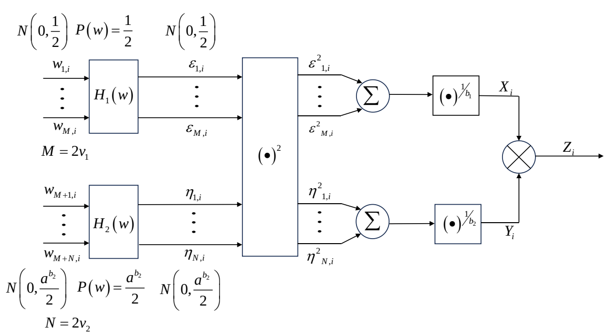


Fig. 5. Simulation flow diagram of generalized compound distributed clutter.

Document [17] derived the relationship between the autocorrelation coefficients r_{ij} and q_{ij} of the correlated Gaussian random variables $\{\varepsilon_i\}$ and $\{\eta_i\}$ under the condition of generalized compound distribution and the autocorrelation coefficient s_{ij} of the output variable $\{Z_i\}$, as shown in (5):

$$s_{ij}=\left\{\left[\left(1-r_{ij}^2\right)^{v_1+\frac{2}{b_1}}{}_2F_1\left(v_1+\frac{k}{b_1},v_1+\frac{l}{b_1},v_1,r_{ij}^2\right)\right.\right.$$
$$\left.\left.{}_2F_1\left(v_2+\frac{k}{b_2},v_2+\frac{l}{b_2},v_2,q_{ij}^2\right)-1\right]\right\}\cdot\frac{\left(1-q_{ij}^2\right)^{v_2+\frac{2}{b_2}}}{\Lambda-1}, \tag{5}$$

$$\Lambda=\frac{\Gamma(v_1)\Gamma(v_2)\Gamma\left(\frac{2}{b_1}+v_1\right)\Gamma\left(\frac{2}{b_2}+v_2\right)}{\Gamma^2\left(\frac{1}{b_1}+v_1\right)\Gamma^2\left(\frac{1}{b_2}+v_2\right)}, \tag{6}$$

$${}_2F_1(a,b,c,d)=\sum_{n=0}^{+\infty}\frac{(a)_n(b)_n}{(c)_n}\cdot\frac{d^n}{n!} \tag{7}$$

where k, l is the order, $|d|<1$.

Due to the autocorrelation coefficients r_{ij} and q_{ij} cannot be determined directly according to s_{ij}, it needs to be determined based on different shape parameters v_1, v_2 and power parameters b_1, b_2. In actual simulation process, it is often assumed that $q_{ij}=r_{ij}$ or $q_{ij}=r_{ij}^{10}$. Considering to reduce the arithmetic complexity of clutter simulation, this paper selects $q_{ij}=r_{ij}$ in clutter simulation experiments.

3.2 Power Spectrum Model

The power spectrum characteristic of clutter is another important parameter to describe radar clutter, and the power spectrum distribution of clutter is directly related to the design of filter. The Gaussian power spectrum model is the most commonly used power spectrum model in sea clutter modelling and simulation. A large variable of measured and experimental data shows that the Gaussian power spectrum model can approximate the sea clutter power spectrum of most radars. The expression is as follows [22]:

$$S(f)=S_0\exp\left[-\left(\frac{(f-f_{\mathrm{d}})^2}{2\sigma_{\mathrm{f}}}\right)\right] \tag{8}$$

where S_0 is the average clutter power, f_{d} is the Doppler frequency and σ_{f} is the broadening of the power spectrum reflected by the standard deviation of the power spectrum.

3.3 Parameter Estimation Method

The model parameters are estimated based on moments from measured data. The least squares parameter estimation method (LMS) can estimate parameters based on the derived expressions of various moments, and then determine the expression of the generalized compound distributed model. This method can directly obtain the estimated values of each parameter, and the cost function of the LMS for the generalized compound distribution is as follows [23]:

$$A(Z;a,b_1,b_2,v_1,v_2)=\sum_{k=1}^{5}\left|E(Z^k)-m_k\right| \tag{9}$$

where m_k is the K-th moment of the measured data, and $E(Z^k)$ is a nonlinear function with five unknown parameters determined by (4). In order to minimize the cost function $A(Z;\ a,\ b_1,\ b_2,\ v_1,\ v_2)$, we can take partial derivatives of $A(Z;\ a,\ b_1,\ b_2,\ v_1,\ v_2)$ for a, b_1, b_2, v_1, v_2 respectively, and set their vaues to 0. The simultaneous equation yields:

$$\begin{aligned}
&\partial A(a,b_1,b_2,v_1,v_2)/\partial a=0,\\
&\partial A(a,b_1,b_2,v_1,v_2)/\partial b_1=0,\\
&\partial A(a,b_1,b_2,v_1,v_2)/\partial b_2=0,\\
&\partial A(a,b_1,b_2,v_1,v_2)/\partial v_1=0,\\
&\partial A(a,b_1,b_2,v_1,v_2)/\partial v_2=0.
\end{aligned} \tag{10}$$

Substituting (4) and the moments of each order of the measured data into (10), the estimated values of the five parameters of the generalized compound distribution can be obtained by solving this system of five nonlinear equations.

4. Improved Generalized Compound Distributed Clutter Simulation Method

4.1 Improved Generalized Compound Gaussian Distributed Clutter Modeling

The traditional ZMNL method shown in Fig. 5 requires the shape parameters to be integer or semi-integer, which cannot effectively simulate generalized compound distributed clutter with non-integer or non-semi-integer shape parameters. Due to the fact that the shape parameters of the simulated clutter are determined by the intermediate $G\Gamma$ distributed variables, in order to extend the shape parameters of the simulated generalized compound distributed clutter from integers or semi-integers to general positive real numbers, it is necessary to improve the generation method of the GΓ distributed variables in Fig. 5. Propose to use the additivity of the second parameter of the Gamma distribution,

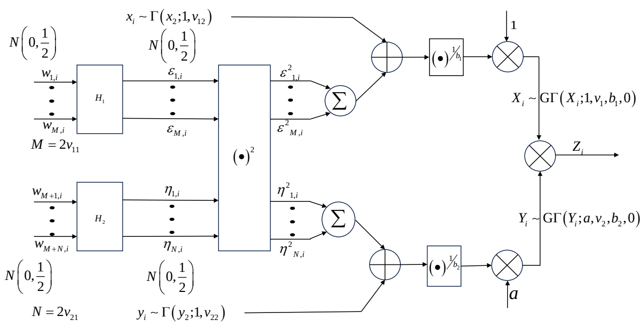


Fig. 6. Flow diagram for improved generation of generalized compound distributed clutter by ZMNL method.

and add Gamma distributed branches to the traditional generalized compound distributed clutter simulation flow diagram. The improved generalized compound distributed clutter generation ZMNL process is shown in Fig. 6. The principle is as follows:

Assuming that V follows the GΓ distribution, i.e., $V \sim \mathrm{G}\Gamma(a, v, b, u = 0)$, if T follows the scale parameter of 1 and the shape parameter of v, i.e., $T \sim \Gamma(1,v)$, then we have (11) [24]:

$$V = aT^{\frac{1}{b}} + u \sim \mathrm{G}\Gamma\left(a, v, b, 0\right). \tag{11}$$

For the generalized compound distribution, X and Y are mutually independent GΓ distributions, where $X \sim \mathrm{G}\Gamma(X_i; a_1, v_1, b_1, 0)$ and $Y \sim \mathrm{G}\Gamma(Y_i; a_2, v_2, b_2, 0)$. Combining with (11), it can be obtained that:

$$X = a_1 x^{\frac{1}{b_1}} + u_1, \tag{12}$$

$$Y = a_2 y^{\frac{1}{b_2}} + u_2 \tag{13}$$

where $a_1 = 1$, $u_1 = u_2 = 0$, $x \sim \Gamma(x; 1, v_1)$, $y \sim \Gamma(y; 1, v_2)$, due to the fact that the GΓ distribution does not have the additivity, but through the nonlinear transformation relationship between the GΓ distribution and the Gamma distribution in (11), it can be obtained that the shape parameters of the GΓ distributed random variables X and Y are determined by the shape parameters of the Gamma distributed random variables x and y.

From this conclusion, by combining with the Gamma distributed random variables generation method and using the additivity of the Gamma distribution, v_1 is separated into the integer or semi-integer part v_{11} and the non-integer or non-semi-integer part v_{12}, and v_2 is separated into the integer or semi-integer part v_{21} and the non-integer or non-semi-integer part v_{22}, i.e., $v_1 = v_{11} + v_{12}$, $v_2 = v_{21} + v_{22}$. The non-integer or non-semi-integer parts of Gamma distribution are generated by increasing the branches of the Gamma distribution, so that then GΓ distributed random variables with arbitrary shape parameters can be obtained through the specific nonlinear transformation relationship between the Gamma distribution and the GΓ distribution.

4.2 Improved Gamma Distributed Random Variables Generation Method

According to the nonlinear transformation relationship between the GΓ distribution and the Gamma distribution, simulating a more accurate Gamma distributed random variable can obtain a more accurate GΓ distribution variable. Therefore, we propose to generate Gamma distributed random variables with shape parameter v_{12} and v_{22} directly by differential variance solution to reduce the deviations of clutter simulation. The PDF of Gamma function is transformed into the second-order nonlinear ordinary differential equation [25], [26], and generating Gamma random variables under arbitrary parameters by the power series expansion method, the mathematical derivation is as follows:

The Gamma distribution PDF is shown in (14):

$$f\left(y \mid a, v\right) = \frac{\alpha^{v} y^{v-1}}{\Gamma(v)} \exp(-\alpha y) \tag{14}$$

where $\Gamma(v)$ is the Gamma function, v is the shape parameter, and a is the scale parameter.

Convert the PDF of (14) to the following form:

$$\frac{1}{f(y)} = \frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\Gamma(v)}{\alpha^{v}} y(t)^{1-v} \, \mathrm{e}^{\alpha y(t)} \tag{15}$$

where y is a function of t, and t is a random variable uniformly distributed in closed interval from 0 to 1, differentiating (15) with respect to t yields

$$\begin{aligned}\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} = \frac{\Gamma(v)}{\alpha^{v}} \Big[& y^{1-v}(t) \alpha \, \mathrm{e}^{\alpha y(t)} \\ & + (1-v) y^{-v}(t) \, \mathrm{e}^{\alpha y(t)} \Big] \frac{\mathrm{d}y}{\mathrm{d}t}.\end{aligned} \tag{16}$$

Substituting (15) into (16) and simplifying, yields

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} = \left[\alpha + \frac{1-v}{y} \right] \left(\frac{\mathrm{d}y}{\mathrm{d}t} \right)^2, \tag{17}$$

$$y \frac{\mathrm{d}^2 y}{\mathrm{d}t^2} - \left[y\alpha + 1 - v \right] \left(\frac{\mathrm{d}y}{\mathrm{d}t} \right)^2 = 0 \tag{18}$$

and $y(0) = 0$, $y(t) \sim \alpha[t\Gamma(v + 1)]^{1/v}$ with $t \to 0$.

Applying the transformation

$$z = \left[t\Gamma(v+1) \right]^{1/v}, \tag{19}$$

substituting (19) into (18), we obtain

$$y \left(\frac{\mathrm{d}^2 y}{\mathrm{d}z^2} + \frac{1-v}{z} \frac{\mathrm{d}y}{\mathrm{d}z} \right) - \left(y\alpha + 1 - v \right) \left(\frac{\mathrm{d}y}{\mathrm{d}z} \right)^2 = 0. \tag{20}$$

Assume that the solution of (20) is given by the infinite power series

$$y(z) = \alpha \sum_{n=1}^{\infty} c_n z^n, \tag{21}$$

substituting the series solution into (20), we find

$$\begin{aligned} n(n+v)c_{n+1} = & \sum_{k=1}^{n} \sum_{l=1}^{n-k+1} c_k c_l c_{n-k-l+2} l (n-k-l+2) \\ & - \Delta(n) \sum_{k=2}^{n} c_k c_{n-k+2} k \left[k - v - (1-v)(n+2-v) \right] \end{aligned} \tag{22}$$

where $\Delta(n) = 0$ if $n < 2$ and $\Delta(n) = 1$ if $n >> 2$. In the following we list some c_n:

$$\begin{aligned} c_1 &= 1, \\ c_2 &= \frac{1}{1+v}, \\ c_3 &= \frac{1}{2}\frac{5+3v}{(1+v)^2(2+v)}, \\ c_4 &= \frac{1}{3}\frac{31+33v+8v^2}{(1+v)^3(2+v)(3+v)}. \end{aligned} \tag{23}$$

In summary, the approximate solution of the differential equation function can be obtained as follows:

$$y(t) = \alpha \sum_{n=1}^{\infty} c_n \left(\left[t\Gamma(v+1) \right]^{1/v} \right)^n \tag{24}$$

where the coefficients c_n are given by (23).

Therefore, the approximate solutions of the Gamma distributed random variables x_2 and y_2 corresponding to the non-(semi)-integer parts v_{12} and v_{22} are:

$$x_2 = \sum_{n=1}^{\infty} c_n ([t\Gamma(v_{12}+1)]^{1/v_{12}})^n \sim \Gamma\left(x_2;1,v_{12}\right), \tag{25}$$

$$y_2 = \sum_{n=1}^{\infty} c_n ([t\Gamma(v_{22}+1)]^{1/v_{22}})^n \sim \Gamma\left(y_2;1,v_{22}\right). \tag{26}$$

5. Comparison and Analysis

There are many types of statistical distributed models for sea clutter, but due to the influence of radar parameters and sea surface conditions on the characteristics of sea clutter, no amplitude distributed model is universal. The generalized compound distribution can degenerate into other distributions under specific parameters, which has wide applicability and strong flexibility, and can accurately describe the sea clutter distribution of high-resolution radar.

In order to further verify the strong adaptability of generalized compound distributed clutter, we use the measured data of IPIX radar to analyze the statistical characteristics of sea clutter. The measured data were taken from a publicly available dataset of X-band radar working on the east coast of Canada in 1993 at a small rubbing angle. The range resolution in 30 m, f_r = 1000 Hz. The IPIX radar data files selected in this article are shown in Tab. 1, all of which are matrices of 131 072 × 14, meaning that the data packet contains 14 distance units, each of which contains 131 072 consecutive sampling points.

The amplitude fitting analysis was performed on the measured sea clutter in the second distance unit of the IPIX radar dataset # 311 file (Low sea conditions, Level 2). The fitting results of different models on the amplitude distribution of IPIX radar data are shown in Fig. 7. It can be seen that the Log-normal distributed model has a particularly concentrated energy, forming a sharp peak in the lower amplitude area. After that, the PDF of the amplitude rapidly decreases, but when it reaches a certain degree, the rate of decline becomes slow, forming a relatively long tail with

Data number	#311
Sea condition level	Low sea conditions (Level 2)
Wave height (m)	0.91
Polarization mode	VV
Wind direction (°)	310
Wind speed (km/h)	33

Tab. 1. Relevant information of IPIX sea radar measured data set.

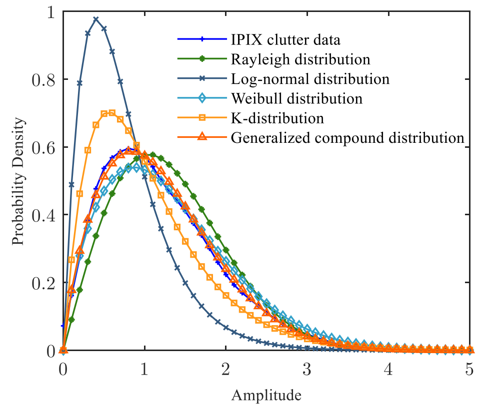


Fig. 7. Fitting different statistical models.

Distribution type	K-S
Rayleigh distribution	7.6%
Log-normal distribution	12.6%
Weibull distribution	2.2%
K-distribution	5.5%
Generalized compound distribution	1.4%

Tab. 2. The fitting deviation of various models.

a relatively long width of the main peak. The distribution curves of the K-distribution, Weibull distribution, and generalized compound distribution fall between the Log-normal distribution and Rayleigh distribution. From the amplitude fitting curve, it can be seen that the generalized compound distributed curve is closest to the statistical results of the measured data, and the K-S (Kolmogorov Smirnov) test method is used to verify the fitting degree of different statistical distributed models with the IPIX measured data. The K-S test expresses the similarity between two independent statistical samples, and the smaller the value of K-S, the higher the approximation of the fitting. The goodness of fit test K-S values between different statistical distribution models and measured data are shown in Tab. 2. It can be seen that the generalized compound distribution has the smallest K-S value, thus verifying the universality of the generalized compound distributed model.

In [14], Zhu generated the specific Gamma distribution with non-integer or non-semi-integer shape parameter by the product of Beta random variables and exponential random variables, which overcame the shortcoming of the traditional ZMNL. Now, we compare the proposed method with Zhu's method. In the experiment, we set the Gamma shape parameter v = 0.15, a = 1. Sampling frequency is set to 1000 Hz and the total simulation variable is 20 000. The Gamma branches whose shape parameters v_{12} and v_{22} ($v_{12} = v_{22} = 0.15$) are generated by the method of in [14] and the proposed method respectively. The comparison of results are shown in Fig. 8. Obviously, it can be seen from the locally enlarged curve that the fitting degree between the average histogram of simulated data and the theoretical PDF curve of the proposed method is higher than that of Zhu's method.

To illustrate the performance of the proposed method, the mean squared difference (MSD) is used to test the fitting degree of the simulated data. Simulation was done 100 times. The MSD value comparison is shown in Fig. 9.

The proposed method has improved the fitting degree on the generation of the GΓ distributed random variable. To further verify the effect of non-integer shape parameters, four sets of generalized compound distributed sea clutter variables with the data length of 20 000 and the radar pulse repetition frequency of 1000 Hz were generated using the method proposed in Sec. 4. The simulated power spectrum is Gaussian spectrum with a bandwidth of 60 Hz. The specific simulation parameters are shown in Tab. 3. Figures 10 to 12 show the simulation results of the generalized compound distribution degenerated into Weibull distribution, K-distribution, and Generalized K-distribution, respectively, and Figure 13 shows the simulation results of the ordinary generalized compound distribution. Among them, Figure 10(a) shows the comparison between the simulated Weibull distribution clutter and the theoretical amplitude probability density curves. When degenerating to the Weibull distribution, the shape parameter can only be fixed values, i.e., $v_1 = 1$, $v_2 = 0.5$, which does not involve the problem of approximating shape parameters. Figure 10 shows the Weibull distribution clutter variable simulated by this generalized compound distributed model fits well with the theoretical values. Figures 11(a)–13(a) show the comparison of amplitude probability density curves for the K-distribution, Generalized K-distribution and generalized compound distribution of traditional and improved methods, where the shape parameters of the traditional method are taken down to integers or semi-integers, i.e., $v = 1.5$. In Fig. 11(a) to Fig. 13(a), the probability density curves of the K-distribution, Generalized K-distribution, or generalized compound distributed clutter simulated by the traditional ZMNL method have some deviations from the theoretical values. Compared with the improved generalized compound clutter simulation method, especially when the amplitude value is small, the proposed method has a higher fitting degree than the traditional method. Figures 11(b)–13(b) show the comparison of the power spectral density curves between traditional and improved methods. In Figs. 11(b)–13(b), the power spectrum characteristics fit well with the ideal Gaussian spectrum curve within the effective bandwidth, and the newly added branch has a negligible impact on the power spectral density of the generalized compound clutter simulation.

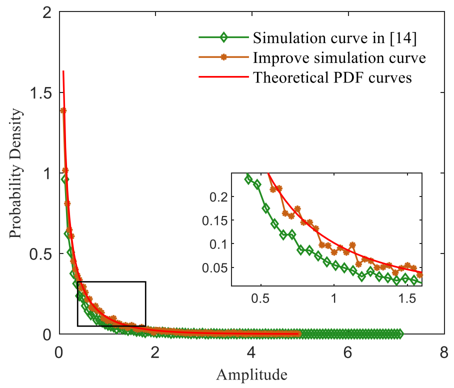


Fig. 8. Probability density comparison.

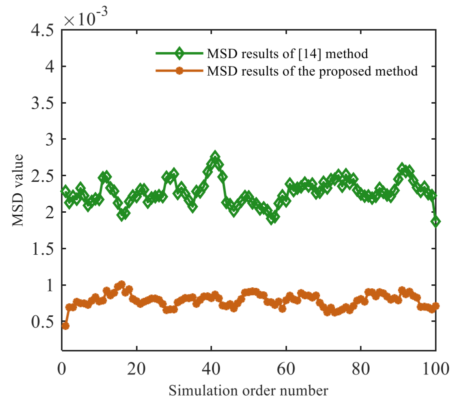


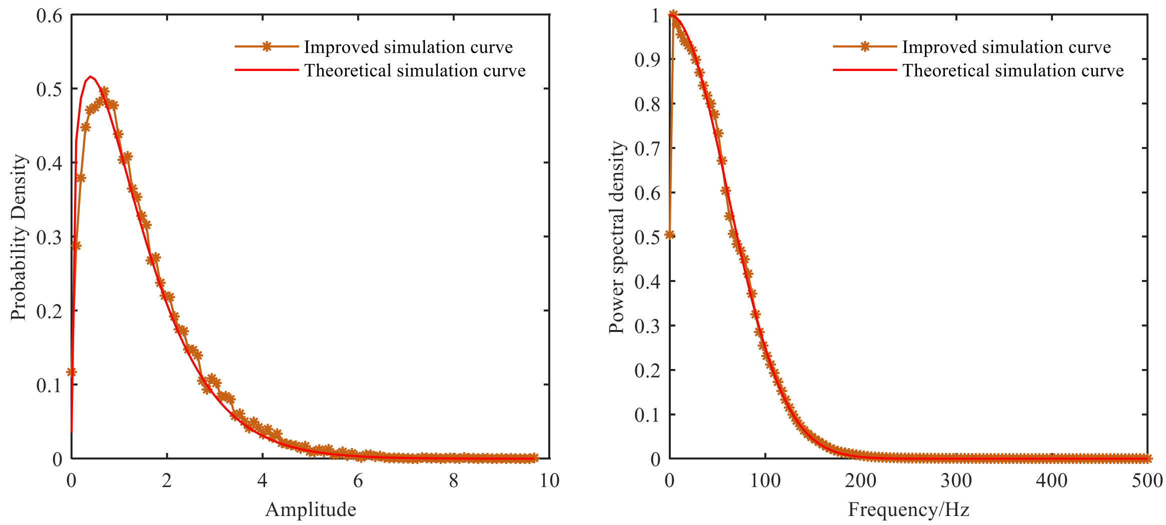
Fig. 9. MSD value comparison.

a	v_1	v_2	b_1	b_2	Distribution type
2.5	1	0.5	2.5	2.5	Weibull distribution
2.5	1	1.65	2	2	K-distribution
2.5	1.65	1.65	2	2	Generalized K-distribution
2	1.65	1.65	2	3	Generalized compound distribution

Tab. 3. Simulation parameters for amplitude distribution.

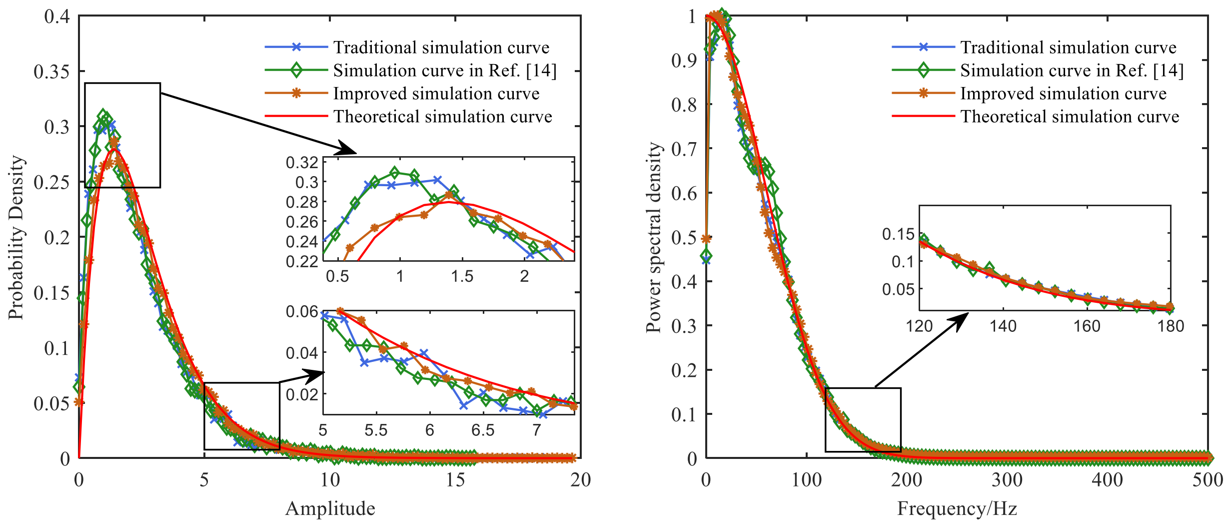
6. Conclusion

The generalized compound distributed model can effectively simulate clutter with special distribution characteristics. Using the generalized compound distributed model can more effectively estimate the distribution characteristics of actual clutter, and contribute to improving the detection probability and reducing false alarms for subsequent detection. However, the traditional ZMNL method is limited to integer or semi-integer shape parameters when simulating compound distributed clutter, which has limitations in practical applications. To address this issue, this paper proposes an improved ZMNL method based on the characteristics of the GΓ distribution. By combining it with the Gamma distribution and using the additivity of the second parameter of the Gamma distribution, the shape parameters of the simulated generalized compound distributed sea clutter are extended to the general real variable range. Through simulation experiments, we found that the PDF curves generated by the improved ZMNL method fits better with the theoretical curves, improving the amplitude characteristic simulation performance of generalized compound distributed sea clutter.



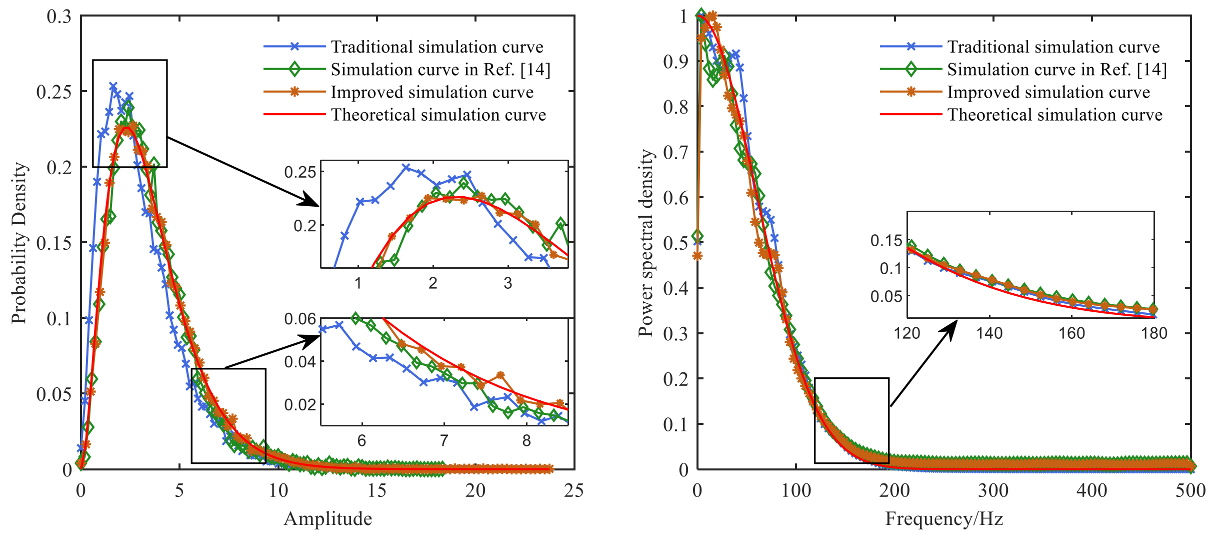
(a) Amplitude probability density curve (b) Power spectral density curve

Fig. 10. Probability density function and power spectral density function of Weibull distributed variable.



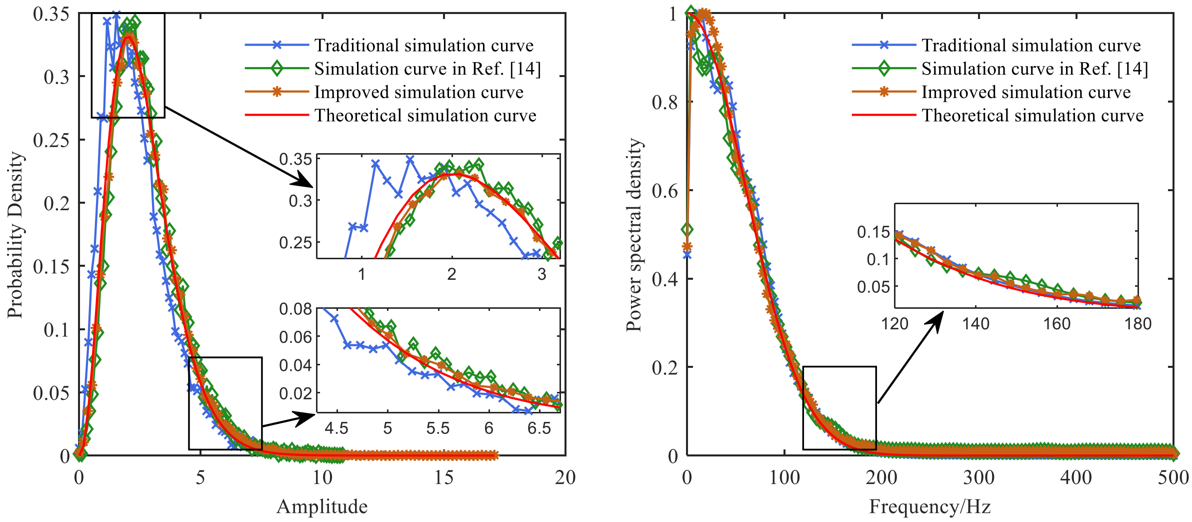
(a) Amplitude probability density curve (b) Power spectral density curve

Fig. 11. Probability density function and power spectral density function of K-distributed variable.



(a) Amplitude probability density curve (b) Power spectral density curve

Fig. 12. Probability density function and power spectral density function of Generalized-K distributed variable.



(a) Amplitude probability density curve (b) Power spectral density curve

Fig. 13. Probability density function and power spectral density function of generalized compound distributed variable.

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